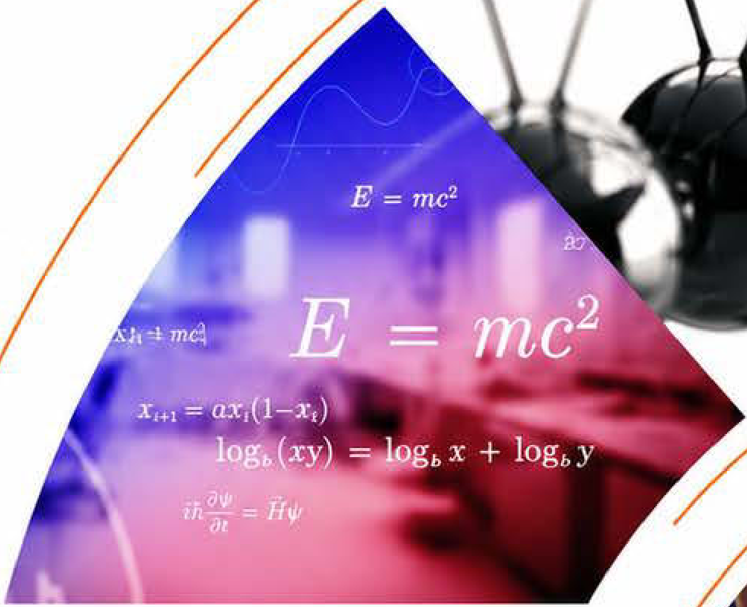


Physics

part 1

Grade 11

SPRIX

Physics

Egyptian Baccalaureate

2nd Year

Advised by
INTERNATIONAL BACCALAUREATE®

The International Baccalaureate™

has reviewed the pedagogical and assessment approaches underpinning this textbook.

Ministry of Education and Technical Education

New Administrative Capital

Cairo, Egypt

Introduction

Physics is the search for the smallest number of ideas that explain the largest number of things. A ferry crossing the Nile, a ball kicked across a courtyard, a pendulum swinging in a stairwell, the colors spread by a compact disc — all of them yield to a handful of principles. Putting those principles within your reach is the work of this book.

This Grade 11 Physics textbook has been developed in alignment with the vision of the Egyptian Baccalaureate System and in accordance with the International Baccalaureate framework. It is designed to build not only what you know, but how you come to know it: to observe carefully, to predict before you are told, to measure with your own hands, and to name an idea only after you have experienced it.

This part you will travel through two connected landscapes. In Mechanics you will describe motion in two dimensions, resolve and recombine velocities, and follow a projectile through its flight. You will turn forces about a pivot, balance them into equilibrium, and account for the energy they transfer. You will meet momentum and impulse, and discover why the total momentum of an isolated system never changes. You will follow a body around a circle, and end by looking outward — to Kepler's laws and the gravitation that holds the planets in their orbits.

In Oscillations and Waves you will study motion that repeats. You will find the same mathematics in a loaded spring and a swinging pendulum, then watch that motion travel outward as a wave. You will see waves add and cancel, bend as they change speed, and reveal themselves in sound, in light, and in the fringes of Young's double slit.

Each lesson follows the same deliberate sequence. It opens with a question worth answering and a phenomenon worth explaining, and asks you to commit a prediction in writing before you read on. It then hands you the apparatus: you explore, collect your own data, and look for the pattern in it. Only then is the model named, stated precisely, and put to work in examples you can follow line by line. Practice follows in graded steps — a first attempt, then a new context, then an exam-style question — so that understanding is built rather than memorized.

Two habits run through every chapter. The first is the Nature of Science: short accounts of how a result was actually won — who measured it, what they got wrong, and how the correction came. The second is the discipline of saying where a model stops. A projectile traces a parabola only when air resistance is neglected; the normal force equals the weight only under stated conditions; the small-angle approximation remains an approximation. Knowing the limits of an equation is part of knowing the equation.

You will also find physics where you live. The problems in this book are set on the Nile and on Lake Nasser, in a metro car and along the North Coast, on the strings of an oud and in the optical fibers that come ashore at Alexandria. The intention is simple: the laws you are studying are not confined to a laboratory.

Mathematics is used here as physicists use it — as a language for reasoning, not a list of formulas to recall. Symbols are defined before they are used, units are carried through every calculation, and each worked example closes by asking whether the answer is reasonable.

Approach this book with a pencil in your hand. Write the prediction even when you are unsure of it: a wrong prediction that you examine will teach you more than a correct answer you were handed. Take your own measurements seriously enough to ask why they scatter. Physics rewards that kind of attention, and it is well within your reach.

How to use this book

Every lesson follows the same sequence: Observe, Predict, Measure, Name.

1 **Lesson question**
The big idea to answer

2 **You will learn to**
Clear goals for this lesson

3 **The phenomenon – predict first**
See it, then commit a prediction

4 **Explore**
Try it and notice patterns

5 **Worked example (why?)**
Method shown with reasoning

6 **Now you try**
Practice the idea yourself

7 **Key fact**
What to remember

8 **Quick application**
Apply the idea in a new context

9 **Linking question**
Connect ideas across the lesson

10 **Equation box**
Key equations at a glance

11 **Nature of Science**
Explore how scientific ideas develop

12 **Exam-style**
Apply your learning in exam contexts

What makes this book different

It does not simply tell you the physics — it trains you to think like a physicist.

- Predict before the explanation, then test your ideas against evidence.
- Understand why each method works, uncover misconceptions, and connect ideas across the lesson.

How learning works here

1. Observe
A real thing happens, and it breaks what you expected.

2. Predict
Record your prediction before reading on.

3. Measure
do it with your own hands and collect the data

4. Name
Learn the scientific term after building the idea — not by memorizing it first.

Start with the phenomenon.
Let evidence build the explanation.

Tools:

1. Experimental techniques
2. Technology
3. Mathematics

Inquiry:

1. Exploring and designing
2. Collecting and processing
3. Concluding and evaluating

The content

Chapter 1 – Mechanics

Chapter 2 – Oscillations and Waves

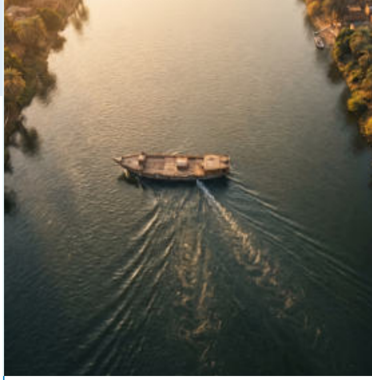
CHAPTER 1
Mechanics

Lesson questions

How can any motion be described and predicted?

What do forces and momentum tell us about how motion changes?

Why is the total energy of an isolated system conserved?



Lesson 1-1 — Velocity Vectors and Relative Velocity

❓ **Lesson question**: Why does a ferry pointed straight across the Nile land downstream — and why does falling rain appear to streak sideways on a moving bus window?

You will learn to

By the end of this lesson, you will be able to:

1. **Explain** how two velocities combine into one resultant velocity.
2. **Calculate** the perpendicular components of a velocity and recombine them.
3. **Determine** the velocity of an object as seen from a moving observer.

🔍 The phenomenon — predict before you continue

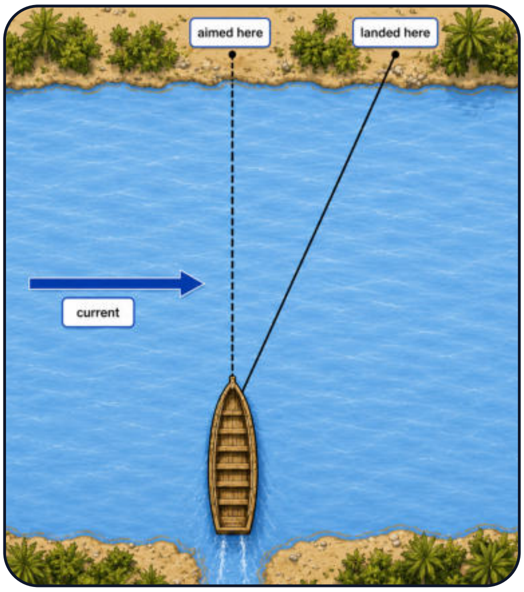


Figure 1-1-1 · The ferry's track ends downstream though its bow points across.

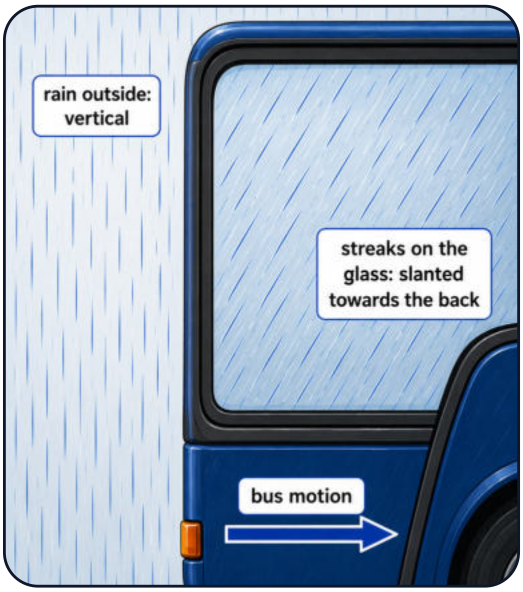


Figure 1-1-2 · Rain falling vertically streaks backward on the moving bus window.

The Nile ferry aims its bow straight at the opposite landing. The engine setting and the heading are both unchanged, yet the ferry lands well downstream of its target. On a moving bus, rain that falls vertically in the ground frame appears as slanted streaks on a side window. Both observations follow from the same principle.

Predict before you continue — commit in writing:

(a) The ferry's engine setting never changes. If the river current became faster, would the crossing take more time, the same time, or less time?

(b) From the moving bus, do the slanted streaks make the rain seem to come from ahead of the bus, or from behind it?

⭐ **Key fact**

Right triangles: $a^2 + b^2 = c^2$

Perpendicular velocity vectors form a right triangle.

Explore — draw the crossing

What you'll do: measure lengths and identify a pattern.

Time: 12 min

You will need an A4 sheet of paper, a ruler, a pen, and a pencil.

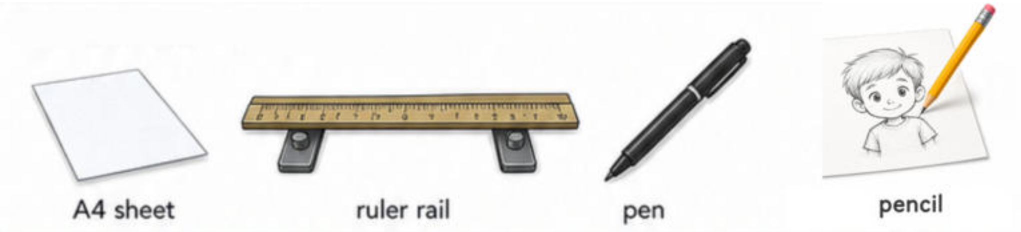


Figure 1-1-3 · The four items needed for the activity.

Steps:

1. Partner A holds the sheet flat under the ruler, which acts as a rail. On the shared signal, A pulls the sheet straight toward their chest — at a right angle to the rail — at a slow, steady speed.
2. On the same count, Partner B slides the pen along the rail at a slow, steady speed, drawing the whole time.
3. Stop together. Measure how far the pen moved along the rail (the **across** distance) and how far the sheet was pulled (the **pull** distance). Then measure the length of the slanted trace left on the sheet.
4. Repeat twice more: once pulling the sheet faster, once sliding the pen faster.

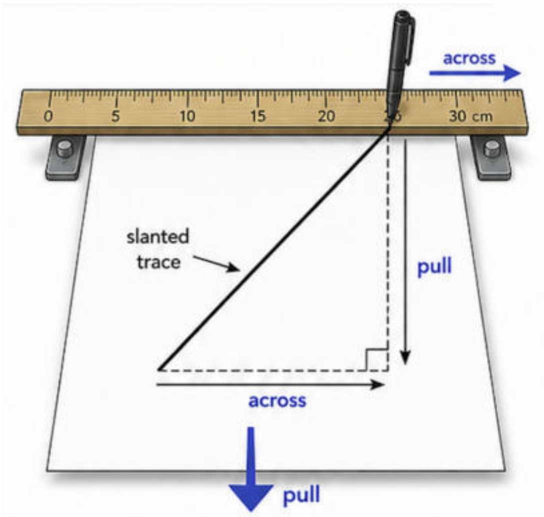


Figure 1-1-4 · One slanted trace: the pen is pulled across while it slides along the rail.

Run	Across (cm)	Pull (cm)	Trace measured (cm)	$\sqrt{(\text{across}^2 + \text{pull}^2)}$ (cm)
1				
2 (Pull the sheet faster)				
3 (Move the pen faster)				

Analyze what you collected:

(a) Compare the last two columns, run by run. What pattern do the numbers show?

(b) The pen moves only along the rail, while the sheet is pulled straight down. How do these two motions combine to produce the slanted trace?

Write your explanation in two lines before you read the next page.

Measurement note

Move slowly, use one shared signal, trust the pattern more than a single trial.

From observation to model

In the Explore, the pen moved only along the rail, yet its trace on the sheet was slanted. The measured length of the trace approximately matched $\sqrt{(\text{across}^2 + \text{pull}^2)}$, not the simple sum of the across and pull distances. You might expect the two motions to add directly, but your own results show otherwise. The model that follows explains these observations.

Velocity is a vector quantity

A velocity describes both how fast an object moves and the direction in which it moves — it is a vector, drawn as an arrow. Sometimes one body has two velocity components: a boat is pushed by its engine while the river carries it along. The two arrows then combine **tip to tail** into a single **resultant velocity**: $\vec{v} = \vec{v}_1 + \vec{v}_2$. This is the addition of velocities. **Velocity vectors must be combined geometrically to determine the resultant; scalar addition of magnitudes is only valid when the vectors are collinear.**

Resolving a velocity into components

Any velocity can be resolved into two perpendicular components: $v_x = v\cos\theta$ along one axis and $v_y = v\sin\theta$ along the other. For a river of fixed width, the velocity component across the river determines the crossing time; the downstream drift equals the downstream velocity component multiplied by that time. Work each axis separately, then recombine with $v = \sqrt{v_x^2 + v_y^2}$.

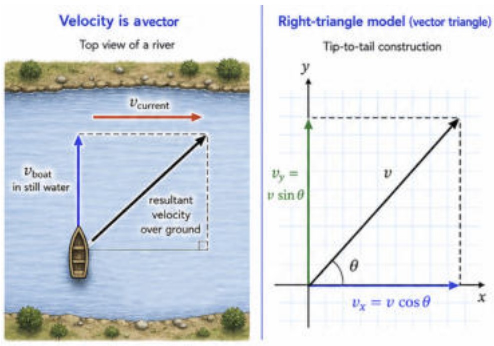
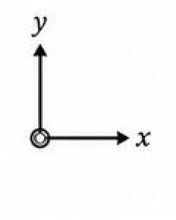
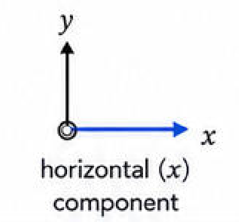


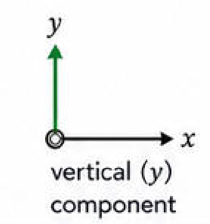
Figure 1-1-5 · The same velocity drawn as one arrow and as two perpendicular components.

Term

Resultant velocity: the single velocity vector with the same effect as two combined velocity vectors.







Point

- Let v_1 [m/s] be the velocity of a boat in still water and v_2 [m/s] the velocity of the current; the velocity of the boat relative to the bank is $v = v_1 + v_2$ — the arrows are combined geometrically, not the magnitudes. The velocity v is called the resultant velocity of v_1 and v_2, and finding it is called the addition of velocities. Conversely $\mathbf{v}$ can be resolved into v_1 and v_2: this is called the resolution of a velocity into components.
- Resolve v into mutually perpendicular x- and y-directions. Let v be the magnitude, θ the angle between v and the x-axis: $v_x = v\cos\theta$, $v_y = v\sin\theta$, and $v = \sqrt{v_x^2 + v_y^2}$. A positive or negative sign is assigned to the magnitude of each component to indicate its direction along the coordinate axis.
- Let the velocity of A be v_A and the velocity of B be v_B. The velocity of B seen from A — the relative velocity of B with respect to A — is $v_{AB} = v_B - v_A$: observed minus observer.

Equation box

Components of a velocity	$v_x = v\cos\theta$ $v_y = v\sin\theta$	Magnitude from components	$v = \sqrt{(v_x^2 + v_y^2)}$

What a moving observer sees

Now consider an observer on a moving body. To obtain an object's velocity relative to the bus, subtract the bus's ground-frame velocity from the object's ground-frame velocity. The **relative velocity** of B with respect to A is the velocity of B minus the velocity of A $\vec{v}_{AB} = \vec{v}_B - \vec{v}_A$, observed minus observer. Both arrows are drawn from a **common origin**. In the ground frame, the rain falls straight down; subtracting the bus's velocity from the rain's ground-frame velocity gives the rain's velocity relative to the bus, which points downward and backward.

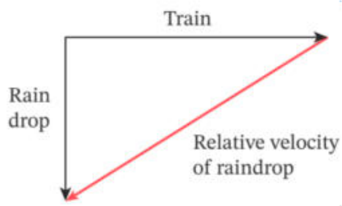


Figure 1-1-7 · The vector triangle that gives the raindrop's velocity relative to the train.

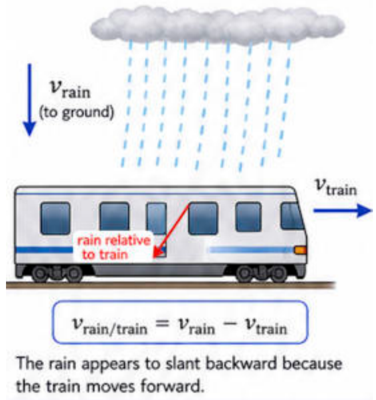


Figure 1-1-6 · In the train's frame the rain arrives slanted, because the train's velocity is subtracted.

Equation box

Relative velocity of B with respect to A	$\vec{v}_{AB} = \vec{v}_B - \vec{v}_A$

Worked example

Situation	A boat has a speed of 4.0 m/s in still water. It moves in a straight river with a current of 3.0 m/s. The boat's bow points perpendicular to the riverbank. Determine the speed of the boat relative to the riverbank.
Given and required	**Given:** Boat's speed in still water = 4.0 m/s and river current speed = 3.0 m/s (perpendicular). **Required:** speed of the boat relative to the riverbank.
Representation	Draw a right-angled vector triangle with the boat velocity (4.0 m/s) and river current (3.0 m/s) as perpendicular sides. The hypotenuse represents the resultant boat velocity.
Strategy	Steps for resolving a velocity: Resolve the velocity into mutually perpendicular x-axis and y-axis directions, and calculate the sum or difference for each component using $v = \sqrt{v_x^2 + v_y^2}$.
Annotated solution	Along the riverbank: $v_x = 3.0\ \mathrm{m/s}$. Across the river: v_y = 4.0 m/s $v = \sqrt{(3.0)^2 + (4.0)^2} = \sqrt{25} = 5.0\mathrm{ms^{-1}}$ Therefore, the speed of the boat relative to the riverbank is 5.0 m/s.
Reasonableness check	The answer 5.0 m/s is greater than the larger single component, 4.0 m/s, but less than the simple sum, 7.0 m/s. This is expected because the two velocity components are perpendicular. The numbers also form a 3-4-5 right-angled triangle.
Explain it yourself	In your own words, why is the resultant speed 5.0 m/s and not 7.0 m/s?

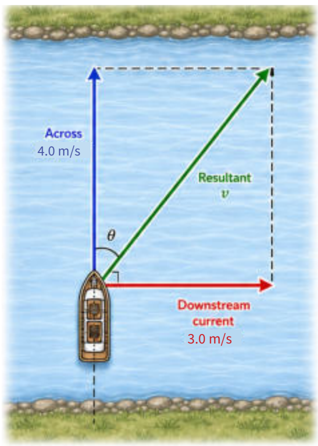


Figure 1-1-8 · The across and downstream velocities add to the resultant velocity over the ground.

Quick application:

An airplane moves at a constant speed of 60 m/s at 30° above the horizontal. Determine the height it gains in 5.0 s.

Now you try: To a passenger on a metro train crossing an open-air section at 5.0 m/s, vertically falling rain appears to fall at 30° to the vertical. Calculate the speed of the raindrops.

Try

(a) A boat with a speed of 6.0 m/s in still water crosses a straight river with a current of 3.0 m/s, with its bow perpendicular to the current.

(i) Determine the angle θ that the resultant velocity of the boat makes with the riverbank.

(ii) If the river is 60 m wide, determine the time it takes to cross.

(b) A cyclist traveling west at 5.0 m/s observes an apparent wind of 5.0 m/s coming from the north. Determine the direction and speed of the wind relative to the ground.

In a new context — landing across the wind

On a gusty afternoon at Borg El Arab Airport, the wind blows across the runway as a pilot lines up to land. If the aircraft were pointed straight down the runway, the crosswind would push it sideways off the centerline. This is the same effect as in the ferry problem. So the pilot points the aircraft slightly into the wind. The aircraft's velocity through the air, tip to tail with the wind's velocity, gives a resultant directed straight along the runway. With an approach airspeed of 220 km/h and a 40 km/h crosswind, the required heading offset (the crab angle) is about 10°.

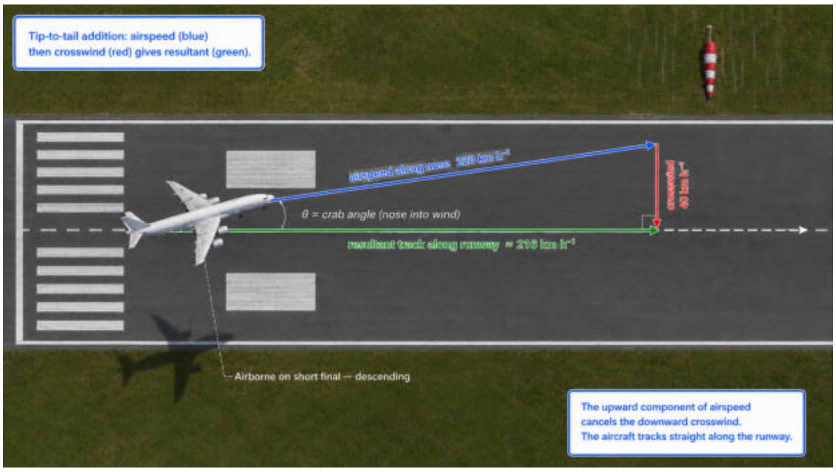
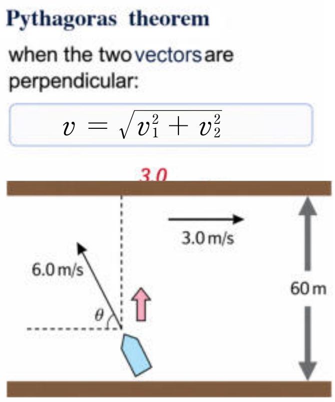


Figure 1-1-10 · The aircraft heads into the wind, so its track stays along the runway.

Your turn: Explain why the pilot must point the aircraft's nose away from the runway's direction even though the aircraft must travel exactly along the runway.

Figure 1-1-9 · Perpendicular components combine through the Pythagorean relation.



Key fact

3-4-5 triangle: the perpendicular sides 4 (across) and 3 (along) give a hypotenuse of 5.

Exam tip

Velocity is a vector quantity that involves magnitude and direction. Do not give only a number when direction is required.

Data booklet

Resolve, combine, and recombine.

Nature of Science: Observations — uniform motion cannot be detected from inside the moving frame

In 1632, Galileo imagined a cabin below deck on a smoothly sailing ship: dripping water, fish in a bowl, drifting butterflies. No observation inside reveals whether the ship is at rest or moving with uniform velocity. What an observer measures depends on the observer's own motion — the same lesson as the rain on your bus window. A velocity is meaningful only when the reference frame is specified.

Your turn: suggest how two careful observers can record different velocities for the same raindrop without either of them being wrong.

Lesson question, answered

Why does a ferry pointed straight across the Nile land downstream — and why does falling rain appear to streak sideways on a moving bus window? The ferry's velocity over the ground is the tip-to-tail sum of its still-water velocity and the current's. So its track slants downstream although its bow never turns. The velocity of the rain relative to the bus equals the rain's ground-frame velocity minus the bus's ground-frame velocity, so rain that falls vertically in the ground frame appears to fall at an angle to an observer on the bus.

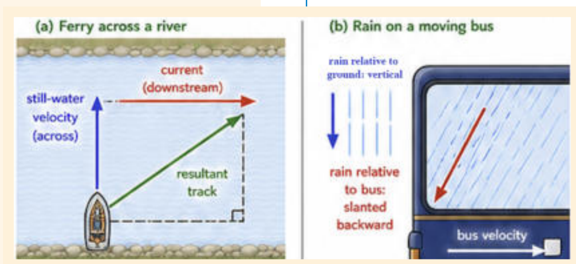
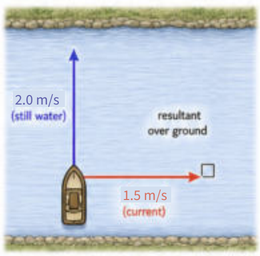


Figure 1-1-11 · Ferry and rain: one method for both situations.

Exercise

(a) A boat with a speed of 2.0 m/s in still water moves in a straight river that has a current of 1.5 m/s, with its bow pointing perpendicular to the riverbank. Determine the speed of the boat relative to the riverbank.

Figure 1-1-12 · Still-water velocity and current give the resultant.



(b) An airplane is climbing at a constant speed of 50 m/s at an angle of 30° to the horizontal. Determine the height gained by the airplane in 10 s.

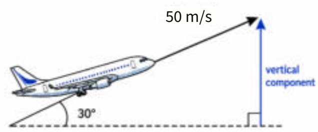


Figure 1-1-13 · Resolving 50 m/s at 30°.

(c) To an observer looking out of the window of train A, which is moving on a horizontal track at a speed of 10 m/s, vertically falling rain appears to fall at an angle of 45° to the vertical.

(i) Determine the speed of the raindrops.

(ii) To an observer looking out of the window of train B passing train A, the rain appears to be falling at an angle of 60° to the vertical. Determine the speed of train B.

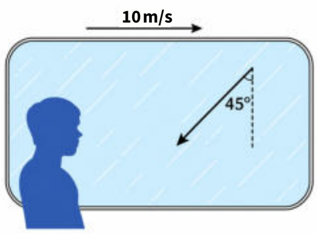


Figure 1-1-14 · Rain seen at 45° to the vertical through the window of a moving vehicle.

(d) Car A travels eastward at 20 m/s and car B travels northward at 20 m/s. Determine the direction and magnitude of the relative velocity of B with respect to A.

Exam-style question

A river flows at 1.2 m/s parallel to its banks. A swimmer whose speed in still water is 0.9 m/s heads directly across the river. **Determine** the magnitude of the swimmer's velocity relative to the bank.

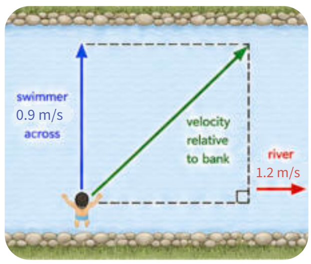


Figure 1-1-15 · The swimmer's velocity across and the river's velocity give the velocity relative to the bank.

Lesson 1-2 — Horizontal Projectile Motion

❓ **Lesson question:** When a coin is projected horizontally off a table while another drops beside it, which hits the floor first — and what determines the result?

You will learn to

By the end of this lesson, you will be able to:

1. **Explain** why the horizontal and vertical motions of a projectile occur independently.
2. **Calculate** the flight time and the range of a horizontally launched projectile.
3. **Determine** the speed of a projectile at any instant by recombining its components.

🔍 The phenomenon — predict before you continue

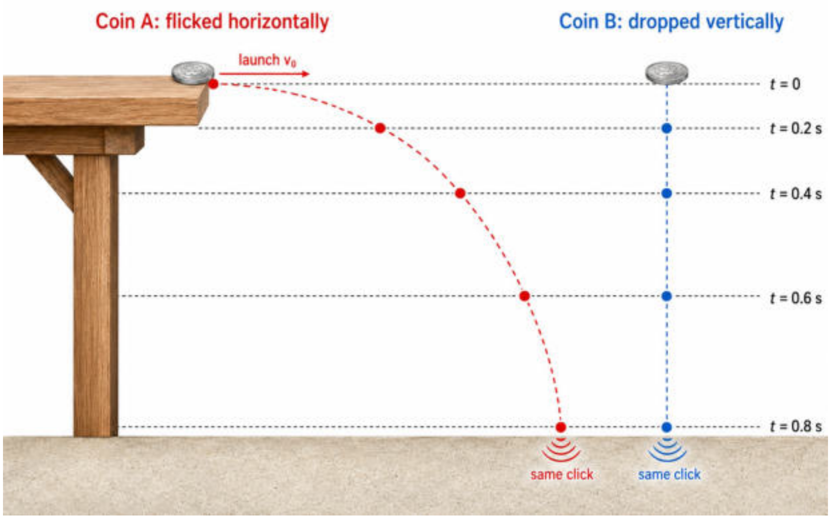


Figure 1-2-1 · One coin flicked sideways and one dropped land at the same instant.

Place two coins at the very edge of a table. Flick one coin horizontally off the table and, at the same instant, nudge the other coin just over the edge so that it drops straight down with no horizontal push. The flicked coin lands some distance away, while the other lands almost directly below the edge.

Predict before you continue — commit in writing:

(a) Which coin hits the floor first — the flicked one, the nudged one, or neither?

(b) If you flick the first coin twice as hard, does it stay in the air for a longer time, a shorter time, or the same time?

Key fact

$v = v_0 + at$

$s = v_0 t + ½at^2$

Free fall:

$v_0 = 0$

$a = g = 9.8\ \mathrm{m/s^2}$

🔍 Explore — compare the motion of two coins

What you'll do: launch everyday objects, time their motion, and identify a pattern.

Time: 10 min

You will need:

- two identical coins
- a pencil
- a flat ruler
- a table in a quiet room.

Steps:

Figure 1-2-2 · Two coins, a flat ruler, a pencil pivot, and a table.

1. Place a plastic ruler flat on a table, with one end projecting beyond the edge. Hold the ruler firmly near its center. Place one coin on the table beside the ruler and a second coin on the projecting end.
2. Flick the projecting end sharply. The ruler launches the first coin horizontally while moving out from beneath the second coin, allowing it to fall vertically. The two coins begin their motion at approximately the same time.
3. Listen: one strike on the floor, or two? Record what you hear.
4. Repeat five times, from gentle flick to the hardest you can. Record each trial.

⭐ **Key fact**

Two clicks closer than about 20 ms are perceived as one sound. Repeat in a quiet room.

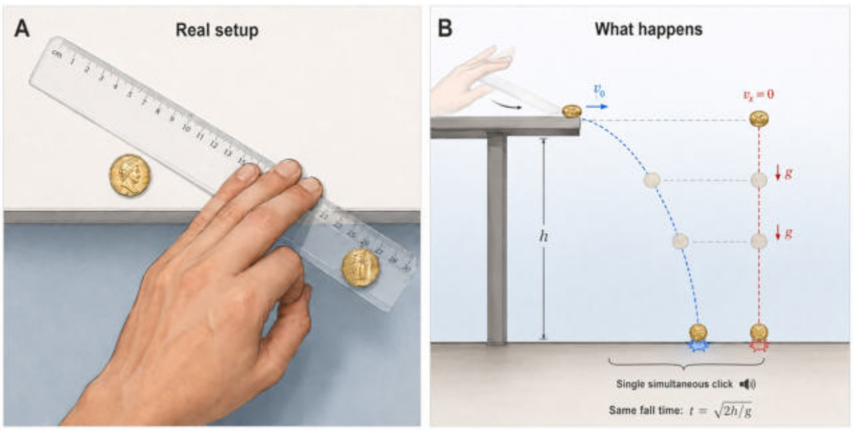


Figure 1-2-3 · The real setup and the motion it produces.

Trial	Flick strength (gentle → strong)	One strike or two?
1		
2		
3		
4		
5		

Analyze what you collected:

(a) Did the landing ever split into two clicks as the flick became harder? What pattern does the column show?

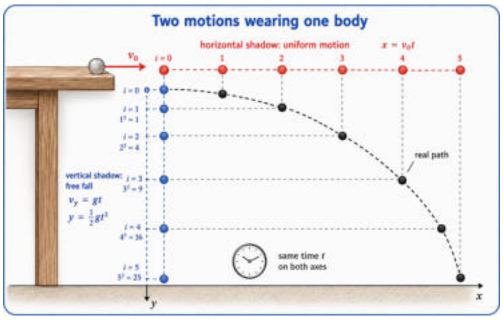
(b) The flicked coin travels a much longer, curved path, but the strikes stay together. Why does the longer path not increase the time taken to reach the floor?

✏️ **Write your explanation in two lines before you read the next page.**

From observation to model

In Explore, the two coins struck the floor at nearly the same time, even when the horizontally launched coin was given different initial speeds. Increasing its horizontal speed increased the horizontal distance traveled but did not change its time of fall. You may have predicted that the faster coin would land later, but the observations did not support this prediction. The following model explains the evidence.

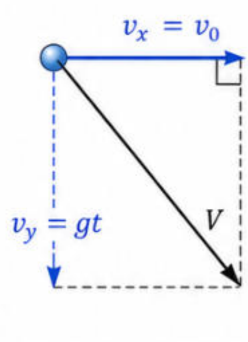
Two motions combined in one body



An object projected horizontally follows a parabolic path. It has two independent motions. Horizontally, no force acts on the object (with air resistance ignored), so keeps a **constant velocity**: horizontal $x = v_0 t$. Vertically, gravity pulls exactly as it pulls any dropped object, so the object is in **free fall**:

$v = gt$ and $y = \frac{1}{2}gt^2$.

The two component motions are **independent** — they share only the same time t.



speed: $V = \sqrt{v_x^2 + v_y^2}$

Figure 1-2-4 · Components combine to give the speed.

The parabola, and what determines the time

Combining the components produces a parabolic path. However, each part of the motion can be analyzed separately. The vertical motion obeys $y = \frac{1}{2}gt^2$, where y is the vertical component of the object's displacement. Therefore, the flight time is given by: $t = \sqrt{\frac{2y}{g}}$. The horizontal motion obeys $x = v_0 t$, where x is the horizontal component of the object's displacement. The velocity of a horizontally projected object is found by combining its horizontal and vertical velocity components.

Point

! The motion of an object thrown horizontally is called **horizontal projectile motion**. It is resolved into **horizontal and vertical components** and treated as two independent linear motions, which share only the same time t.

! The two components

Horizontal component: The object keeps the constant velocity v_0 [m/s], so the horizontal component x [m] of its displacement is $x = v_0 t$.

Vertical component: The object is in free fall. Letting g [m/s^2] be the acceleration due to gravity, the vertical component of the velocity is $v = gt$ and the vertical component y [m] of the displacement is $y = \frac{1}{2}gt^2$.

Equation box

Horizontal motion (uniform motion)	$s = v_0 t + \frac{1}{2}at^2$ with $a = 0 \longrightarrow x = v_0 t$
Vertical motion (free fall from rest)	$v = v_0 + at$ with $v_0 = 0 \longrightarrow v = gt,\ y = \frac{1}{2}gt^2$

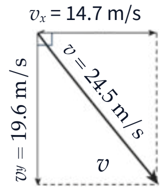
Height sets the time

The time $t = \sqrt{\frac{2y}{g}}$, depends only on the vertical height (y) and the gravitational acceleration (g). The initial horizontal speed does not appear because it has no effect on the time of flight. Increasing the launch speed increases the horizontal distance traveled, but it does not change the time taken for the projectile to reach the ground.

Term

Projectile: a body in flight whose motion after launch is governed by gravity alone (air resistance neglected).

Worked example

Situation	An object was thrown horizontally at a speed of 14.7 m/s from a height of 19.6 m above the ground. Assuming the acceleration due to gravity is 9.8 m/s², determine the following: **(a)** the time taken for the object to reach the ground in s. **(b)** the horizontal range of the object in m. **(c)** the speed of the object just before it hits the ground in m/s
Given and required	**Given:** $v_0 = 14.7\ \mathrm{m/s}$, $y = 19.6\ \mathrm{m}$, $g = 9.8\ \mathrm{m/s^2}$. **Required:** t, x, and v.
Representation	The motion is split into two parts. Horizontally, the object moves with constant speed $v_0 = 14.7\ \mathrm{m/s}$. Vertically, it falls freely through $y = 19.6\ \mathrm{m}$ under gravity. The path is a parabola.
Strategy	First, use the vertical motion to find the time: $y = \frac{1}{2}gt^2$. Then use the horizontal motion to find the range: $x = v_0 t$. Finally, find the vertical component using $v_y = gt$, then the impact speed using $v = \sqrt{(v_0^2 + v_y^2)}$.
Annotated solution	**(a)** Since the object is in free fall in the vertical direction, let t be the time taken to fall 19.6 m. $y = \frac{1}{2}gt^2$ $19.6 = \frac{1}{2} \times 9.8 \times t^2$ $19.6 = 4.9\ t^2$ $t^2 = 4$. Since $t > 0$, $t = 2.0$ s **(b)** The horizontal velocity is constant, so: $x = v_0 t = 14.7 \times 2.0 = 29.4$ m, $x \approx 29$ m **(c)** At $t = 2.0$ s, the horizontal component of the velocity is $v_x = v_0 = 14.7\ \mathrm{m/s}$ The vertical component of the velocity is $v_y = gt = 9.8 \times 2.0 = 19.6\ \mathrm{m/s}$ Let the speed of the object just before hitting the ground be v, $v = \sqrt{(v_x^2 + v_y^2)} = \sqrt{(14.7)^2 + (19.6)^2} = 24.5\ \mathrm{m/s}$ $= 24.5 \approx 25$ m/s  Figure 1-2-7 · Resultant speed 24.5 m/s.
Reasonableness check	The object falls for 2.0 s, travels 29.4 m horizontally, and its final speed is greater than its initial speed because gravity increases only the vertical component of velocity.
Explain it yourself	In your own words, explain the results obtained.

Now you try: an object projected horizontally at a speed of 4.0 m/s from a certain height reaches the ground in 2.0 s. Calculate the initial height of the object and its horizontal range.

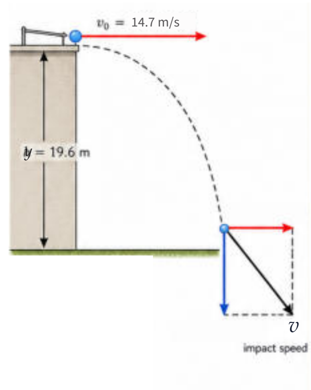


Figure 1-2-6 · Impact speed after a 19.6 m fall.

Quick application

A phone slips from a window ledge 4.9 m above the pavement. Determine its fall time.

Try

(a) An object is projected horizontally at a speed of 19.6 m/s from a height of 58.8 m above the ground.

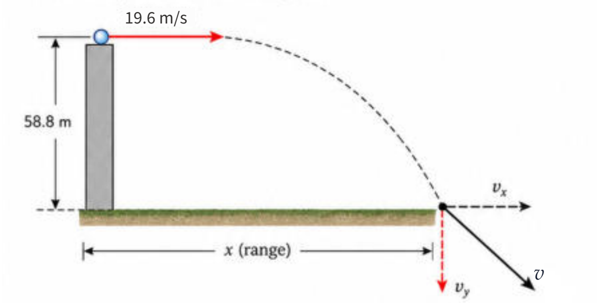


Figure 1-2-8 · A stone thrown at 19.6 m/s from a height of 58.8 m.

(i) Determine the time taken (in seconds) for the object to reach the ground.

(ii) Determine the horizontal range of the object in m.

(iii) Determine the speed of the object in m/s just before it hits the ground.

(b) An object projected horizontally at a speed of 3.0 m/s from a certain height reaches the ground in 3.0 s.

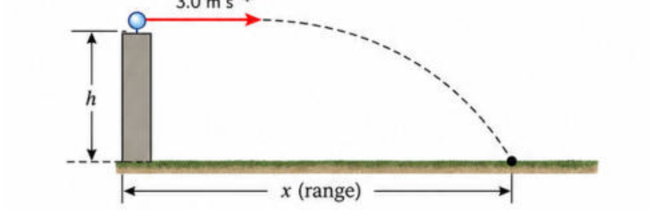


Figure 1-2-9 · Range of a ball launched at 3.0 m/s.

(i) Determine the initial height of the object in m.

(ii) Determine the horizontal range of the object in m.

Key fact

Height determines flight time. Horizontal speed determines range. Combine the velocity components to find the landing speed.

In a new context — Dropping before the target

A relief aircraft flies level at 60 m/s, 80 m above a flood-cut village, free-dropping double-bagged grain sacks onto a marked field. The pilot cannot release over the mark. Each sack leaves the aircraft with the aircraft's forward velocity, maintains that velocity horizontally (air resistance aside), and falls vertically under gravity meanwhile. The fall takes $t = \sqrt{\frac{2 \times 80}{9.8}} \approx 4.0$ s, in which the sack moves forward 60 × 4.0 ≈ 240 m. So the release must occur about 240 m before the aircraft reaches the target.

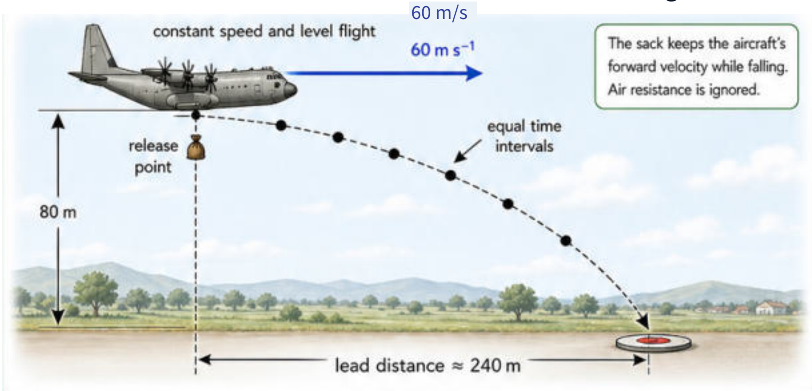


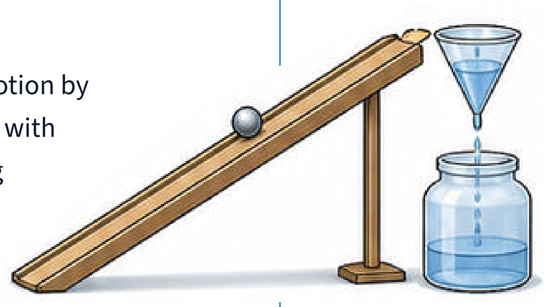
Figure 1-2-10 · The sack keeps the aircraft's forward velocity as it falls.

Linking question

Can projectile motion be solved by energy instead of kinematic equations?

If the aircraft maintains its constant horizontal velocity, the sack lands almost directly below the aircraft — yet far ahead of where it was released. Explain how both statements are true at once.

Nature of Science: Experiments — slowing the effect of gravity so the motion could be timed



Free fall was too fast for Galileo to time directly, so he slowed the motion by rolling balls down smooth, gently inclined grooves and timing them with a water clock. In Two New Sciences (1638), he showed that, ignoring air resistance, a projectile combines uniform horizontal motion with vertical motion under gravity, forming a parabola. Your two-coin experiment follows the same idea.

Your turn: Explain why studying motion down a gentle, frictionless incline is analogous to studying direct vertical free fall.

Lesson question, answered

Figure 1-2-11 · The coin experiment, revisited.

When a coin is projected horizontally off a table while another coin is dropped beside it, which hits the floor first — and what determines the result? Neither arrives first: they strike together. When air resistance is neglected, gravity determines the vertical motion independently of the horizontal motion. Increasing the horizontal speed increases the horizontal distance traveled but does not change the time of flight from the same height.

Exercise

(a) An object is projected horizontally at a speed of 9.8 m/s from a height of 14.7 m above the ground.

(i) Determine the time taken (in seconds) for the object to reach the ground.

(ii) Determine the horizontal range of the object in m.

(iii) Determine the speed of the object in m/s just before it hits the ground.

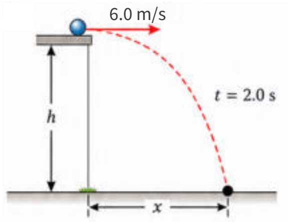


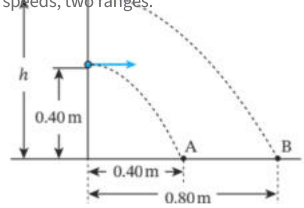
Figure 1-2-12 · A 6.0 m/s launch falling for 2.0 s.

(b) An object projected horizontally at a speed of $6.0\ \mathrm{m/s}$ from a certain height reaches the ground in 2.0 s.

(i) Determine the initial height of the object in m.

(ii) Determine the horizontal range of the object in m.

Figure 1-2-13 · Same height, two speeds, two ranges.



(c) An object is projected horizontally from a height of 0.40 m and lands at point A, which is at a horizontal distance of 0.40 m.

(i) Determine the time in s from when it is thrown until the object reaches the ground.

(ii) Determine the initial speed of the object in m/s.

(iii) Determine the required initial height in m to make the object fall to point B, 0.80 m away in the horizontal direction, assuming the same initial speed.

Exam tip

Use unrounded time into later parts; rounding too early can reduce the accuracy of later answers.

Exam-style question

A ball rolls off a horizontal bench of height 1.25 m with a speed of 2.0 m/s.

(a) **Calculate** the time the ball takes to reach the floor.

(b) **Determine** the horizontal distance from the bench at which the ball lands.

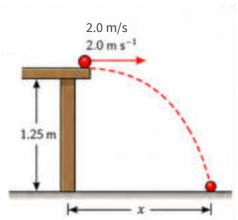
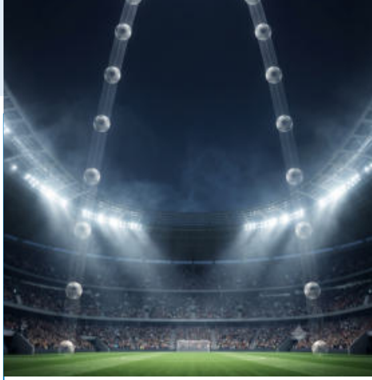


Figure 1-2-14 · Launch at 2.0 m/s from 1.25 m.



Lesson 1-3 — Projectile Motion at an Angle

❓ **Lesson question:** At the highest point of a ball's angled projectile motion, is the ball momentarily at rest?

You will learn to

By the end of this lesson, you will be able to:

1. **Explain** how a projectile launched at an angle can be resolved into two independent component motions.
2. **Calculate** the time of flight, maximum height and range of a projectile launched at an angle.
3. **Describe** the motion of a projectile at the highest point, including its horizontal and vertical velocity components.

The phenomenon — predict before you continue

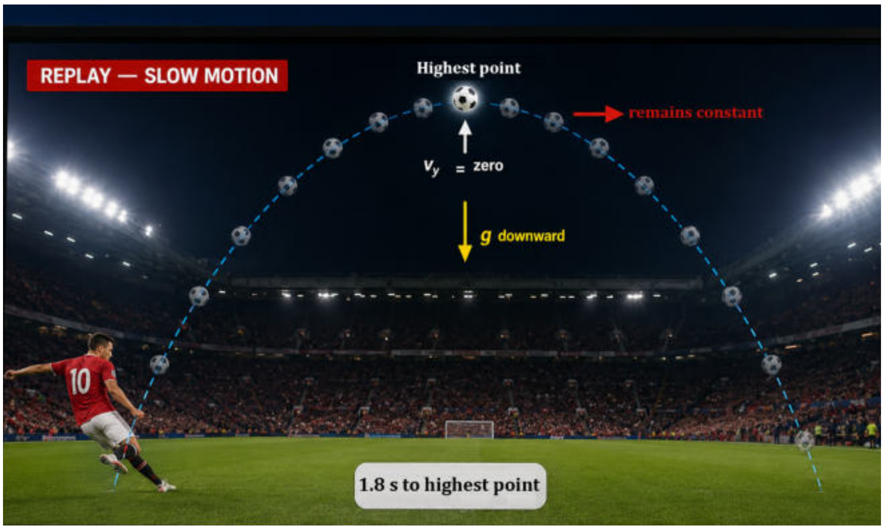


Figure 1-3-1 · A kicked ball: at the highest point the vertical velocity is zero.

Watch a slow-motion replay of a free kick. The ball leaves the player's foot, rises, and appears to pause at the highest point of its trajectory before descending toward the goal. Commentators sometimes describe this as 'hang time'. Watch the replay again and focus on the highest point: is the ball truly at rest at that instant?

Predict before you continue

(a) At the highest point, is the ball's velocity zero, minimum -but non-zero, or unchanged?

(b) The ball takes 1.8 s to reach the highest point — will the return back to launch height take more, less, or the same time?

Key fact

the horizontal and vertical components of motion are independent.

🔍 Explore — Pause at the highest point

What you'll do: use a phone's slow-motion video to track a projectile launched at an angle and look for patterns in its motion.

Time: 12 min

Figure 1-3-2 · A phone at 240 fps, a paper ball, and a doorway with a straight edge.

You will need:

- A phone with slow-motion video.
- A crumpled-paper ball.
- A doorway with a clear vertical edge.
- A ruler

Steps:

1. Toss the paper ball at about 45° so it follows a curved path across an open doorway; one partner films it in slow motion against the door edge.
2. Step through the video frames near the highest point of the trajectory, one at a time. Against the fixed door edge, watch the two components of motion separately.
3. Mark the center of the ball in each frame. Using the scale, measure the horizontal and vertical distances between consecutive positions.
4. Repeat with a steeper launch angle and record your observations.

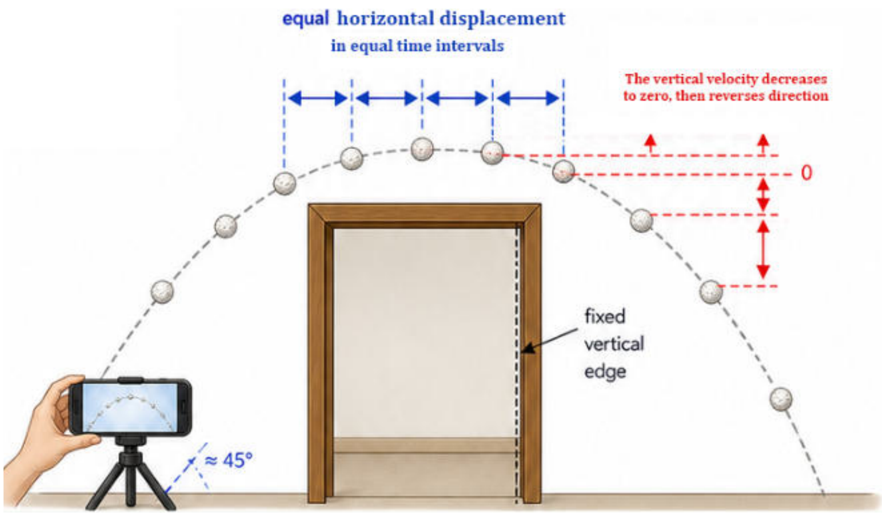


Figure 1-3-3 · Equal horizontal steps in equal times as the vertical velocity reverses.

launch angle	horizontal spacing	Vertical spacing
about 45°		
steeper		

Analyze what you collected:

(a) For each launch angle, did the horizontal displacement per frame remain approximately constant as the ball passed through the highest point?

(b) How did the vertical motion change as the ball passed through the highest point?

Write your explanation in two lines before you read the next page.

⭐ **Measurement note**

Equal frame numbers mean equal time intervals. Compare horizontal displacement over the same count of frames.

From observation to model

In Explore, as you examined the video frame by frame, the ball continued to move horizontally through the highest point in approximately equal steps. Meanwhile, the vertical displacement between successive frames decreases as the ball rises, becomes approximately zero at the highest point, then reverses direction and increases as the ball falls. You may have predicted that the ball would stop completely at the highest point, but the video evidence shows otherwise. The following model explains your observations.

Resolving projectile motion into components

Resolve the launch velocity v_0 into two components. The horizontal component is $v_x = v_0 \cos\theta$; with air resistance neglected it stays constant. For the vertical component, the ball behaves exactly like an upward throw — it starts with $v_y = v_0 \sin\theta$ and gravity pulls it back, so $v_y = v_0 \sin\theta - gt$ and $y = v_0 \sin\theta \times t - \frac{1}{2}gt^2$. The horizontal and vertical motions are independent only when air resistance is neglected and gravity is uniform, and they share the same time, t.

At the highest point, only the vertical component of velocity becomes zero. The horizontal component remains constant, so the ball is never truly at rest.

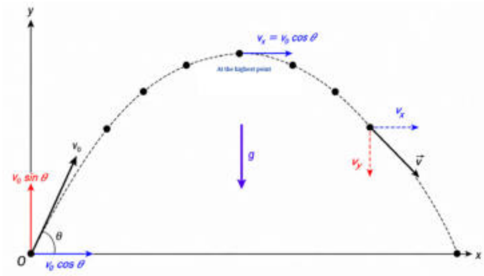


Figure 1-3-4 · The launch velocity resolved into horizontal and vertical components.

Point

- The motion of an object projected at an angle to the horizontal is called **projectile motion**. It follows a parabolic path, and the motion in the vertical plane is resolved into **horizontal and vertical components** and treated as two independent linear motions.
- The two components

  Horizontal component: Neglecting air resistance, the object moves with a constant velocity equal to the horizontal component of the initial velocity v_0 [m/s]: $v_x = v_0 \cos\theta$ and $x = v_0 \cos\theta \times t$.

  Vertical direction: It undergoes the same motion as a vertical upward throw. Constant acceleration $-g$, initial vertical velocity $v_0 \sin\theta$. $v_y = v_0 \sin\theta - gt$ and $y = v_0 \sin\theta \times t - \frac{1}{2}gt^2$.
- Only for launch and landing at the same height, and with air resistance neglected, the motion is symmetric about the vertical line through the highest point. If the time taken to reach the maximum height is 1.2 s, the total time of flight is 2.4 s; if the horizontal distance to the maximum height is 3.0 m, the total horizontal range is 6.0 m.

Equation box

Horizontal motion	$a = 0 \rightarrow x = v_0 \cos\theta \times t$
Vertical motion	$v_y = v_0 \sin\theta - gt$ $y = v_0 \sin\theta \times t - \frac{1}{2}gt^2$

Time of flight

Total launch-to-landing time; on level ground it is twice the time taken to reach the highest point

Quick application

A ball is kicked at 14 m/s at 30° above the horizontal. Determine the two components of its initial velocity

Symmetry about the highest point

The motion is symmetric about the highest point only for launch and landing at the same height, with no air resistance. As a result, the time taken to rise to the maximum height is equal to the time taken to fall back to the launch level: $t_{flight} = 2t_{top}$

Since there is no horizontal acceleration (neglecting air resistance), the horizontal velocity remains constant throughout the motion. Therefore, the horizontal range is given by: $R = v_0 \cos\theta \times t_{\text{flight}}$

For instance, if a projectile reaches its highest point after 1.2 s, it will return to its initial height after 2.4 s. If the horizontal displacement at the highest point is 3.0 m, then the total horizontal range is 6.0 m, since the horizontal motion is uniform.

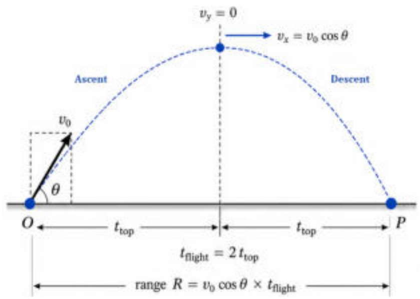


Figure 1-3-5 · Ascent and descent take equal times, so the range follows.

Key fact

the equations of motion. First, resolve the projectile's motion into horizontal and vertical components. Neglecting air resistance $v_0y = v_0 \sin\theta$ and $a = -g$

Worked example

Situation	A small ball is projected from the ground with an initial speed of 19.6 $\mathrm{m/s}$ at an angle of 30° above the horizontal. The motion occurs under constant gravitational acceleration $g = 9.8\ \mathrm{m/s^2}$ with no air resistance.
Given and required	Given: $v_0 = 19.6\,\mathrm{m\,s^{-1}}$, $\theta = 30^\circ$, $g = 9.8\,\mathrm{m\,s^{-2}}$ $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$ Required: (a) Time to reach maximum height (b) Horizontal range
Representation	Resolve velocity into components. $v_0x = v_0 \cos\theta$, $v_0y = v_0 \sin\theta$ At maximum height: $v_y = 0$ Projectile motion is symmetric (upward time = downward time).
Strategy	Decompose the initial velocity into horizontal and vertical directions, use constant-velocity motion formulas for the horizontal, and vertical constant-acceleration equations for the vertical.
Annotated solution	(a) Time to reach maximum height At maximum height: $v_y = 0$ $v_y = v_0 \sin\theta - gt$ $0 = 9.8\ \mathrm{m/s} - (9.8\ \mathrm{m/s^2})t$ $9.8t = 9.8$ $t = 1.0\ \mathrm{s}$ Total time of flight: $2t = 2.0\ \mathrm{s}$ **(b)** Horizontal range $R = v_0 \cos\theta \cdot t_{flight}$ $= 19.6 \times \cos 30° \cdot 2.0$ $= 33.948 \approx 34\ \mathrm{m}$

Reasonableness check	Time 1.0 s is consistent with v_{oy}/g. Range ≈ 34 m is reasonable for a 19.6 m/s projectile at 30°. Motion is symmetric (upward time = downward time).
Explain it yourself	Why does resolving the motion into horizontal and vertical components make it possible to determine the flight time and range separately, even though the projectile follows a curved path?

Now you try: the same ball is kicked steeper — 9.8 $\mathrm{m/s}$ at 60° to the horizontal. **Calculate** the time to reach maximum height and the maximum height.

Try

A small ball is projected from the ground at a speed of 19.6 $\mathrm{m/s}$ at an angle of 60° to the horizontal. Determine the following to two significant figures, assuming the acceleration due to gravity is 9.8 $\mathrm{m/s^2}$ and $\sqrt{3} \approx 1.73$.

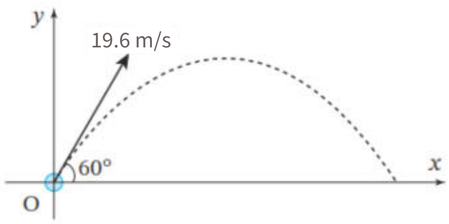


Figure 1-3-6 · A launch of 19.6 m/s at 60° to the horizontal.

(a) Determine the horizontal component of the initial velocity in $\mathrm{m/s}$.

(b) Determine the time taken (in seconds) for the ball to reach its maximum height.

(c) Determine the maximum height reached by the ball in m.

(d) Determine the horizontal range in m.

In a new context — the firework pattern that remains spherical

Figure 1-3-7 · Every fragment follows its own parabola under the same acceleration.

A firework shell bursts high above the city, ejecting glowing pellets, called stars, in all directions. Each star then moves as an independent projectile and experiences the same downward gravitational acceleration, (g). Neglecting air resistance, each star follows a parabolic path relative to the ground. Because gravity gives every star the same acceleration, it does not change their motion relative to one another. If the stars are ejected at approximately equal speeds in all directions, they form an expanding, approximately spherical pattern. The center of this pattern follows a projectile path, while the pattern increases in size but remains approximately spherical.

Explain why the expanding pattern of stars remains approximately spherical, even though each star follows a parabolic path.

Linking question

How does a gravitational field affect the motion of an object?

Figure 1-3-8 · At the apex only the vertical velocity is zero.

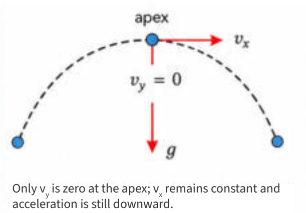
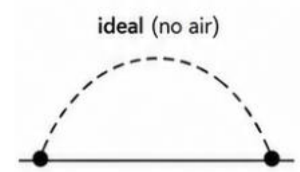


Figure 1-3-9 · Ideal path, no air.



Nature of Science: Models — the model's author identified its limits

In Two New Sciences (1638), Galileo showed that, in the absence of air resistance, a projectile follows a parabolic path. This is the model used throughout this lesson. In the same work, however, he pointed out that air resistance causes the actual path to deviate from a parabola, especially at high speeds. Every physical model has a range of validity, and Galileo himself identified the limitations of the projectile model he developed.

Your turn: suggest one observation from a real football match that the parabolic model neglecting air resistance cannot explain.

Lesson question, answered

At the highest point of a ball's angled projectile motion, is the ball momentarily at rest? Never truly at rest. The vertical component of the velocity becomes momentarily zero, while the horizontal component of the velocity remains constant (ignoring air resistance). The acceleration remains downward throughout the motion. The ball only appears to pause because its vertical motion slows to zero before reversing direction; its horizontal motion never stops.

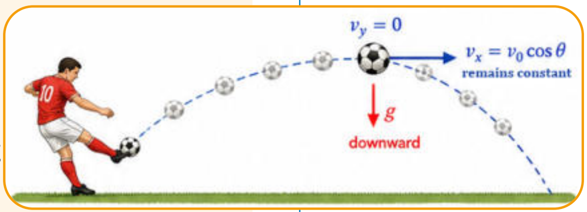


Figure 1-3-10 · The kick revisited: the horizontal velocity never changes.

Exercise

(a) A small ball is projected from the ground at a speed of $39.2\ \mathrm{m/s}$ at an angle of 30° to the horizontal.

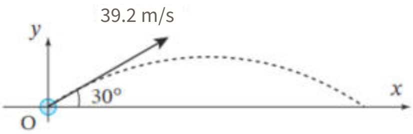


Figure 1-3-11 · A launch of 39.2 m/s at 30°.

(i) Determine the vertical component of the initial velocity in $\mathrm{m/s}$.
(ii) Determine the horizontal component of the velocity after $1.5\ \mathrm{s}$ in $\mathrm{m/s}$.
(iii) Determine the time taken (in seconds) for the ball to reach its maximum height.
(iv) Determine the horizontal range in m.

(b) A small ball is projected from the ground at a speed of $20\ \mathrm{m/s}$ at an angle of 60° to the horizontal.

Figure 1-3-12 · A launch of 20 m/s at 60°.

(i) Determine the horizontal component of the initial velocity in m/s.
(ii) Determine the vertical component of the velocity after 0.50 s in m/s.

(c) A small ball is projected from the top of a building $39.2\ \mathrm{m}$ above the ground at a speed of $19.6\ \mathrm{m/s}$ at an angle of 30° above the horizontal.

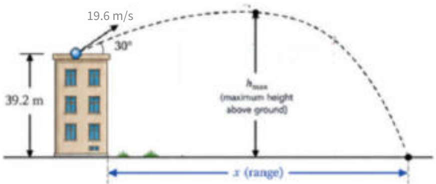


Figure 1-3-13 · Launch at 19.6 m/s and 30° from a 39.2 m building.

(i) Determine the time taken (in seconds) for the ball to reach its maximum height.
(ii) Determine the maximum height reached by the ball above the ground in m.
(iii) Determine the time it takes (in seconds) for the ball to hit the ground.
(iv) Determine the horizontal range in m.

Exam-style question

A stone is thrown from ground level at $12\ \mathrm{m/s}$ at $30°$ above the horizontal.

(a) **State** the horizontal component of its velocity at the highest point.

(b) **Calculate** the time for the stone to return to the ground.

Lesson 1-4 — Moment of a Force

❓ **Lesson question:** Push a door with the same force in two places — why does it rotate easily at the handle but barely move near the hinge?

You will learn to

By the end of this lesson you will be able to:

1. **Calculate** the moment of a force as the product of the force and its perpendicular distance from the point of rotation.
2. **Determine** the perpendicular moment arm for a force acting at an angle.
3. **Assign** a positive or negative sign to a moment for its rotation direction.

🔍 The phenomenon — predict before you continue

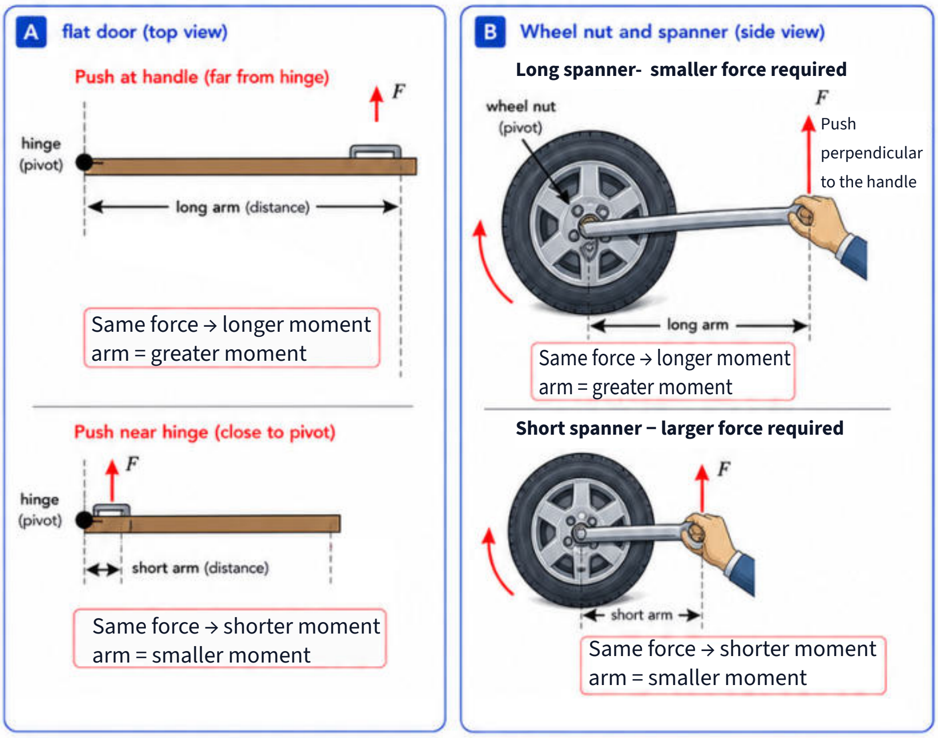


Figure 1-4-1 · Door and spanner: the same force gives a larger moment on a longer arm.

A door opens easily when pushed at the handle but is much harder to open when pushed near the hinge. Likewise, a longer wrench loosens a tight wheel nut more easily than a shorter one. In both cases, the force is the same—the distance from the pivot is what changes.

> ✏️ **Predict before you continue**
>
> **(a)** Does the magnitude of the force alone decide its turning effect?
>
> **(b)** Does pushing the door close to the hinge make the door easier to open because the force is closer to the pivot?

⭐ **Key fact**

A force is a push or a pull with a magnitude and a direction. This lesson focuses on the turning effect of a force about a point.

🔍 Explore — Investigating the Effect of Distance on Balance

What you'll do: place identical coins at measured distances on both sides of a pivot and record the arrangements that balance the ruler.

Time: 12 min

You will need:

- A straight ruler.
- A pencil to act as a pivot beneath the ruler's midpoint.
- A small group of identical coins.
- A flat table.

Figure 1-4-2 · A straight ruler, a pencil pivot, identical coins, and a flat table.

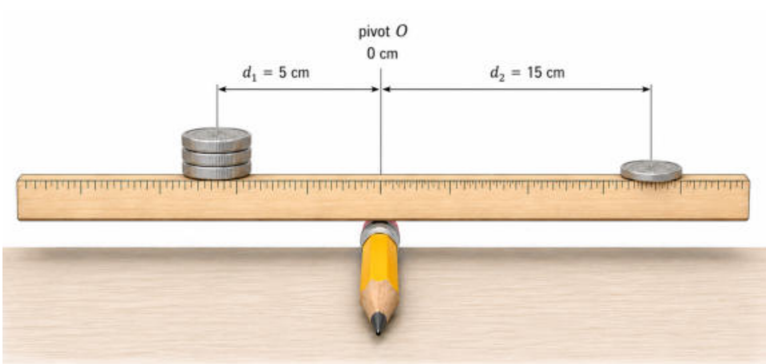


Figure 1-4-3 · A ruler balanced on a pencil pivot, with coins on both sides.

Steps:

1. Rest the ruler across the pencil until it balances horizontally, with the pencil under the midpoint mark.
2. Place one coin near the right end and observe the ruler tilt downward on that side.
3. Now add coins on the left side, close to the pencil, until the ruler balances again.
4. Count how many coins near the pivot (n) are needed to balance the single far coin, and measure both distances.
5. Move the single coin closer to the pencil and repeat the balancing procedure.

d_1	Number of coins on the left side	d_2

Analyze what you collected:

(a) When the single coin was placed farther from the pivot, did you need more coins near the pivot or fewer to balance it?

(b) Multiply the number of coins on each side by its distance from the pivot. How do the two products compare at balance?

✏️ **Write your explanation in two lines before you read the next page.**

⭐ A measurement note

Read each distance to the nearest scale division. Repeat the balancing twice, as the coin count may vary by one near the balance point.

From observation to model

In Explore, a coin far from the pivot was balanced by several coins close to the pivot. At balance the force multiplied by its distance matched on both sides, not the force alone. You may well have predicted the heavier group would tip the ruler downward. Here is the model your evidence points toward.

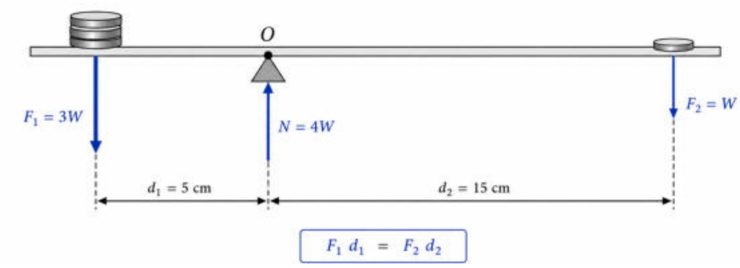


Figure 1-4-4 · At balance the two moments about the pivot are equal.

Moment of a force

A force does not just push a body — it can turn it about a point. The magnitude of its turning effect depends on two things together: the magnitude of the force, and how far its line of action is from the pivot. This turning effect is called the moment of a force about the point. Measure the force in newtons and the perpendicular distance in meters; the moment is then in newton meters (N m). The moment of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.

> ⭐ **Moment of a force**
>
> the turning effect of a force about a point; its unit is (N m).

Direction of rotation — the sign of a moment

A force can produce a moment about a certain point in one of two senses: counterclockwise or clockwise. We distinguish between these directions using a sign convention, taking a counterclockwise turn as **positive** and a clockwise turn as **negative**. So every moment carries both a magnitude and a sign.

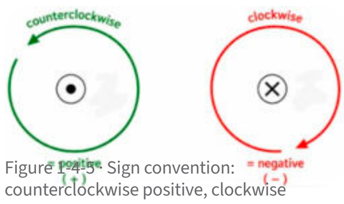


Figure 1-4-5 · Sign convention: counterclockwise positive, clockwise negative.

When the force is inclined — the moment arm

When a force is inclined, the moment arm is determined by the perpendicular distance from the point O to the force's line of action. For a force F as shown in Figure 1-4-6, since $d = L \sin\theta$, the moment of force about point O is given by $M = FL \sin\theta$.

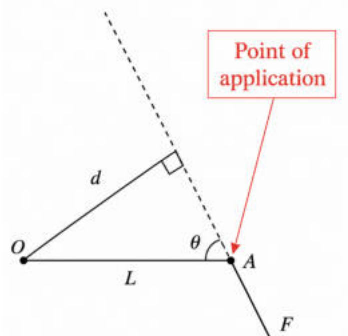


Figure 1-4-6 · The moment arm is the perpendicular distance to the line of action.

> ⭐ **Exam tip**
>
> Before calculating the moment about point O, draw the perpendicular distance from point O to the force's line of action.

Point

- ! An ideal extended object that does not deform when a force is applied is called a rigid body. The turning effect of a force about point O is determined by the product of the magnitude of the force and the perpendicular distance **from point O to the line of action**. This is called the **moment of the force** about point O, and its unit is the **newton meter (N m)**. The perpendicular distance from point O to the line of action is called the **moment arm**.
- ! Letting the moment of force be M [N m], the magnitude of the force be F [N] and the length of the moment arm be d [m], it is expressed as $M = Fd$. For a force F acting at an angle, since $d = L \sin\theta$, the moment about point O is $M = FL \sin\theta$.
- ! A force can rotate the object **counterclockwise or clockwise** about a point, and these are distinguished by assigning a positive **or negative sign** to the moment. So the moment of a force about a point on its own line of action is **0 N m**.

  There the perpendicular distance to the line of action is 0 m.

Equation box

Moment of a force	$M = Fd$
Moment of an inclined force	$M = FL \sin\theta$

Worked example

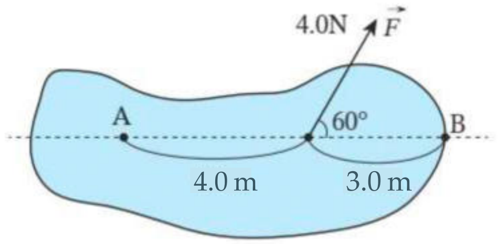


Figure 1-4-7 · A 4.0 N force applied at 60° to the line AB.

Situation	A force of 4.0 N acts as shown. Determine the moment of the force about points A and B. Take counterclockwise as positive.
Given and required	Given: F = 4.0 N, θ = 60°, distance to A = 4.0 m, distance to B = 3.0 m. Required: Find the moment about A and B.
Representation	Moment = Force × perpendicular distance → $M = F\,d$
Strategy	- Find the perpendicular distance from the reference point to the line of action of the force. - Apply $M = F\,L\sin\theta$. - Assign sign (+ counterclockwise, − clockwise).
Annotated solution	**(a)** About point A $M = F\,L\sin\theta$ $= 4.0 \times 4.0 \times \sin 60°$ $= 13.9 \approx 14$ N m Rotation is counterclockwise → $M = +14$ N m **(b)** About point B $M = -\,F\,L\sin\theta$ $= -4.0 \times 3.0 \times \sin 60°$ $= -10.4 \approx -10$ N m
Reasonableness check	The same force acts in both cases, but point A has a larger perpendicular distance from the force's line of action than point B, so the moment magnitude at A is larger.
Explain it yourself	Explain why the moment about point A is larger in magnitude than about point B, even though the same force is applied.

Now you try: the same 4.0 N force acts on the same body. Take a point C on the body, 2.0 m from the point where the force is applied, and it turns the body clockwise about C. **Determine** the moment of the force about C in N m.

Quick application

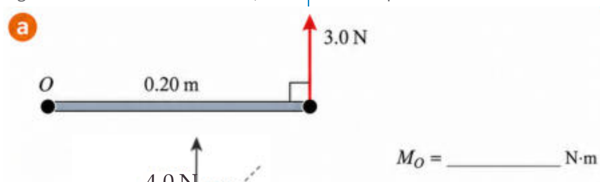
A force of 5.0 N acts perpendicular to a rod 0.40 m long, at one end of the rod. Calculate the moment about the other end using $M = Fd$.

Figure 1-4-8 · A 5.0 N force acting 0.40 m from the pivot.

Try

For each figure, determine the moment of the force about point O. Take the counterclockwise direction as positive.

Figure 1-4-9 · Two forces on a rod, taken about the pivot O.



(a) O is at the left end of a horizontal bar. A horizontal arm 0.20 m long reaches out from O. A force of 3.0 N acts perpendicular at the far end. (The force is perpendicular to the arm — the arm length is the perpendicular distance.)

(b) An inclined bar has a point O at its lower end. The bar is inclined at some angle to the horizontal. A force of 4.0 N is applied at the upper end, 0.30 m from O along the bar. The force acts at an angle of 45° to the bar. Determine the moment of the force about point O.

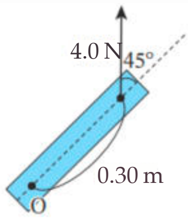
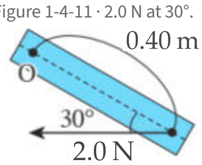


Figure 1-4-10 · 4.0 N at 45°.

(c) An inclined bar has O at its left end. A force of 2.0 N is directed leftward at the far end, 0.40 m from O. The force acts at an angle of 30° to the bar.

Figure 1-4-11 · 2.0 N at 30°.



(d) Point O is at the top left end of a bar. A force of 4.0 N acts 0.20 m from O, acting as shown in Figure 1-4-12 and rotating the bar clockwise.

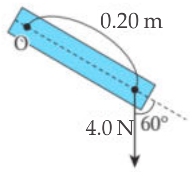


Figure 1-4-12 · A 4.0 N force at 60° on a 0.20 m arm.

A working note

In **(b)**, **(c)** and **(d)** the force is inclined, so find the perpendicular distance (moment arm) $d = L \sin\theta$ first, then multiply by the force, then assign the sign for the rotation.

In a workshop context — using a torque wrench

In a car or aircraft workshop, a wheel nut must be tightened to produce the correct turning effect. If the nut is not tightened enough, it can loosen in service; if it is tightened too much, the bolt may be damaged. Mechanics therefore use a torque wrench: a tool that produces an audible click when the required moment of the force is reached. As the mechanic pulls, the turning effect depends on the force and the perpendicular distance (moment arm). A longer wrench gives a larger arm, so a smaller force can produce the same moment. The click is triggered by the set moment — often called torque in engineering contexts — not by the force alone.

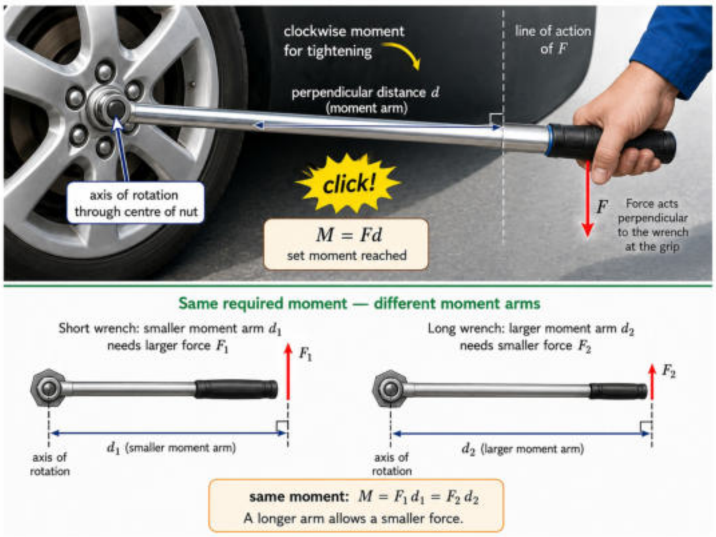


Figure 1-4-13 · The same moment from a longer arm needs a smaller force.

Your turn: Explain why a longer torque wrench reaches the same preset moment with a smaller force, using the relationship: force × perpendicular distance (moment arm).

Key fact

The unit of moment is N m. Although this has the same base units as the joule, it is not used in the same way: N m for moment is used to measure a turning effect, whereas J for energy is used to measure transferred or stored energy.

Nature of Science: Archimedes and the lever — the early law of mechanics

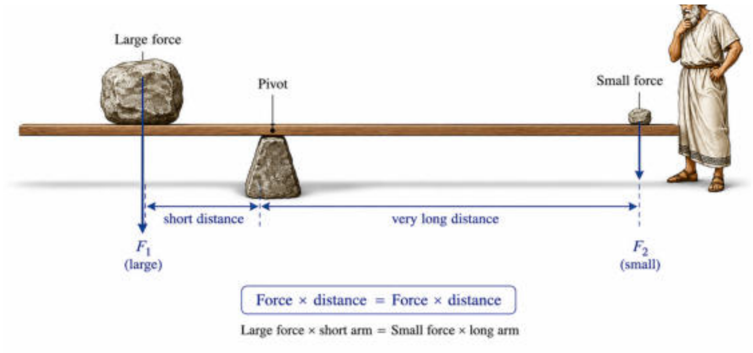


Figure 1-4-14 · A long arm lets a small force balance a large one.

More than two thousand years ago, Archimedes formulated the law of the lever. A small weight far from the pivot can balance a large weight close to the pivot, when weight × distance is equal on both sides. He is said to have declared, "Give me a place to stand and I will move the Earth." It was one of the earliest known precise, quantitative rules for the rotational effect of a force. It is the same product — force × moment arm — that you used throughout this lesson. (Source: Archimedes, On the Equilibrium of Planes, c. 250 BCE.)

Your turn: suggest one everyday tool that uses the law of the lever — a large moment from a small force applied far from the pivot.

Lesson question, answered

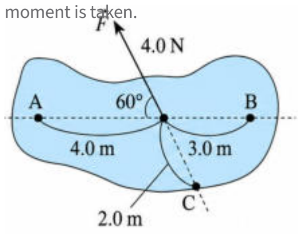
Push a door with the same force in two places — why does it rotate easily at the handle but barely move near the hinge? Because the moment depends on both the force and its perpendicular distance from the hinge, not on the force alone. At the handle the arm is long, so the moment is large; near the hinge the arm is small, so the moment is small. Assign a sign to that moment — counterclockwise positive, clockwise negative — and the moment is fully described for this planar sign convention. **The moment of a force is the product of the force and the perpendicular distance from the point to the force's line of action: apply the force farther from the pivot, and the same force produces a much larger moment.**

Exercise

(a) As shown in Figure 1-4-16, when a force F of magnitude 4.0 N is applied, determine the moment of the force about each of the following points.
Take the counterclockwise direction as positive.

(i) Point A (ii) Point B (iii) Point C

Figure 1-4-16 · The same body with three marked points about which the moment is taken.



(b) For each of the following figures, determine the moment of the force about point O.

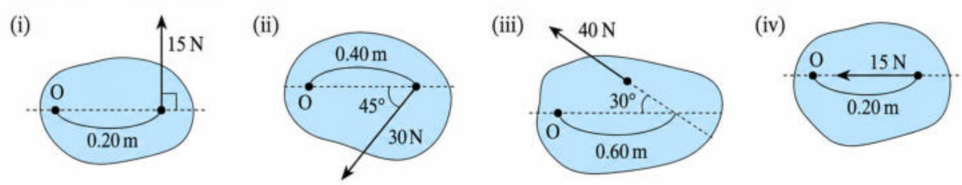
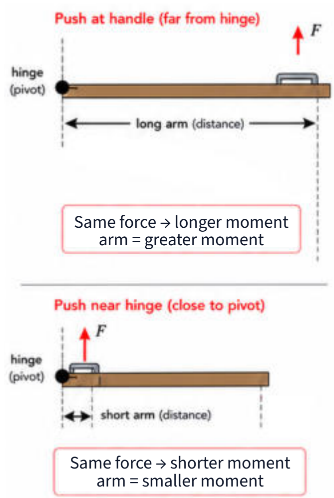


Figure 1-4-15 · The door revisited: the far push turns it more easily.



Key fact

A force whose line of action passes through the pivot has zero moment arm, so it produces zero moment — distance, not just force, determines the moment.

Exam-style question

A force of 6.0 N acts at the end of a lever 0.50 m long, at 30° to the lever. **Determine** the moment of the force about the other end of the lever.

Lesson 1-5 — Equilibrium of Forces

❓ **Lesson question**: A book remains at rest on a table and a shop sign is suspended by two chains — what is true of the resultant force acting on each object?

You will learn to

By the end of this lesson, you will be able to:

1. **State** the condition of equilibrium for a body at rest or moving with constant velocity.
2. **Draw** a free-body diagram, isolating the body and showing all external forces acting on it, identifying the source of each force.
3. **Determine** an unknown force by resolving the forces into components along two perpendicular axes and applying the equilibrium conditions to each axis.

🔍 The phenomenon — predict before you continue

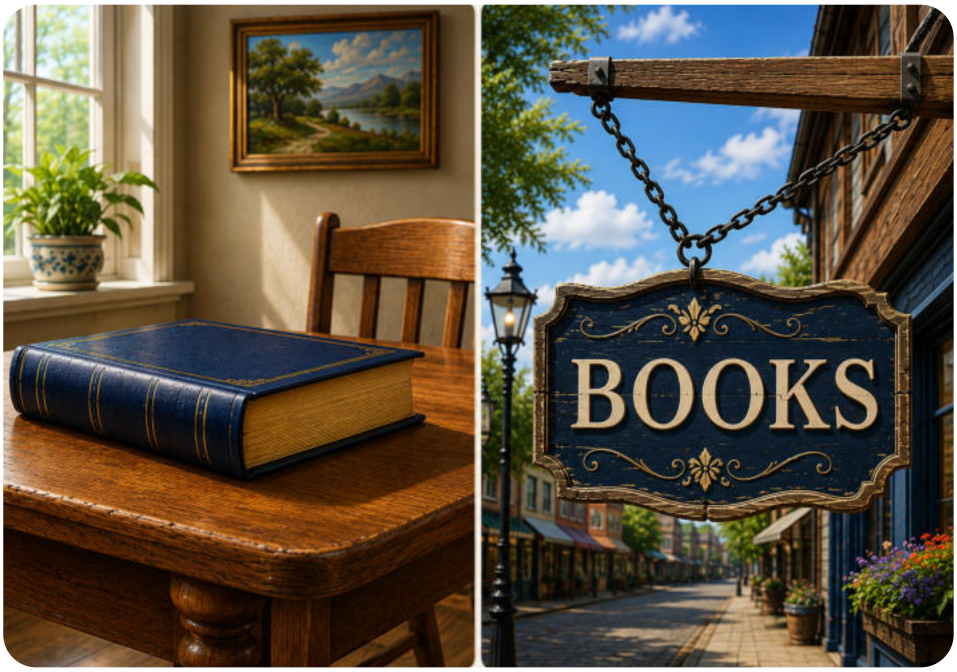


Figure 1-5-1 · A resting book and a hanging sign: both are in equilibrium.

A heavy book lies at rest on a table. Its weight acts downward due to gravity, yet the book remains at rest on the table. Nearby, a heavy shop sign hangs motionless from two chains inclined at different angles: one steep, the other closer to the horizontal.

Predict before you continue

(a) What force, if any, does the table exert on the book?

(b) The sign is suspended by two chains at different angles. Which chain has the greater tension: the steeper chain or the chain that is closer to the horizontal?

Explore — increase the angle, measure the extension

What you'll do: Measure the extension of two identical rubber bands as their angle with the vertical is changed, while keeping the load constant.

Time: 12 min

You will need:

- A filled water bottle of known weight.
- Two identical rubber bands.
- A ruler.
- A protractor.
- A horizontal bar or two hooks that can be moved apart.

Steps:

1. Suspend a bottle using two identical rubber bands. Adjust the hooks so that each rubber band makes a small angle with the vertical.
2. Measure the extension of each rubber band and record the angle each band makes with the vertical.
3. Keep the same bottle and move the upper attachment points farther apart so that the bands form a larger angle with the vertical.
4. Measure the extension of each rubber band again.
5. Repeat the procedure for several larger angles, without exceeding the elastic limit of the rubber.

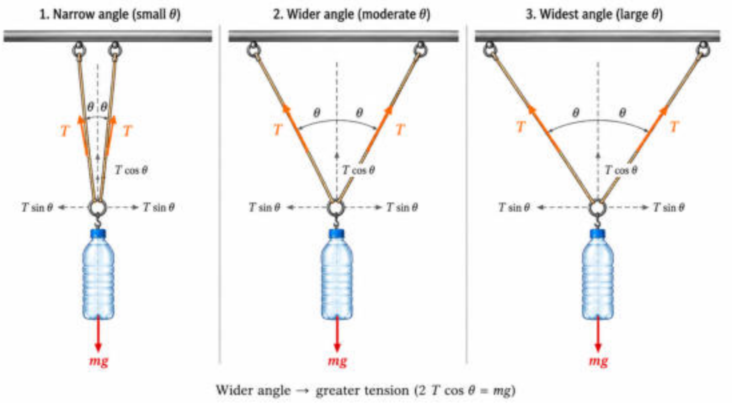


Figure 1-5-2 · The wider the angle, the greater the tension in each band.

Trial	Did the bottle's weight change?	Extension of right band (cm)	Extension of left band (cm)	Angle of each band from the vertical, θ (°)	average extension (cm)
1					
2					
3					

Analyze what you collected:

(a) As the angle from the vertical increased, what happened to the extension of each band and, therefore, to the tension in it?

(b) The bottle's weight remained constant, so why did the tension in each band increase? What happened to the horizontal components of the two tension forces as the bands moved farther apart?

Write your explanation in two lines before you read the next page.

A measurement note

Measure each band's extension twice and calculate the mean. Use the same bands throughout the experiment, and compare the average extension for different angles rather than against a fixed value.

From observation to model

In the Explore experiment, the bottle's weight remained constant, but increasing the angle each rubber band made with the vertical increased the mean extension of the bands, indicating an increase in the tension in each band.

You might have expected the tension to remain constant because the weight did not change. However, the observations show that the tension required to support the bottle depends not only on its weight but also on the angle each band makes with the vertical. The evidence supports the following model.

Isolate the body — draw a free-body diagram

A force is a push or a pull exerted by one object on another. To analyze a body, first isolate it from its surroundings. Show only the external forces acting on the body, not the forces that the body exerts on other objects. Represent each force by an arrow indicating its direction and label it according to its source. Where appropriate, draw each force at its point of application. The body's weight is represented by a single downward force acting through its center of gravity.

A free-body diagram isolates one body and shows all the external forces acting on it, with each force identified by its source.

Force	Symbol	Source and direction
Normal force	F_N	A contact surface; acts perpendicular to the surface. For a body in vertical equilibrium on a horizontal surface with no other vertical forces, $F_N = mg$.
Tension	T	A taut rope or cable; acts along the rope, directed away from the body.
Elastic force	$F_e = -kx$	A stretched or compressed spring; acts opposite to the displacement from equilibrium, within the limit of proportionality.
Weight	$F_g = \mathrm{mg}$	The gravitational field of the Earth (or a planet); acts vertically downward through the center of gravity.

Apply the equilibrium condition

A body is in translational equilibrium when it stays at rest or moves at constant velocity — that is, when the resultant force on it is zero:

$\Sigma F = 0$

A force at an angle is resolved into components along two perpendicular axes, and the condition is applied independently along each: $\Sigma F_x = 0$ and $\Sigma F_y = 0$

The horizontal components of the two tensions are equal and opposite, so they cancel. The vertical components act upward and together balance the weight of the bottle. Therefore, the resultant force on the bottle is zero, and it remains in equilibrium.

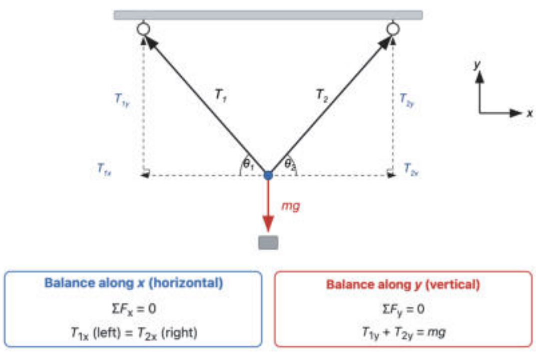


Figure 1-5-4 · Horizontal components cancel while vertical components carry the weight.

Point

- If a rigid body remains at rest or moves with constant velocity under the action of several forces, it is said to be in equilibrium. To analyze it, first isolate the body from its surroundings and draw a free-body diagram showing only the external forces acting on it. The body's weight is a single downward force acting through its center of gravity.
- **F**1 + **F**2 + **F**3 + … = 0. The equilibrium equations are set up by resolving the forces into components along two perpendicular axes: ΣF_x = 0 and ΣF_y = 0.

Quick application

A 5.0 kg lamp hangs from two cables that each make 40° with the horizontal. Taking $g = 9.8\ m/s^2$, calculate the vertical component each cable must supply.

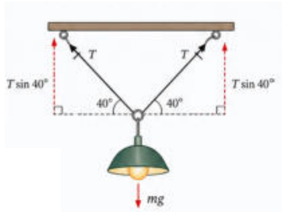
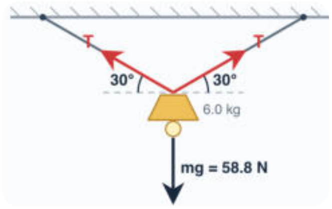


Figure 1-5-3 · A lamp hung from two cables at 40° to the horizontal.

Equation box

Condition for Translational Equilibrium	$\Sigma F = 0$
Resolved along two perpendicular axes	$\Sigma F_x = 0,\ \Sigma F_y = 0$

Worked example

Situation	A lamp of mass 6.0 kg is suspended in equilibrium by two identical cables, each making an angle of 30° with the horizontal. Calculate the tension in each cable.
Given and required	Given: m = 6.0 kg, g = 9.8 m/s² and θ = 30°. Required: tension T in each cable.
Representation	 Figure 1-5-5 · A 6.0 kg load on two cables at 30°.
Strategy	Apply the equilibrium conditions: ΣFx = 0, ΣFy = 0. Use the vertical balance: the sum of the vertical components of tension equals the weight.
Annotated solution	$2T\ \sin 30° = W$ $T = W/(2\ \sin 30°)$ $= 58.8/(2 \times 0.5)$ $= 58.8\ \mathrm{N} \approx 59\ \mathrm{N}$
Reasonableness check	The tension is equal to the lamp's weight because at 30° only half of each tension acts vertically to support the lamp.
Explain it yourself	Why does each cable carry a tension equal to the lamp's full weight, even though two cables support it?

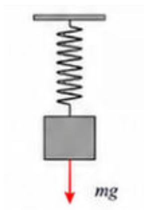
Now you try: The same 6.0 kg lamp is suspended by two identical cables, each making an angle of 50° with the horizontal. Determine the tension in each cable (N), and compare it with the value obtained in the previous example.

Try

(a) A streetlamp above a public square hangs from two cables that make angles of 30° and 60° with the horizontal. The lamp has a mass of 20 kg. Determine the tension in each cable, in N, and state which cable is under the greater tension.

(b) A sign of mass 5.0 kg over a street hangs from two cables that each make 40° with the horizontal. Determine the tension in each cable, in N.

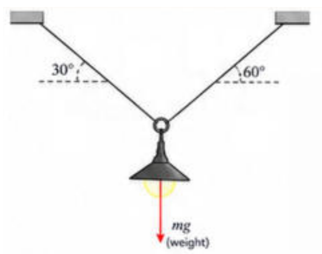
(c) A metal block hangs at rest from a spring with a spring constant of 200 N/m, stretching it by 0.15 m. Determine the weight of the block, in N.



Exam tip

For a body in equilibrium supported by two cables, resolve both tensions. Their horizontal components cancel, and their vertical components add to balance the weight.

Figure 1-5-6 · A lamp on two cables at 30° and 60°.



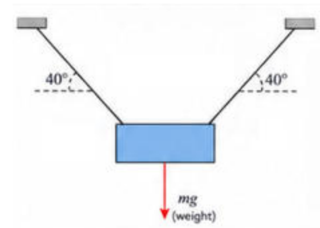


Figure 1-5-7 · Two cables at 40°.

In a new context — power lines between poles

Traveling along a country road, you may notice that power lines sag between poles rather than forming a straight horizontal line. The same principle of equilibrium applies to a load supported by two cables.

The weight of each span is distributed continuously along the wire. At the two supports, the vertical components of the tension forces together balance the total weight of the span, while the horizontal components are equal in magnitude and opposite in direction.

As the sag decreases, the wire becomes more nearly horizontal at the supports. The vertical component then becomes a smaller fraction of the tension, so greater tension is required to support the same weight.

In the idealized limit of a perfectly horizontal wire supporting a non-zero weight, the required tension would tend to infinity. Power lines are therefore installed with a controlled amount of sag.

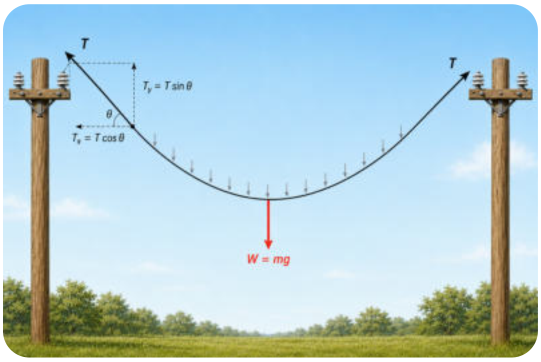


Figure 1-5-9 · A cable sags until its tension balances the weight.

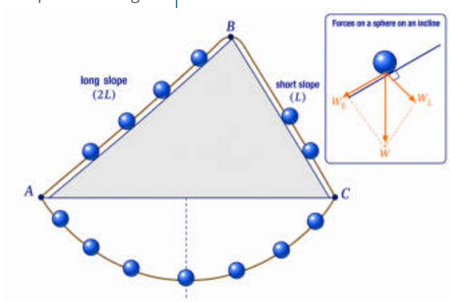
Your turn: Using vertical components, explain why reducing the sag in a power line increases the tension in the line.

Linking question

"How are concepts of equilibrium and conservation applied to understand matter and motion from the smallest atom to the whole universe?"

Nature of Science: Models — an early law of equilibrium before Newton

Figure 1-5-10 · Beads on two slopes: the incline angle sets the component along it.



Before Newton formulated the laws of motion, Simon Stevin (1586) studied equilibrium on inclined planes using a thought experiment known as the "wreath of spheres." He imagined a closed chain of equally spaced spheres placed around two inclined planes. If one side produced a greater turning effect than the other, the chain would move continuously while returning to the same arrangement, producing perpetual motion. Stevin concluded that the effects on two slopes must balance. This idea helped establish the law of equilibrium on an inclined plane and the principles of resolving forces into components.

Your turn: Why could Stevin trust this conclusion even without modern equations of motion?

Lesson question, answered

A book remains at rest on a table and a shop sign suspended by two chains — what is true of the resultant force acting on each object?

In both cases, the bodies are in translational equilibrium, so the vector sum of all forces acting on each body is zero: $\sum F = 0$

For the book, the upward normal force balances its weight. For the sign, the horizontal components of the two tension forces cancel, while their vertical components together balance its weight. Because the chains make different angles with the horizontal, the tensions are generally unequal, and the steeper carries the greater tension.

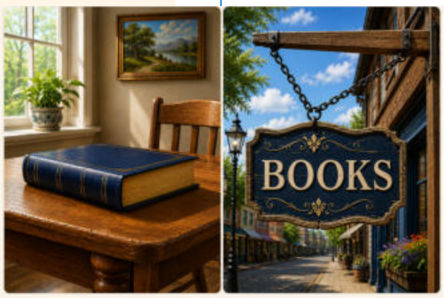


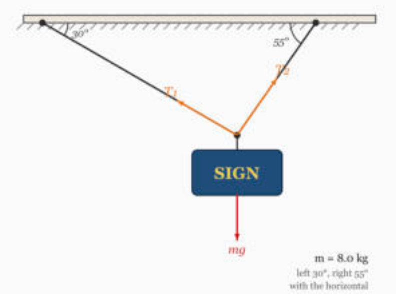
Figure 1-5-11 · The book and the sign revisited.

Exercise

Work to two significant figures; take $g = 9.8$ m/s^2.

(a) A shop sign of mass 8.0 kg hangs in equilibrium from two light cables attached to a horizontal beam. The left cable makes an angle of 30° with the horizontal, and the right cable makes an angle of 55° with the horizontal.

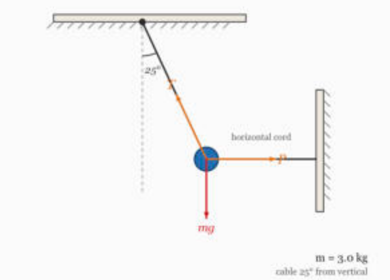
Determine the tension in each cable, in N, and state which cable carries the greater tension.



Two cables at different angles — resolve T_1 and T_2.

Figure 1-5-12 · An 8.0 kg sign held by two cables at different angles.

Figure 1-5-13 · Three forces hold a 3.0 kg lamp in place.



Three forces act on the lamp — balance components along x and y.

(b) A lamp of mass 3.0 kg is held in equilibrium by a light cable fixed to the ceiling and a light horizontal cord. The cable makes an angle of 25° with the vertical.

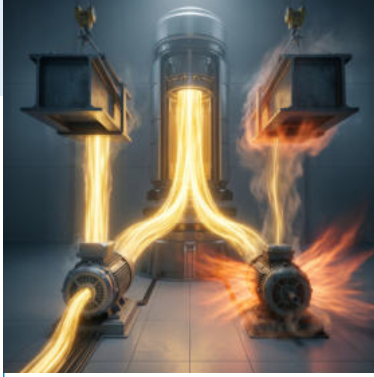
Determine (i) the tension T in the cable, and (ii) the tension P in the horizontal cord, both in N.

(c) A block of mass 1.5 kg hangs in equilibrium from two identical light springs. Each spring makes an angle of 40° with the vertical and has a spring constant of 60 N/m.

Determine (i) the force in each spring, in N, and (ii) the extension of each spring, in m.

Exam-style question

A 6.0 kg lamp hangs at rest from two cables that make equal angles of 40° with the horizontal. Draw a free-body diagram of the lamp, then determine the tension in each cable.



Lesson 1-6 — Power and Efficiency

❓ **Lesson question**: Two lifts raise the same load through the same vertical height, but one completes the lift in half the time. Do both lifts perform the same useful work? Which lift has the greater power?

You will learn to

By the end of this lesson, you will be able to:

1. **Calculate** the power of a machine from the work done and the time taken, or from force and constant velocity.
2. **Determine** the efficiency of a device from its useful output energy and total input energy, expressed as a percentage.
3. **Interpret** energy transfer in a Sankey diagram and identify where the wasted energy is dissipated.

🔍 The phenomenon — predict before you continue

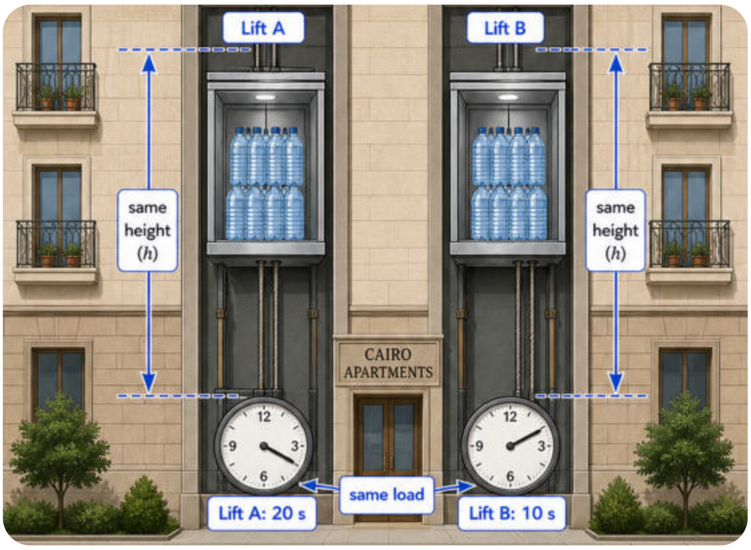


Figure 1-6-1 · Two lifts raise the same load through the same height in different times.

Two lifts rise side by side in one building. Each carries the same load to the same floor — the same height. Lift A takes 20 s; lift B takes 10 s, in half the time. People sometimes say that the faster lift "does more work." Is that true, and which lift has the greater power?

Predict before you continue

(a) Did the faster lift perform more useful work on the load, or did both lifts perform the same useful work?

(b) A salesperson claims, "The more powerful motor uses less electrical energy." Could this be true?

⭐ **Key fact**

Work: $W = F\,s\cos\theta$.
A force does work only when it causes a displacement in the direction of the force or its component along the displacement.

🔍 Explore — climb the school stairs

What you'll do: use a scale and a stopwatch to measure the power produced while climbing stairs.

Time: 12 min

You will need:

- A digital weighing scale.
- A stopwatch or phone timer.
- A measuring tape.

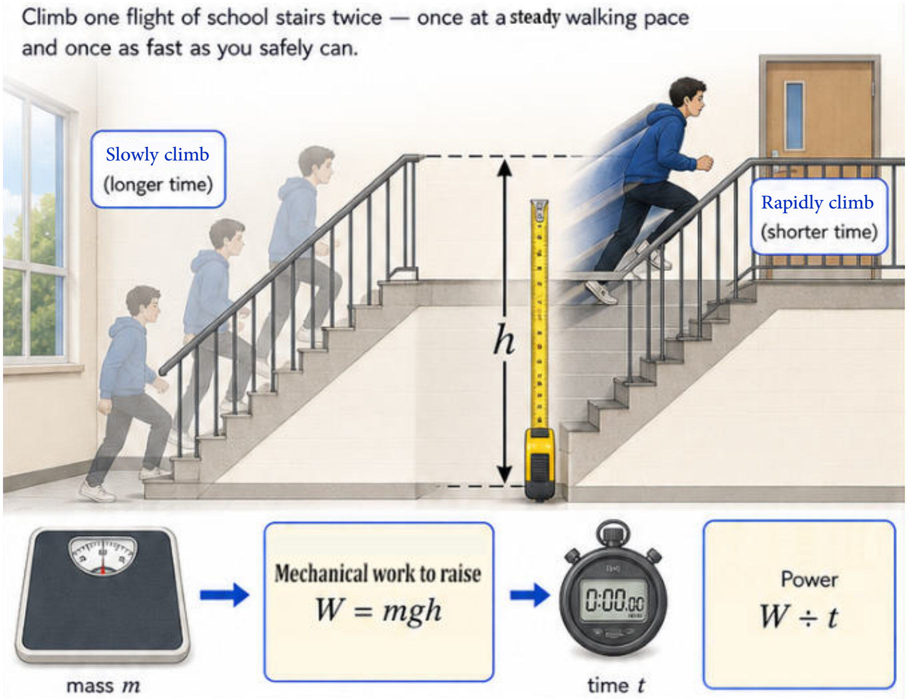


Figure 1-6-2 · The same climb made slowly and quickly: same work, different power.

Steps:

1. Measure your mass (m) and record it in kilograms.
2. Measure the vertical height h of one flight of stairs, in meters — measure the vertical rise, not the length of the stairs.
3. Calculate the work done in lifting yourself through one flight $W = mgh$, taking $g = 9.8\ m/s^2$.
4. Climb the flight slowly while your partner measures the time. Record the result.
5. Repeat the climb as rapidly as is safely possible and record the shorter time.

Climb	Time	Work done = mgh	Work ÷ time
Slowly			
Rapidly			

Analyze what you collected:

(a) Did the work you did change between the two climbs, or did it remain the same? Explain your answer.

(b) For which climb was the value of (W/t) greater? What does this quantity represent?

✏️ **Write your explanation in two lines before you read the next page.**

⭐ A measurement note

Measure the time for each ascent twice and take the average value. Human reaction time when starting and stopping the stopwatch can introduce an uncertainty of a few tenths of a second. This uncertainty has a greater percentage effect on the faster ascent because its measured time is shorter.

From observation to model

In Explore you did the same amount of work in both climbs — your mass and the vertical height were the same, so ($W = mgh$) was unchanged. However, the faster climb completed this work in less time and therefore gave a greater value of (W/t). This quantity is called average power.

Power — rate of doing work

Two machines can perform the same amount of work or transfer the same amount of energy, but take different times to do so. Power measures this difference: it is the rate at which work is done or energy is transferred.

The average power over a time interval is: $P = \Delta W/\Delta t$

where ΔW is the work done during the time interval. The unit of power is the watt: $1\ \mathrm{W} = 1\ \mathrm{J/s}$

Larger values of powers are commonly expressed in kW and MW:

$1\ kW = 10^3\ W,\ 1\ MW = 10^6\ W$

Power can also be related to force F and speed v. If a force acts in the direction of motion: $P = Fv$

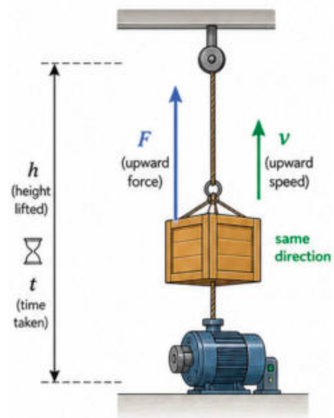


Figure 1-6-3 · Raising a load steadily: the force and the motion point the same way.

Point

- The work done by a constant force F [N] moving a body through a displacement s [m] at an angle θ to the force is $W = Fs\cos\theta$. Two machines can do the same work but take different times: the rate of doing work is called the average power, and for work ΔW [J] done in a time Δt [s] it is $P = \Delta W/\Delta t$, measured in watts (W).
- When a force F acts on a body moving at a speed v [m/s], the power can be written without measuring time: $P = Fv\cos\theta$.
- The fraction of the input energy that leaves a device as useful output is called its efficiency. It is a dimensionless quantity, written as a decimal between 0 and 1 or as a percentage.

  No real device is 100% efficient: some of the input energy is always transferred to the surroundings as waste.

  A Sankey diagram shows this visually — one input arrow splits into useful output and waste, each arrow's width proportional to the energy it carries.

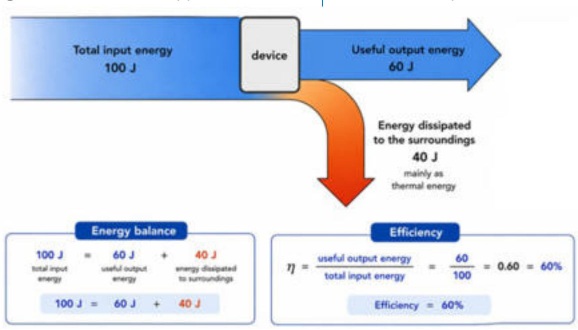
Equation box

Work	$W = F\,s\cos\theta$
Power, from work and time	$P = \Delta W/\Delta t$
Power, from force and speed	$P = Fv\cos\theta$

Efficiency is a dimensionless quantity, expressed as a decimal (between 0 and 1) or as a percentage. No real device is 100% efficient because some input energy is always transferred to the surroundings.

A Sankey diagram represents energy transfers visually: a single input arrow splits into useful output and waste energy pathways, with the width of each arrow proportional to the energy it represents. Sankey diagrams are interpreted by comparing the relative widths of the arrows.

Figure 1-6-4 · Of 100 J supplied, 60 J is useful and 40 J is dissipated.



Quick application

An office lift carries its load upward at a constant speed of 2.5 m/s. The lifting cable exerts an upward force of 7840 N on the lift. Determine the useful power delivered to the lift by the motor.

Equation box

Efficiency	η = useful output energy ÷ total input energy

Worked example

Situation	A building-site hoist lifts a 5.0×10^2 kg pallet vertically upward through a height of 6.0 m at constant speed in 12 s. Neglect air resistance. Determine: **(a)** the work done by the hoist, in J **(b)** the average power developed by the hoist. **(c)** check using $P = Fv$
Given and required	Given: $m = 500$ kg, $h = 6.0$ m, $t = 12$ s, $g = 9.8$ m/s^2. Required: **(a)** work W, **(b)** power P, **(c)** check using $P = Fv$
Representation	Vertical lift at constant speed, F = mg.
Strategy	**1)** Use $W = mgh$ **2)** Use $P = W/t$ **3)** Find speed $v = h/t$ and check using $P = mgv$.
Annotated solution	**(a)** $W = 500 \times 9.8 \times 6.0 = 2.94 \times 10^4$ J $\approx 2.9 \times 10^4$ J **(b)** $P = (2.94 \times 10^4) / 12 = 2.5 \times 10^3$ W $= 2.5$ kW **(c) v = 6.0 / 12 = 0.50 m/s** $P = mgv = 500 \times 9.8 \times 0.5 = 2450$ W $= 2.5$ kW
Reasonableness check	The two methods give the same positive value for the power.
Explain it yourself	If the hoist lifts the same load but in half the time, how does this affect the power and why?

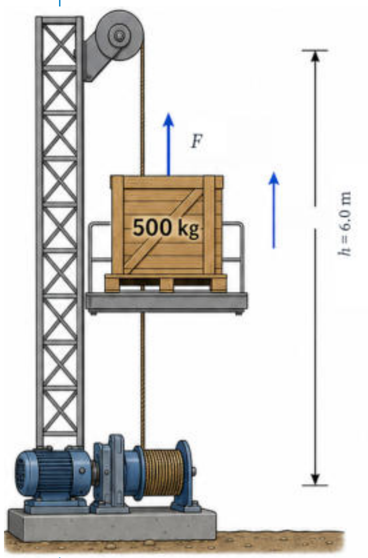


Figure 1-6-5 · A 500 kg load raised through 6.0 m.

Key fact

For a vertical lift at constant speed:

$P = Fv = mgv = mgh / t$

Use $P = Fv$ when the speed is given, or $P = mgh / t$ when the height and time are given.

Now you try: a lift cabin of mass 400 kg rises vertically through 9.0 m at a constant speed in 15 s. Take $g = 9.8$ m/s^2. Calculate the power output of its motor, in kW, then **check** your answer using $P = Fv$.

Efficiency — fraction of input energy usefully transferred

No device converts all of its input energy into useful output; some energy is always dissipated, usually as thermal energy. Efficiency measures the fraction of input energy converted into useful output: η = useful output energy / total input energy

Try

Give each efficiency as a percentage.

(a) An electric pump on a Nile Delta farm draws 2000 W of electrical power and delivers 1500 W of useful output power lifting irrigation water. Determine the pump's efficiency.

Figure 1-6-6 · A pump: 2000 W supplied, 1500 W useful.

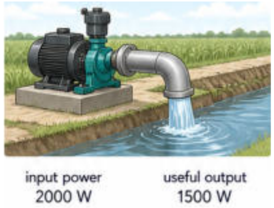


Figure 1-6-7 · A heater: 2400 W supplied, 2040 W useful.

(b) A small electric water heater takes in 2400 W and delivers 2040 W of useful thermal power to the water. Determine its efficiency.

(c) A lamp is supplied with 100 J of electrical energy. It emits 25 J of light, and the remainder is dissipated as heat. Determine the energy dissipated as thermal energy and the efficiency of the lamp.

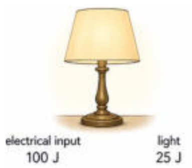


Figure 1-6-8 · A lamp: 100 J supplied, 25 J as light.

(d) Sketch a simple Sankey diagram for the lamp in (c): draw one input arrow representing 100 J, splitting into a useful light output and a wasted heat output. Make the widths of the arrows proportional to the energies they represent.

In a new context — choosing an efficient lamp

Replacing a tungsten filament lamp with an LED lamp can reduce the electrical energy required to light a room. For every 100 J of electrical energy supplied, a filament lamp may transfer only about 10 J as useful light. The remaining 90 J is dissipated to the surroundings, mainly as thermal energy. An LED lamp may transfer about 40 J of the same 100 J input as useful light, while about 60 J is dissipated to the surroundings.

The LED is therefore more efficient because a greater fraction of its input energy is transferred as useful light. A Sankey diagram illustrates this difference: the LED has a wider useful-light arrow and a narrower wasted-energy arrow than the filament lamp, even though both receive the same 100 J input.

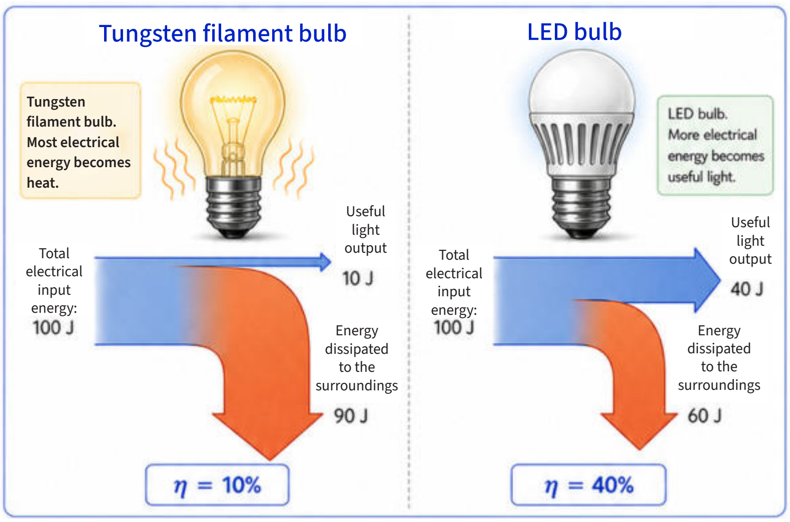


Figure 1-6-9 · A filament lamp wastes as heat what an LED delivers as light.

Explain, from the two Sankey diagrams, why an LED lamp needs less electrical energy than a tungsten filament lamp to produce the same amount of useful light.

Key fact

Reading a Sankey diagram: The total input equals the sum of the useful output and all wasted outputs. Therefore, a missing output can be calculated by subtracting the sum of the known outputs from the total input.

Exam tip

Show efficiency as a percentage only when the question asks for it. Efficiency may be written as a decimal or a percentage.

Linking question

"How is the efficiency of a device related to the energy it dissipates to the surroundings?"

Nature of Science: Measurement — the origin of horsepower

When James Watt marketed his improved steam engines in the late eighteenth century, customers needed a familiar way to compare their power with that of working horses. Watt used an estimate of the sustained power of a working horse as a practical basis for comparison. He defined one horsepower as a rate of doing work of 33,000 foot-pounds-force per minute, equivalent to approximately 746 W. This allowed engine power to be expressed in terms that customers could readily understand. Horsepower is therefore a conventional unit of power, not the exact power produced by every horse.

Your turn: Suggest one advantage of expressing engine power in horsepower rather than introducing a completely new unit.

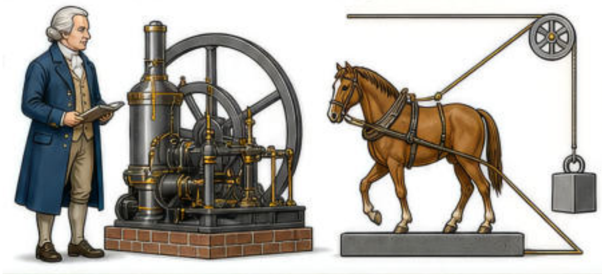
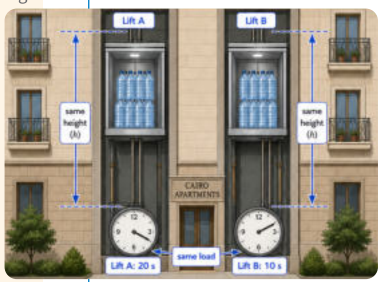


Figure 1-6-10 · Watt compared his engine with the horse it replaced.

Lesson question, answered

Two lifts raise the same load through the same vertical height, but one completes the lift in half the time. Do both lifts perform the same useful work? Which lift has the greater power? Both lifts perform the same useful work because the same load is raised through the same vertical height ($W = mgh$). The lift that completes the motion in less time has the greater power ($P = W/t$). If their input energies differ, the more efficient lift is the one that converts a larger fraction of input energy into useful lifting and dissipates less energy to the surroundings. A Sankey diagram would therefore show a wider useful-output arrow and a narrower wasted-energy arrow for the more efficient lift.

Figure 1-6-11 · The two lifts revisited.



Exercise

(1) A goods lift raises a load of mass 1.5×10^3 kg vertically through a height of 18 m at a constant speed, taking 30 s.

- **(a) Calculate the work done by the lift on the load.**
- **(b)** Determine the power output of the lift, in kW.
- **(c)** Show that the constant speed of the load is 0.60 m/s.
- **(d)** Using $P = Fv$, confirm your answer to (b), and explain briefly why the two methods must give the same result.

(2) Two lifts, A and B, each raise a load of mass 600 kg through the same vertical height of 9.0 m at a constant speed. Lift A completes the lift in 24 s; lift B completes the same lift in 12 s.

- **(a)** Explain why both lifts do the same useful work on the load, and state its value.
- **(b)** Calculate the power output of lift A and of lift B, in kW.
- **(c)** State the ratio P^B: P^A and describe how it is related to the ratio of the times taken.

(3) An electric winch is supplied with 5.0×10^4 J of electrical energy. It does 3.5×10^4 J of useful work in raising a load; the remaining energy is dissipated.

- **(a)** Calculate the energy dissipated by the winch.
- **(b)** Determine the efficiency of the winch, expressed as a percentage.
- **(c)** Draw a Sankey diagram for this energy transfer, with the width of each arrow proportional to the energy it represents.
- **(d)** State the main form of energy into which the dissipated energy is transferred.

(4) An electric hoist raises a load of mass 250 kg through a vertical height of 6.0 m at a constant speed in 8.0 s. While it does so, the hoist is supplied with electrical energy at a constant rate of 2.5 kW.

- **(a)** Calculate the useful power output of the hoist, in kW.
- **(b)** Determine the efficiency of the hoist, expressed as a percentage.
- **(c)** Calculate the energy dissipated by the hoist during the 8.0 s lift, and state the main form of energy to which it is transferred.

Exam-style question

A small crane lifts a load of mass 600 kg vertically through a height of 8.0 m at a constant speed in 16 s. Take $g = 9.8$ m/s².

Calculate

the work done by the crane on the load.

the mechanical power output of the crane, in kW.

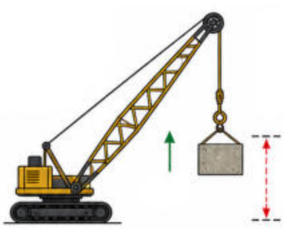


Figure 1-6-12 · A crane raising a load at steady speed.

Lesson 1-7 — Momentum and Impulse

❓ **Lesson question:** Why is a phone less likely to break when it lands on a bed than when it lands on a tiled floor, even if it is dropped from the same height?

You will learn to

By the end of this lesson, you will be able to:

1. **Describe** momentum as the product of mass and velocity, $p = mv$.
2. **Determine** the impulse of a force and relate it to the change in momentum, $J = F\Delta t = \Delta p$.
3. **Explain** why increasing the time over which a momentum change occurs reduces the average resultant force.

⭐ **Key fact**

Momentum and impulse are vector quantities. Two key relationships are used in this lesson: p $= mv$ and J = $F\Delta t$.

🔍 The phenomenon — predict before you continue

Figure 1-7-1 · The same fall on a soft bed and on hard tiles.

A phone is dropped from the same height onto two different surfaces. It lands on a bed and comes to rest undamaged. It then lands on a tiled floor from the same height, and the screen cracks. In both cases, the phone reaches the surface with the same mass and speed, so the change in momentum is the same. The difference lies in the interaction during the impact, not in the fall itself.

✏️ **Predict before you continue**

(a) What does the bed change: the change in momentum, the stopping time, or both?

(b) Does a ball bouncing off a wall experience a greater, smaller, or the same force as a ball that sticks to the wall, for the same initial speed and interaction time?

Explore — Testing the impulse–momentum theorem

What you'll do: You will measure the impulse acting during a collision and the change in momentum it produces, then investigate the relationship between them.

Time: 40 min

You will need:

- A dynamics cart and a level track.
- A force sensor fixed to one end of the track, with a stiff and a soft bumper.
- A motion sensor (or motion-encoder cart) to measure the cart's velocity.
- A data logger or computer with graphing software.
- A balance.

Steps:

1. Measure the cart's total mass m. Fix the force sensor and its spring bumper at one end of a level track, with the motion sensor at the other end.
2. Zero the force sensor with no force on it. Set the software to plot force and velocity against the same time axis.
3. Push the cart so that it moves at an approximately constant speed before striking the bumper and rebounds; keep your hand clear during the collision.
4. From the velocity–time graph, read the velocity on the constant-velocity sections just before (v_i) and just after (v_f) the collision. Taking the initial direction as positive makes v_f negative, so $\Delta p = m(v_f - v_i)$.
5. From the force–time graph, determine the impulse J as the area under the pulse, and record the peak force and the contact time Δt.
6. Repeat for two more runs: first with the same bumper at a higher incident speed, then with the softer bumper at approximately the same incident speed as in Run 1.

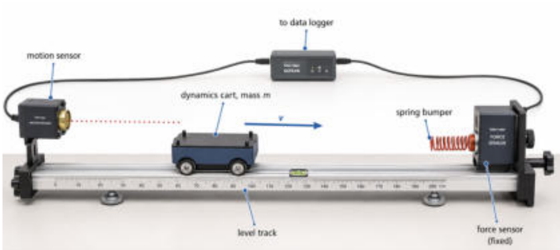


Figure 1-7-2 · A cart on a level track, with a motion sensor and a force sensor.

trial	v_i (m/s)	v_f (m/s)	Δt(s)	$\Delta p = m(v_f - v_i)$ (N s)	Impulse J (N s)
1					
2					
3 (soft bumper)					

Analyze what you collected:

(a) For each trial, compare the impulse J (area under the force–time graph) with the change in momentum $\Delta p = m(v_f - v_i)$. Are they equal in magnitude within experimental uncertainty? This tests the impulse–momentum theorem.

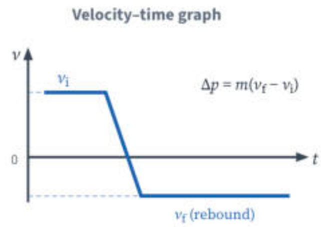
(b) The softer bumper increases the contact time. For a similar change in momentum, explain how this affects the force during the collision.

Write, in two lines, why the area under your force-time graph should equal m(v_f- v_i), before reading the next page.

Measurement note

Read v_i and v_f from the flat (constant-velocity) parts of the velocity-time graph, just before and just after the collision – never during it. Find the impulse from the area under the force pulse only, and re-zero the force sensor before each run.

Figure 1-7-3 · The velocity-time graph of a rebound gives the change in momentum.



From observation to model

In Explore, when Runs 1 and 3 are compared, the cart has the same mass and approximately the same incident speed. Yet the softer bumper exerted a smaller average force over a longer time, while the firmer bumper exerted a larger average force over a shorter time. You may have predicted that the softer bumper reduces the cart's change in momentum. It does not — for the same initial speed and the same rebound speed, the change in momentum is the same; the interaction time, and therefore the average force, is different.

Momentum and impulse

A body of mass m moving with velocity v has momentum p = mv.

Momentum is a vector quantity in the same direction as velocity and is measured in $\mathrm{kg\ m/s}$.

A change in momentum is produced by a resultant force acting over a time interval. For a constant resultant force, $J = F\Delta t$ and $J = \Delta p$.

Therefore, Newton's second law in momentum form is $F = \dfrac{\Delta p}{\Delta t}$.

For the same change in momentum, increasing the interaction time reduces the average resultant force, whereas decreasing the interaction time increases the average resultant force.

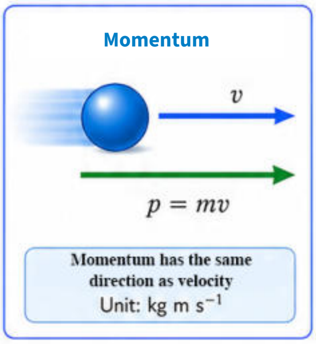


Figure 1-7-4 · Momentum points the same way as velocity.

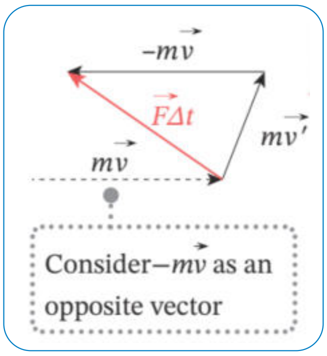


Figure 1-7-5 · The impulse turns the initial momentum into the final one.

Point

- The product of an object's mass m [kg] and its velocity v [m/s] is called momentum, which represents the quantity of motion. Its unit is the kg m/s, and it is expressed as $p = mv$.
- The product $F\Delta t$ [N s] is called impulse, which represents the change in momentum. Its unit is the N s. If the momentum before the change is mv and the momentum after the change is mv', then $mv' - mv = J = F\Delta t$, so Newton's second law in momentum form is $F = \dfrac{\Delta p}{\Delta t}$.
- A longer interaction time reduces the average force, for the same change in momentum. The average force is impulse ÷ time.

Equation box

Momentum	$p = mv$
Impulse	$J = F\Delta t = \Delta p$
Newton's second law (momentum form)	$F = \dfrac{\Delta p}{\Delta t}$

Impulse

The product of a constant resultant force and the time interval during which it acts.

$J = F\Delta t$; it is a vector in the direction of the resultant force, measured in $\mathrm{N\ s}$.

Figure 1-7-6 · A soft surface lengthens the contact time and lowers the force.

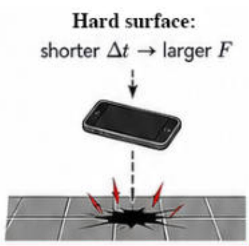
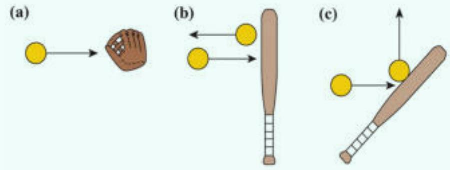
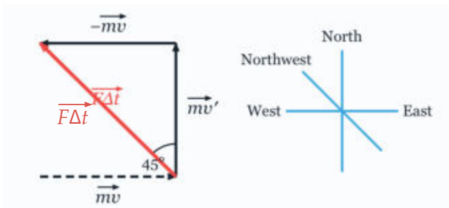


Figure 1-7-7 · A hard surface shortens the contact time and raises the force.

Worked example

Situation	A ball of mass 6.0×10^{-2} *kg* moves eastward at a speed of 20 m/*s*. For each of the following cases, determine the direction and magnitude of the impulse applied to the ball. (a) when the ball is caught; (b) when the ball is hit back westward at the same speed; (c) when the ball is hit back northward at the same speed. 
Given and required	Given: $m = 6.0 \times 10^{-2}$ kg; $v = 20$ m/s (east) Required: impulse
Representation	Draw the initial momentum vector, $p_i = m\,v_i$, in the direction of the ball's initial motion. For each case, determine the final momentum vector, $p_f = m\,v_f$, in the direction of the ball's final motion. The impulse is equal to the vector change in momentum: $J = \Delta p = p_f - p_i = m(v_f - v_i)$.
Strategy	Apply the impulse–momentum theorem: $F\Delta t = mv' - mv$ Steps to find momentum from impulse: Align the starting points of the momentum vector before the change and the impulse vector, and add them using the parallelogram rule to find the momentum vector after the change.
Annotated solution	Use the impulse–momentum relation. **(a)** Take the eastward direction as positive. $F\Delta t = (6.0 \times 10^{-2}) \times 0 - (6.0 \times 10^{-2}) \times 20$ • $F\Delta t = mv' - mv$ • When the ball is caught, its speed becomes zero. $= -1.2$ N s, i.e. 1.2 N s westward — East is positive, so west is negative. **(b)** Assume the eastward direction is positive. $F\Delta t = (6.0 \times 10^{-2}) \times (-20) - (6.0 \times 10^{-2}) \times 20$ $= -2.4$ N s, i.e. 2.4 N s westward **(c)** Considering it as a vector, $\vec{F}\Delta t$ will be as shown in Figure 1-7-9. The magnitude of $\vec{F}\Delta t$ is $\sqrt{2} \times 6.0 \times 10^{-2} \times 20 = 1.70 \approx 1.7$ N s 1.7 N s directed northwest.  Figure 1-7-9 · The impulse resolved with the compass directions of the motion.
Reasonableness check	The largest impulse occurs when the direction reverses (case b). Stopping gives a smaller impulse (case a). Changing the direction of motion to northward gives an intermediate impulse of 1.7 N s directed northwest (case c).
Explain it yourself	Why is the impulse in case (*b*) twice that in case (*a*), even though the speed is the same in both?

Now you try: A constant force of 5.0 N acts on an object for 2.0 s in the direction of its motion. Determine the magnitude of the impulse applied to the object, in N s.

Key fact

If an object moving at speed (v) comes to rest, the magnitude of its momentum change is (mv). If it rebounds at the same speed in the opposite direction, the magnitude of its momentum change is ($2mv$). Reversing the momentum, rather than simply reducing it to zero, doubles the impulse.

Try

(a) A force of 5.0 N is applied to an object of mass 4.0 kg moving at 2.0 m/s for 2.0 seconds in the direction of its motion.

(i) Determine the magnitude of the impulse applied to the object in N s.

(ii) Determine the increase in the object's momentum in kg m/s.

(iii) Determine its final speed in m/s.

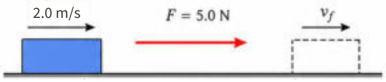


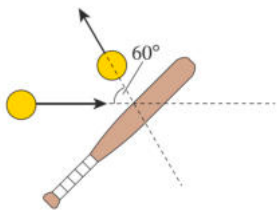
Figure 1-7-10 · A block pushed by a 5.0 N force.

(b) A ball of mass 0.15 kg moving horizontally at a speed of 30 m/s is hit back by a bat at the same speed at an elevation angle of 60° above the horizontal.

(i) Determine the magnitude of the impulse applied to the ball.

(ii) If the contact time between the ball and the bat is 2.0×10^{-2} s, determine the magnitude of the average force exerted on the ball in N.

Figure 1-7-11 · A ball meeting the bat at 60°.



Key fact

In part (b), the ball reverses direction, so draw the momentum vectors first. The impulse is the vector change in momentum, not the difference between the two speeds.

In a new context — How a Helmet Reduces Impact Force

When a motorcyclist falls and their head strikes the road, the change in momentum cannot be avoided because the head must slow down to a stop. However, a helmet increases the time over which this momentum change occurs.

The helmet's energy-absorbing foam compresses during impact, increasing the stopping time and stopping distance of the head. For the same change in momentum:

$$F = \frac{\Delta p}{\Delta t}$$

Increasing the stopping time reduces the average impact force and therefore the average deceleration of the head.

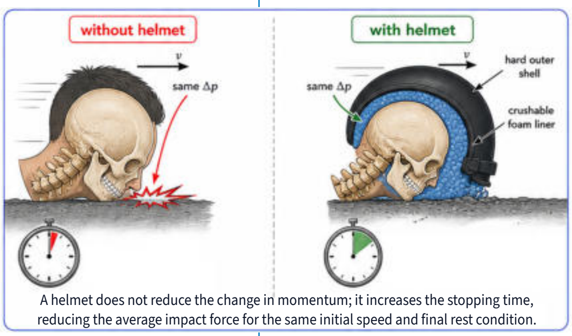


Figure 1-7-12 · A helmet lengthens the stopping time, so the impact force falls.

Your turn: Explain why a thicker, softer helmet liner provides better protection for the skull, even though the magnitude of the head's change in momentum is unchanged.

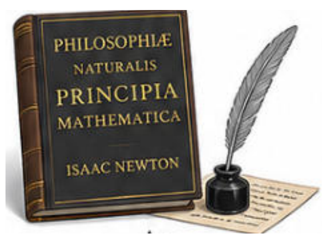
Nature of Science: Laws — Newton's Second Law (Momentum Form)

Newton (1687) did not express his second law as $F = ma$; he stated that the rate of change of motion is proportional to the applied force and occurs in its direction. "Quantity of motion" corresponds to momentum. In modern form: $F_{resultant} = \frac{dp}{dt}$.

Over a time interval: *the average resultant force is* $F_{avg} = \frac{\Delta p}{\Delta t}$.

For constant mass: $F_{resultant} = ma$.

A scientific law describes a consistent relationship between physical quantities and allows predictions to be made under specified conditions. Newton's second law relates the resultant force acting on an object to the rate of change of momentum.



Your turn: Suggest why the momentum form $F = \frac{\Delta p}{\Delta t}$ is considered the more general form.

Lesson question, answered

Why is a phone less likely to break when it lands on a bed than when it lands on a tiled floor, even if it is dropped from the same height? If a phone hits a bed or tiles with the same speed and stops without bouncing, the change in momentum is the same in both cases. The difference is stopping time.

A bed increases the stopping time by deforming, which reduces the average force. A tiled floor stops the phone in a much shorter time, producing a much larger average force and increasing the chance of damage.

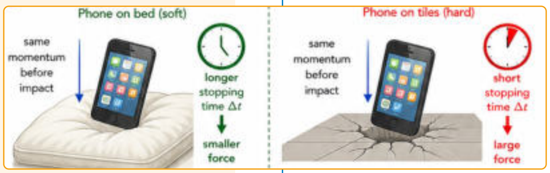


Figure 1-7-13 · The phone comparison revisited: same momentum, different stopping time.

Exercise

Determine the following:

(a) A force of 2.5 N was applied to an object of mass 2.0 kg moving at 1.0 m/s for 0.40 s in the direction of its motion.

(i) Determine the magnitude of the impulse applied to the object in N s.

(ii) Determine the increase in the object's momentum in kg m/s.

(iii) Determine its final speed in m/s.

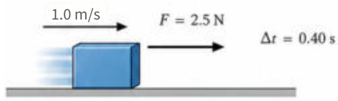


Figure 1-7-14 · A 2.5 N force acting for 0.40 s.

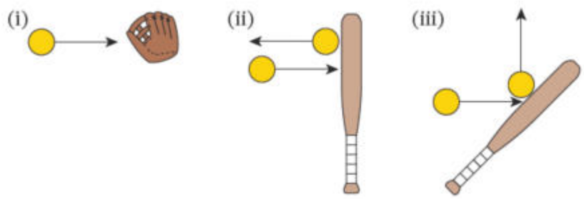
(b) A ball of mass 0.14 kg moving at a speed of 40 m/s is caught with a glove.

(i) Determine the magnitude of the impulse received by the ball in N s.

(ii) If the contact time between the ball and the glove until the ball stops is 2.0×10^{-2} s, determine the magnitude of the average force exerted by the ball on the glove in N.

(c) A ball of mass 3.0×10^{-2} kg is moving eastward at a speed of 10 m/s. For each of the following cases, determine the magnitude and direction of the impulse applied to the ball.

Figure 1-7-15 · Three cases for the exercise.



(i) When the ball is caught.

(ii) When the ball is hit back westward at the same speed.

(iii) When the ball is hit back northward at the same speed.

(d) A ball of mass 0.40 kg moving horizontally at a speed of 10 m/s is hit back at the same speed in a direction that makes an angle of 120° with its initial direction. Determine the magnitude of the impulse received by the ball, and the angle between the direction of the impulse and its initial direction.

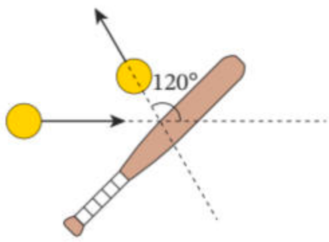


Figure 1-7-16 · A ball meeting the bat at 120°.

Exam-style question

A 0.50 kg ball moving at 8.0 m/s is brought to rest by a glove in 0.040 s. Determine the impulse exerted on the ball.

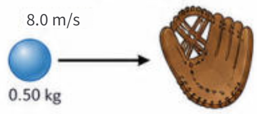


Figure 1-7-17 · A 0.50 kg ball caught at 8.0 m/s.

Lesson 1-8 — The Law of Conservation of Momentum

❓ **Lesson question:** When you step from a stationary small boat onto a dock, why does the boat move backward, and what determines the boat's speed?

You will learn to

By the end of this lesson, you will be able to:

1. **Explain** why the total momentum of an isolated system remains constant.
2. **Apply** conservation of momentum to calculate unknown velocities before and after a collision.
3. **Define** the system, choose an axis and state the positive direction when applying conservation of momentum.

🔍 The phenomenon — predict before you continue

Figure 1-8-1 · Stepping forward off the boat pushes the boat backward.

You finish a trip on a small boat and step forward onto the stone dock. As your foot moves forward, the boat slides backward, increasing the gap between the boat and the dock. Step gently and it moves away slowly, whereas a stronger push gives it a greater backward speed. No one else pushes the boat—yet it still moves backward.

Predict before you continue

(a) During the push-off, do you and the boat undergo equal or different changes in momentum magnitude?

(b) An empty boat and a boat loaded with crates receive the same impulse. Which boat recoils with the greater speed?

Key fact

The momentum of an object is $p = mv$.
The impulse exerted by the resultant force during a time interval equals the change in momentum.

Explore — Recoil when two students push apart

What you'll do: You will measure how far two students on wheeled chairs move after pushing apart, and investigate how the distance each moves depends on their mass.
Time: 20 min

You will need:

- Two wheeled lab chairs that move freely.
- A smooth, level floor with a marked start strip.
- Chalk or tape to mark where each chair stops.
- A heavier and a lighter student for the unequal-mass run.

Steps:

1. Two students sit facing each other on the chairs, palms pressed together, both at rest on the start strip.
2. On a count, both students push firmly once against each other's palms, then keep their hands still as the chairs roll apart. Mark where each chair stops.
3. Repeat with a heavier student facing a lighter one, from the same start strip.
4. Repeat once more, but this time only one student pushes while the other keeps their arms rigid. Mark the stopping points again.

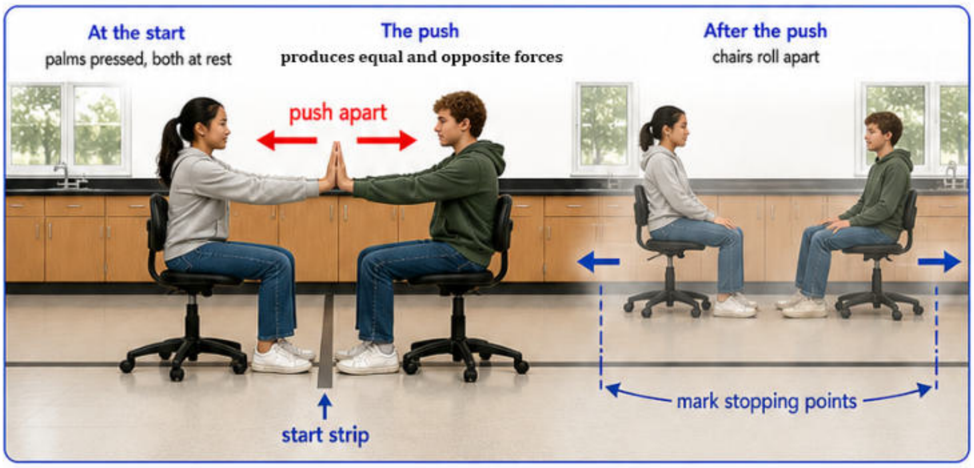


Figure 1-8-2 · Two students push apart on rolling chairs and separate.

Pairing	Which student moved further?
Similar masses	
Heavier vs lighter	
One-sided push	

Analyze what you collected:

(a) When the two students have different masses, which one moves further? How does the distance each moves depend on their mass?

(b) When only one student pushed, did only that chair move, or did both? What does this tell you about the forces during an interaction?

Write, in two lines, why both chairs move even when only one student pushes — before reading the next page.

measurement note

Floor friction varies from one floor position to another, so keep every run on the same marked strip and compare stopping distances only within a single pairing, never between different parts of the room.

From observation to model

In Explore, the chair carrying the lighter student traveled farther, so it left the push with the greater speed, and — notably — both chairs moved apart even when only one student pushed. One might expect that only the chair that is pushed should move, yet the observations show that both chairs move apart.

The two chairs do not act independently; they interact as a pair. Whenever one student pushes on the other, the other pushes back with a force equal in magnitude and opposite in direction, acting for exactly the same time. Each chair therefore receives an impulse equal in magnitude but opposite in direction, so each undergoes an equal and opposite change in momentum; because it has the smaller mass, the lighter chair moves away at the greater speed.

The following model accounts for these observations.

The model: conservation of momentum

A group of objects under consideration is called a system. The forces exerted on each other within the system are called internal forces, and the forces exerted by agents external to the system are called external forces.

The total momentum of the isolated system is the same before and after a collision. This principle is called the law of conservation of momentum.

When object A with mass m_1 and velocity v_1 collides with object B with mass m_2 and velocity v_2, and their respective velocities become v_1' ,and v_2',

$$\underbrace{m_1v_1 + m_2v_2}_{\text{total momentum before the collision}} = \underbrace{m_1v_1' + m_2v_2'}_{\text{total momentum after the collision}}$$

The law of conservation of momentum holds when the system receives no net external impulse.

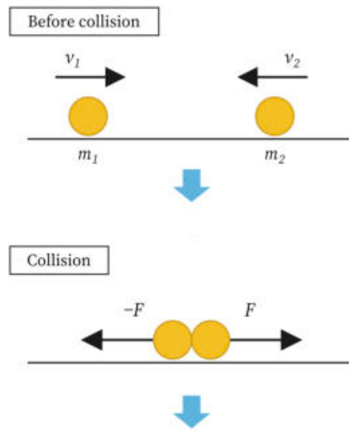
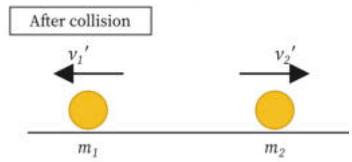


Figure 1-8-4 · In contact: equal and opposite forces.

Figure 1-8-5 · After impact: each mass has a new velocity.



Point

- A group of objects under consideration is called a system. The forces exerted on each other within the system are called internal forces, and the forces exerted by objects outside the system are called external forces.
- The total momentum of an isolated system does not change during a collision. When object A of mass m_1 and velocity v_1 collides with object B of mass m_2 and velocity v_2, and their velocities become v_1' and v_2': $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$.
- It holds when no impulse is received from external forces. It holds too if external forces such as gravity are received but can be ignored For motion in a plane, resolve the vectors into x and y directions.

Equation box

Conservation along an axis (two bodies)	$m_1v_1+m_2v_2=m_1v_1'+m_2v_2'$
Momentum of a body	$p=mv$

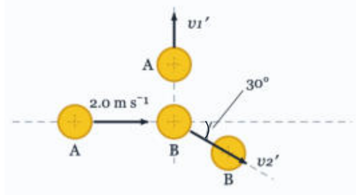
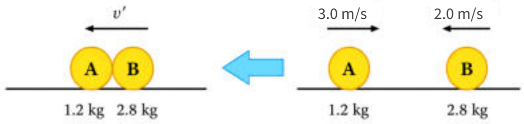
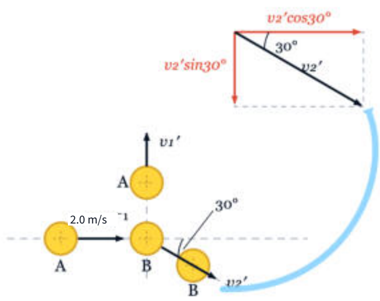
isolated system

A set of objects considered together as one system. The system is isolated during a time interval if net external impulse is zero. Its total momentum therefore remains constant.

Quick application

An object of mass 5 kg, initially at rest, explodes into two objects: a 3 kg object moves east at 4 m/s, while a 2 kg object moves west at 6 m/s. Show that momentum is conserved.

Worked example

Situation	**(a)** Object A of mass 1.2 kg moving in the positive direction at 3.0 m/s and object B of mass 2.8 kg moving in the negative direction at 2.0 m/s along a straight line collide and stick together. Determine the velocity after they stick together in m/s. **(b)** As shown in the figure below, on a smooth horizontal surface, object A of mass 0.20 kg moving at a speed of 2.0 m/s collides with object B of mass 0.80 kg that is at rest. Determine the final speeds of objects A and B after the collision. 
Given and required	**Given (a):** $m_A = 1.2$ kg, $v_A = +3.0$ m/s, $m_B = 2.8$ kg, $v_B = -2.0$ m/s. Required: final velocity v'. **Given (b):** $m_A = 0.20$ kg, $v_A = 2.0$ m/s, $m_B = 0.80$ kg, $v_B = 0$. Required: final speeds of A and B.
Representation	**(a)** One-dimensional motion with a chosen positive direction. **(b)** Two-dimensional motion resolved into perpendicular components; momentum is conserved independently in the x- and y-directions.
Strategy	Apply conservation of momentum. In **(a)**, both objects move together after collision. In **(b)**, resolve motion and use momentum conservation in perpendicular directions.
Annotated solution	**(a)** $1.2 \times (3.0) + 2.8 \times (-2.0) = (1.2 + 2.8)v'$ $3.6 - 5.6 = 4.0v' \rightarrow v' = -0.50$ m/s  Figure 1-8-7 · Two bodies meet head-on and move off together. **(b)** **(i)** x: $0.20 \times 2.0 + 0.80 \times 0 = 0.20 \times 0 + 0.80 \times v'_2\cos30°$ **(ii)** y: $0.20 \times 0 + 0.80 \times 0 = 0.20 \times v'_1 - 0.80 \times v'_2\sin30°$ Solving (i): $v'_2 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0.577\ldots \approx 0.58$ Substituting $v'_2 = \frac{1}{\sqrt{3}}$ into (ii): $v'_1 = \frac{2.0}{\sqrt{3}} = \frac{2.0\sqrt{3}}{3} = 1.15\ldots \approx 1.2$ A: 1.2 m/s; B: 0.58 m/s Figure 1-8-8 · Resolving the final velocity of B into components. 
Reasonableness check	**(a)** Final velocity is negative, consistent with object B having the larger momentum magnitude in the negative direction. **(b)** Both final speeds are less than the initial speed, which is consistent with momentum being shared between the two objects.
Explain it yourself	In part (a), why may you add the two momenta as signed numbers on one line instead of resolving into two axes?

Key fact

Choose the positive direction before applying conservation of momentum.
A negative velocity indicates motion opposite to the chosen positive direction.

Try

(a) Object A of mass 2.0 kg moving in the positive direction at 4.0 m/s and object B of mass 3.0 kg moving in the negative direction at 4.0 m/s on a straight line collide. When the velocity of object B after the collision is 2.0 m/s in the positive direction, determine the velocity of object A after the collision in m/s.

(b) An object of mass 8.0 kg at rest explodes into object A of mass 3.0 kg and object B of mass 5.0 kg. If object A after the explosion moves leftward at 10 m/s, determine the speed and direction of object B after the explosion.

(c) On a smooth horizontal surface, object A of mass 1.0 kg moving at a speed of 4.0 m/s collides with object B of mass 2.0 kg that is at rest. After the collision A is deflected 45° to one side of its original direction and B is deflected 45° to the other side. Determine the final velocity of objects A and B after the collision.

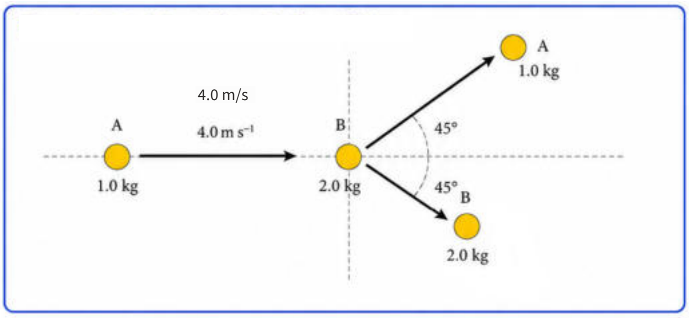


Figure 1-8-9 · A 1.0 kg body strikes a 2.0 kg body, and both leave at 45°.

In a new context — rocket propulsion

A rocket operates by expelling exhaust gases backward at high velocity. The gases gain backward momentum, while the rocket gains equal forward momentum.

For the rocket and exhaust system, if external impulse is negligible, total momentum is conserved: $\Delta p_{rocket} = -\Delta p_{exhaust}$

During launch, gravity acts downward, so the rocket accelerates upward only when its thrust is greater than its weight. (Air resistance is neglected.)

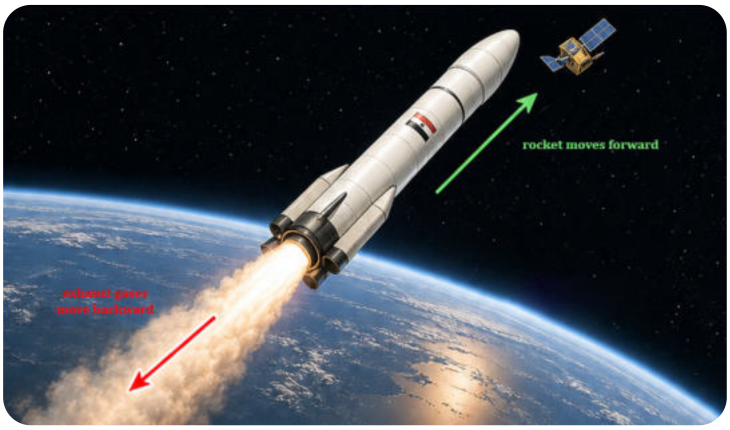


Figure 1-8-10 · A rocket moves forward by pushing gas backward.

Explain why a rocket can accelerate in the vacuum of space even though there is no external medium to push against.

Nature of Science — Evidence — Testing a Hypothesis Against Conservation Laws

In 1930, measurements of beta decay appeared to show that some energy and momentum were missing from the observed products. Rather than immediately abandoning the well-tested conservation laws, Wolfgang Pauli proposed that an undetected neutral particle carried away the missing energy and momentum.

This particle later became known as the neutrino. Pauli's proposal made a testable prediction: if the particle existed, experiments might eventually detect evidence of its interactions. In 1956, Frederick Reines and Clyde Cowan reported the experimental detection of the neutrino

This episode shows that established laws are not accepted without question. Scientists retain them while the evidence strongly supports them, but they also develop and test alternative explanations when new observations appear inconsistent with those laws.

Your turn: Why was proposing a testable hypothesis about a particle more scientifically useful than simply declaring that the conservation laws had failed?

Lesson question, answered

When you step from a stationary small boat onto a dock, why does the boat move backward, and what determines speed?

Treat the person and boat as one system. Initially, total momentum is zero. During the short push-off, external horizontal impulse is negligible, so momentum is conserved:

$$p_{person} + p_{boat} = 0$$

The person and boat gain equal and opposite momentum. Since p = mv, the lighter boat moves faster than a heavier one for the same momentum.

Figure 1-8-11 · The boat revisited.

Exam tip

State the positive direction and write $\Sigma p_{before} = \Sigma p_{after}$ for the axis before any calculation.

Exam-style question

A 1.5 kg trolley moving at 2.0 m/s collides head-on with a stationary 0.50 kg trolley on a smooth track and the two move together. Determine the final speed of the joined trolleys in m/s.

Exercise

(a) Object A of mass 2.0 kg moving in the positive direction at 3.0 m/s on a straight line collides with object B of mass 1.0 kg that is at rest, and they stick together. Determine the velocity after they stick together in m/s.

(b) Cart A of mass 2.0 kg moving at a speed of 4.5 m/s collides on a straight line with cart B of mass 5.0 kg moving at 0.40 m/s in the same direction. If cart B after the collision moves at a speed of 2.6 m/s in its original direction, determine the speed and direction of cart A after the collision.

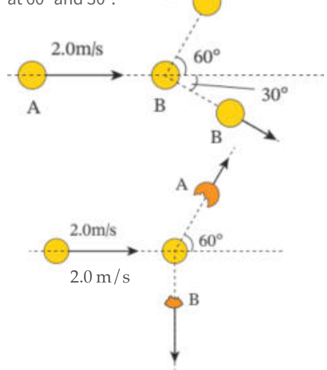
(c) An object of mass 7.0 kg at rest explodes into object A of mass 4.0 kg and object B of mass 3.0 kg. If object A after the explosion moves leftward at 9.0 m/s, determine the speed and direction of object B after the explosion.

(d) An object of mass m at rest explodes into object A and object B. If object A of mass $(\frac{2}{3})$m moves in the positive direction at a velocity of v, determine the speed and direction of object B after the explosion.

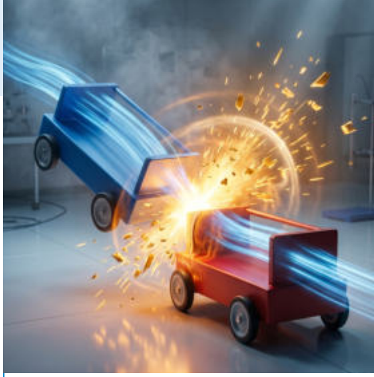
(e) On a smooth horizontal surface, object A of mass 0.20 kg moving at a speed of 2.0 m/s collides with object B of mass 0.60 kg that is at rest. After the collision A is deflected 60° to one side of its original direction and B is deflected 30° to the other side. Determine the final velocity of objects A and B after the collision.

(f) On a smooth horizontal surface, an object of mass 5.0 kg moving at a speed of 2.0 m/s splits into object A of mass 4.0 kg and object B of mass 1.0 kg. After the split A is deflected 60° from its original direction and B moves perpendicular to that direction, on the opposite side. Determine the velocity of objects A and B after the split.

Figure 1-8-12 · Tracks leaving the impact at 60° and 30°.



Lesson 1-9 — Momentum and Mechanical Energy



❓ **Lesson question:** Two trolleys collide, stick together and move more slowly. If their total momentum is conserved, why does their total kinetic energy decrease, and to what forms of energy is it transferred?

You will learn to

By the end of this lesson, you will be able to:

1. **State** when total momentum is conserved and distinguish between elastic and inelastic collisions.
2. **Calculate** the decrease in total kinetic energy during an inelastic collision.
3. **Explain** how kinetic energy is transferred to other forms during an inelastic collision.

🔍 The phenomenon — predict before you continue

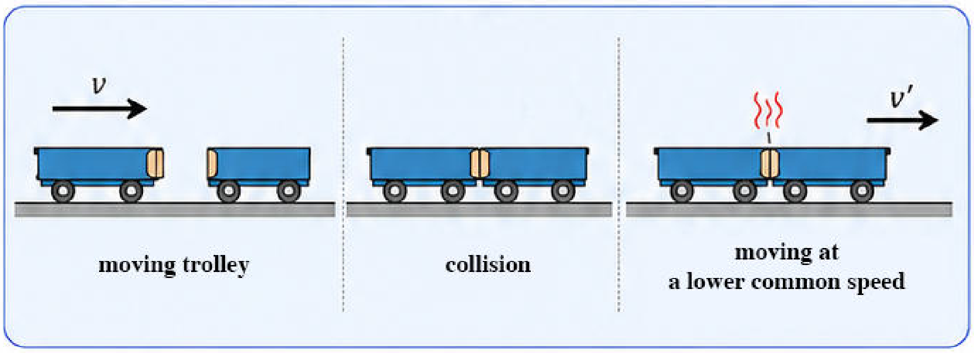


Figure 1-9-1 · The trolleys join and move on at a lower common speed.

A moving laboratory trolley collides with a second trolley that is initially at rest. Adhesive pads on their ends cause the trolleys to stick together and move with a common velocity after the collision. Their common speed is lower than the initial speed of the moving trolley. A short sound may be heard during the collision, and the trolleys and pads may undergo slight deformation. These observations suggest that some kinetic energy is transferred to other forms of energy.

Compare the motion before and after the collision.

Predict before you continue

(a) After they stick together, is the total momentum smaller, the same, or larger than before?

(b) What happens to the total kinetic energy — does it become smaller, stay the same, or become larger?

⭐ **Key fact**

For an isolated system:
$\sum \vec{p}_{before} = \sum \vec{p}_{after}$
For each object:
$E_k = \frac{1}{2}mv^2$

Explore — Momentum and kinetic energy in an inelastic collision

What you'll do: record a slow-motion video of two trolleys colliding, then compare their motion before and after the collision.

Time: 12 min

You will need:

- A phone with slow-motion video.
- Two lab trolleys with a hook-and-loop (adhesive) pad.
- A tape measure placed alongside the track for scale.

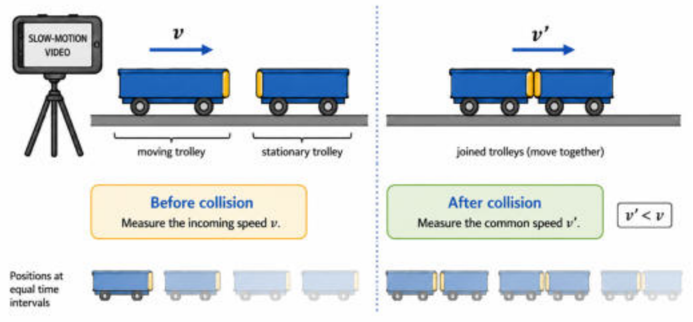


Figure 1-9-2 · Filming the trolleys to measure the speed before and after.

Steps:

1. Measure and record the mass of each trolley. Place trolley B at rest on a level, low-friction track.
2. Release trolley A so that it collides with trolley B. Hook-and-loop pads on the facing ends should cause the trolleys to stick together. Record the entire collision using slow-motion video.
3. Place a measuring tape parallel to the track and close to the path of the trolleys. Use the video frame rate and the distance traveled over several frames to determine: the speed of trolley A immediately before the collision; the common speed of the joined trolleys immediately after the collision.
4. Note whether sound, vibration or visible deformation occurs during the collision.
5. Add a mass securely to trolley B and repeat the procedure. Keep the initial speed of trolley A as nearly the same as possible.

Trial	Mass of trolley A (kg)	Mass of trolley B (kg)	Speed before collision, v (m/s)	Common speed after collision, v' (m/s)	Sound or deformation observed?
Lighter target trolley					
Heavier target trolley					

Analyze what you collected:

(a) For each trial, calculate the total momentum and total kinetic energy before and after the collision. Is each total approximately conserved?

(b) Explain how the observations of sound, vibration or deformation provide evidence that some kinetic energy was transferred to other forms of energy.

Write your explanation in two lines before you read the next page.

A measurement note

At a constant frame rate, equal numbers of frame intervals represent equal time intervals. Measure both speeds over the same number of frame intervals, immediately before and after the collision, so that the comparison is based on equal time intervals.

From observation to model

In Explore, the two trolleys moved together with a common speed lower than the initial speed of the moving trolley. Using the measured masses and speeds shows a more precise result: the total momentum before and after the collision was approximately the same, while the total kinetic energy after the collision was smaller.
The kinetic energy was not destroyed. Some of it was transferred to other forms of energy, including internal energy associated with deformation and vibration, and sound.

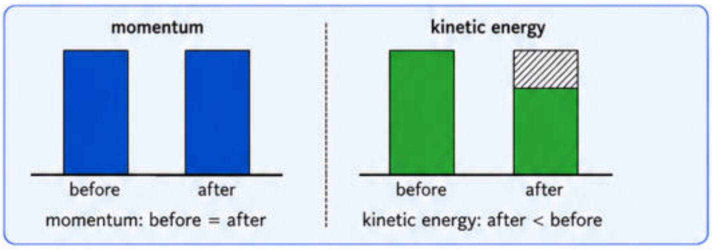


Figure 1-9-3 · Momentum is unchanged, but kinetic energy falls.

Point

- The sum of a system's kinetic energy and potential energy is called mechanical energy. When an object of mass m [kg] moves at a speed v [m/s], its kinetic energy E_k [J] is $E_k = ½mv^2$.
- When objects collide, their relative speed after the collision is less than or equal to their relative speed before it, so for ordinary (non-explosive) collisions the coefficient of restitution e — the ratio of the relative speed after the collision to the relative speed before it — satisfies $0 \leq e \leq 1$. When $e = 1$ it is called a perfectly elastic collision; when $0 \leq e < 1$ an inelastic collision; and the inelastic collision with $e = 0$ is called a perfectly inelastic collision. The bodies become one body after the collision..
- In an elastic collision ($e = 1$) mechanical energy is conserved. In an inelastic collision ($0 \leq e < 1$) mechanical energy decreases: some of it is transferred to internal energy associated with deformation and vibration, and to sound.

Equation box

Kinetic energy of a body	$E_k = \frac{1}{2}mv^2$
Conservation of momentum (isolated system)	$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$
Kinetic energy lost	$\Sigma E_{k,before} - \Sigma E_{k,after}$

The perfectly inelastic case
In the collision, the total kinetic energy of the two trolleys decreased. Mechanical energy of a system is defined as the sum of its kinetic energy and potential energy.

Their gravitational potential energy remained constant. Therefore, the decrease in total kinetic energy corresponds to a decrease in the mechanical energy of the system.

Mechanical energy is not conserved in this inelastic collision. Some of the mechanical energy is transferred to internal energy associated with deformation and vibration, and to sound.

When objects collide, their relative speed after the collision is less than or equal to their relative speed before the collision, so for ordinary (non-explosive) collisions the coefficient of restitution e satisfies $0 \leq e \leq 1$.

When e = 1, the collision is called a perfectly **elastic collision**.

When $0 \leq e < 1$, it is called an **inelastic collision**.

Among inelastic collisions, a collision where e = 0 is specifically called a perfectly

Key fact:

In an inelastic collision, kinetic energy is not conserved. In an elastic collision, kinetic energy is conserved.

Quick application

A single trolley of mass 2.0 kg moves at 4.0 m/s.

Calculate its kinetic energy in J.

inelastic collision; the objects move together after the collision.

- In an elastic collision ($e = 1$), mechanical energy is conserved.
- In an inelastic collision ($0 \le e < 1$), mechanical energy decreases.

When two objects collide, the ratio of the magnitude of the relative speed after the collision to that before the collision is called the coefficient of restitution. The coefficient of restitution depends on the materials of the colliding objects.

Letting the velocities of the two objects before the collision be v1 and v_2, the velocities after the collision be v_1' and v_2', and the coefficient of restitution be e, $v_1' - v_2' = -e(v_1 - v_2)$

Worked example

Situation	A small ball A of mass 1.0 kg moves to the right at 4.0 m/s and collides head-on with a stationary small ball B of mass 1.5 kg. The coefficient of restitution is 0.25. Determine the following: **(a)** Determine the final velocities of the two balls after the collision in m/s. **(b)** Determine the mechanical energy lost in the collision in J. Take the rightward direction as positive.
Given and required	**Given:** $m_A = 1.0$ kg, $v_A = +4.0$ m/s, $m_B = 1.5$ kg, $v_B = 0$ m/s, $e = 0.25$. **Required:** **(a)** Final velocities v'_A and v'_B **(b)** Mechanical energy lost during the collision.
Representation	Let the velocities of small balls A and B after the collision be v'_A and v'_B
Strategy	Use conservation of momentum together with the coefficient of restitution equation to determine the final velocities. Then calculate the total kinetic energy before and after the collision and find the difference.
Annotated solution	**(a) Final velocities**. From the law of conservation of momentum, $1.0 \times 4.0 + 1.5 \times 0 = 1.0 \times v'_A + 1.5 \times v'_B$ $v'_A + 1.5\, v'_B = 4.0$.............. (1) From the coefficient of restitution formula, $v'_A - v'_B = -0.25(4.0 - 0)$ $v'_B - v'_A = 1.0$.............. (2) Solving (1) and (2): $v'_A = 1.0$ m/s, $v'_B = 2.0$ m/s **(b) the mechanical energy lost during the collision:** The sum of kinetic energy before the collision is $\frac{1}{2} \times 1.0 \times 4.0^2 + \frac{1}{2} \times 1.5 \times 0^2 = 8.0$ J The sum of kinetic energy after the collision is $\frac{1}{2} \times 1.0 \times 1.0^2 + \frac{1}{2} \times 1.5 \times 2.0^2 = 3.5$ J The loss in kinetic energy is 8.0 J − 3.5 J = 4.5 J
Reasonableness check	Since ($e = 0.25 < 1$), the collision is inelastic, so the mechanical energy is not conserved. The total kinetic energy decreases − consistent with the 4.5 J loss.
Explain it yourself	Explain why momentum is conserved during the collision even though mechanical energy is not.

Now you try: A small ball A of mass 4.0 kg moves to the right at 6.0 m/s and collides head-on with a stationary small ball B of mass 3.0 kg. After the collision, the two balls stick together and move as a single object.

Determine the final velocity, then **calculate** the kinetic energy lost in J.

Try

(a) On a straight line, a small ball A of mass 2.0 kg moving to the right at 4.0 m/s and a small ball B of mass 1.0 kg moving to the left at 6.0 m/s collide head-on. The coefficient of restitution between small ball A and small ball B is 0.50. Take the rightward direction as positive.

(i) Determine the velocity of each ball after the collision, in m/s.

(ii) Determine the kinetic energy lost in the collision in J.

(b) A small ball A of mass 2.0 kg moving to the right at 4.0 m/s collides head-on with a small ball B of mass 3.0 kg that is at rest, and they stick together. Take the rightward direction as positive.

Figure 1-9-5 · A moving body strikes one at rest, and the two join.

(i) Determine the final velocity of the balls in m/s.

(ii) Determine the kinetic energy lost in the collision in J.

In a new context — the crumple zone

A vehicle collision is largely inelastic. During the impact, the crumple zone deforms, so the vehicle rebounds less and its kinetic energy decreases. The decrease in kinetic energy is transformed mainly into internal energy through permanent deformation, internal vibrations, and heating, while a smaller amount is transferred to the surroundings as sound. The deformation also increases the time over which the vehicle's momentum changes, reducing the average force acting on the driver and passengers.

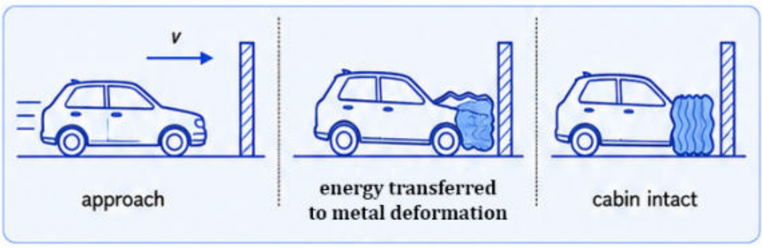


Figure 1-9-6 · A crumple zone absorbs energy while the cabin stays intact.

Your turn: Explain why a car designed to crumple in a crash protects its passengers better than one built to remain rigid.

Nature of Science: Evidence — where the "lost" kinetic energy goes

In 1850, James Joule published his paddle-wheel experiment in the Philosophical Transactions. Falling weights turned a paddle that stirred water, and the mechanical work produced a measurable rise in the water's temperature. This was evidence that the work was not lost but converted into internal (thermal) energy. The kinetic energy "lost" in an inelastic collision changes in the same way: it becomes internal energy, which may be measured as a temperature rise.

Key fact

In part (a), the coefficient of restitution is given rather than the final velocities, so the restitution equation and conservation of momentum must be solved together.
In part (b), the objects move together after the collision and their common velocity is unknown. Use conservation of momentum to calculate this velocity first.

Your turn: suggest why a measured temperature rise is stronger evidence than simply stating that the energy is lost.

Lesson question, answered

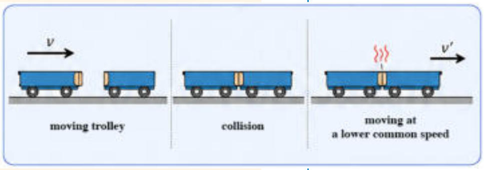


Figure 1-9-7 · The trolleys revisited.

Two trolleys collide, stick together and move more slowly. If their total momentum is conserved, why does their total kinetic energy decrease, and to what forms of energy is it transferred? The total momentum is conserved during the collision. However, the decrease in the total kinetic energy of the colliding bodies is converted mainly into internal energy through deformation, internal vibrations, and heating, while a smaller fraction is carried away by sound waves.

Exercise

(a) A small ball A of mass 2.0 kg moving to the right at 8.0 m/s collides on a straight line with a small ball B of mass 1.0 kg moving to the right at 2.0 m/s. After the collision A moves at 5.0 m/s and B moves at 8.0 m/s, both to the right. Take the rightward direction as positive.

(i) Show that the momentum is conserved in the collision.

(ii) Determine the kinetic energy lost in the collision in *J*.

(b) A small ball A of mass 0.10 kg moving to the right at 3.0 m/s collides head-on with a small ball B of mass 0.20 kg moving to the left at 2.0 m/s. After the collision A moves at 1.0 m/s to the left and B is at rest. Take the rightward direction as positive.

(i) Show that the momentum is conserved in the collision.

(ii) Determine the kinetic energy lost in the collision in *J*.

(c) A small ball A of mass 1.0 kg moving to the right at 4.0 m/s collides head-on with a small ball B of mass 1.5 kg that is at rest, and they stick together. Take the rightward direction as positive.

(i) Determine the velocity of the combined balls in m/s.

(ii) Determine the kinetic energy lost in the collision in *J*.

(d) A small ball A of mass 0.20 kg moving to the right at 10 m/s collides head-on with a small ball B of mass 0.30 kg moving to the left at 5.0 m/s, and they stick together. Take the rightward direction as positive.

(i) Determine the velocity of the combined balls in m/s.

(ii) Determine the kinetic energy lost in the collision in *J*.

Exam-style question

A trolley A of mass 1.0 kg moving at 4.0 m/s collides with a stationary trolley B of mass 1.5 kg, and the two stick together.

(a) Determine the final speed of the combined trolleys.

(b) Calculate the kinetic energy lost in the collision.

Exam tip

Write the conservation of momentum equation in a single line.
Use a table method to calculate the total kinetic energy before and after the collision.

Lesson 1-10 — Uniform Circular Motion and Centripetal Force

❓ **Lesson question**: A ball is whirled in a circle on the end of a string. If the string is cut, in which direction does the ball move?

You will learn to

By the end of this lesson, you will be able to:

1. **Calculate** the period (T), frequency (f), angular velocity (ω) and linear speed (v) of an object in uniform circular motion.
2. **Determine** the centripetal acceleration (a_c) of the object and identify the force that provides it.
3. **Explain** why no outward (centrifugal) force acts on an object moving in a circle.

🔍 The phenomenon — predict before you continue

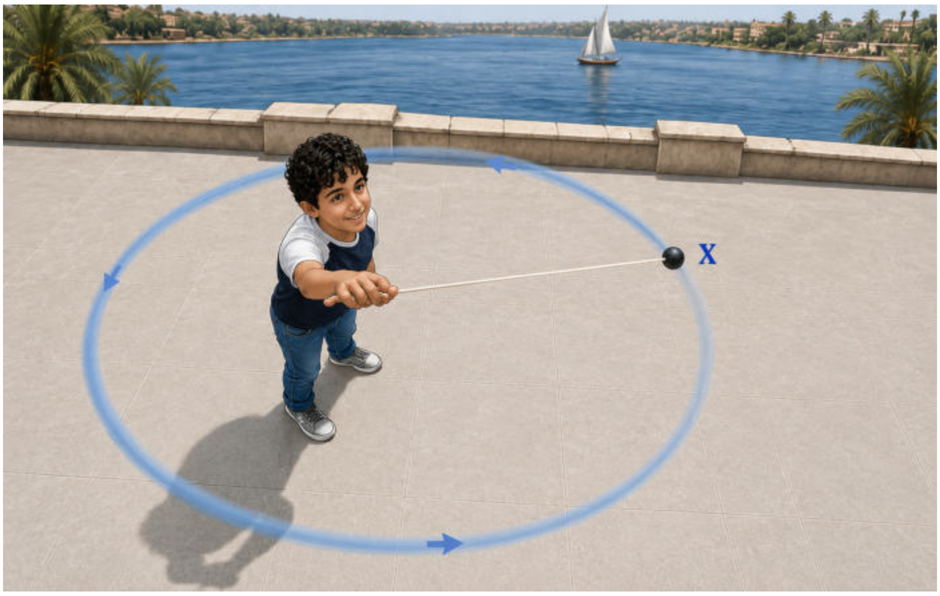


Figure 1-10-1 · A stone whirled on a string moves in a circle at steady speed.

Whirl a ball on a string in a horizontal circle above your head. The string stays taut, and your hand feels a constant inward pull for as long as the ball keeps moving in the circle. Now suppose the string is cut at the instant the ball is at point X. With no force from the string, the ball is free. Predict the path the ball follows after point X.

Predict before you continue

(a) When the string is cut at X, does the ball move: (i) straight outward along the line of the string point, (ii) along the tangent to the circle, or (iii) follow a gradually curving path?

(b) While the ball moves around the circle at constant speed, is there any outward force acting on it? Explain your answer.

⭐ Key fact

Velocity changes when its direction changes, not only when its magnitude changes. So an object moving in a circle at constant speed is still accelerating.
From the data booklet, the equations for uniform circular motion are:
$v = 2\pi r / T = \omega r$
$a_c = v^2 / r = \omega^2 r$

Explore — Testing the equation for centripetal force

What you'll do: You will release an object moving in a circle and find the direction it then travels.

Time: 15 min

You will need:

• Rubber bung tied securely to about 0.5 m of strong string.

• Safety goggles, worn by everyone in the area

• A large, clear space — ideally outdoors or a sports hall

• Chalk or floor tape to mark a grid: a circle, its center,

and the 12, 3, 6 and 9 o'clock points

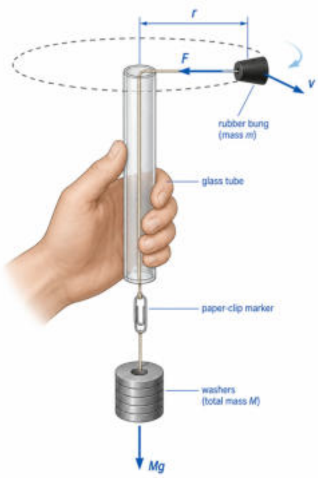


Figure 1-10-2 · The bung is whirled while the hanging washers provide the pull.

Steps:

1. Put on safety goggles and check that the space around you is completely clear. If the floor grid is not already marked, chalk a circle with its center and the 12, 3, 6 and 9 o'clock points.
2. Loop the free end of the string over one finger and whirl the bung gently in a horizontal circle above the grid. Feel how the string tugs your finger, and note the direction of the tug.
3. Keeping the circle steady and slow, choose a release point on the grid (for example, the 3 o'clock mark). The instant the bung passes that point, open your finger to let the string go — releasing only toward clear space.
4. Watch which way the bung first travels: straight outward along the line from the center, or off along the tangent (at right angles to the string). Record the release point and the flight direction.
5. Bring the bung to rest, then repeat for five different release points around the circle, keeping the speed roughly the same each time.
6. Compare your five runs and decide whether the flight direction follows a consistent pattern.

Release point on the grid	Flight direction (outward from the center / along the tangent)

Also record: while the bung circles, which way does the string tug your finger?

Analyze what you collected:

(a) Across your five releases, what is the pattern — does the bung move outward from the center, or off along the tangent?

(b) While it circled, what did your finger experience — a push outward or a pull inward — and which object pulled which?

Write, in two lines, why the string must pull harder on your finger when the bung moves faster at the same radius — before reading the next page.

Safety note

Whirl gently in a clear space, well away from other people, windows and lamps. Use a rubber bung — never a hard object — and check the knot and string are secure before each run. Wear safety goggles.

From observation to model

In Explore, the string pulled the bung steadily inward, toward the center of the circle — never outward. When the bung was released, the tension disappeared, and the bung moved in a straight line tangent to the circle at the point of release. You might have expected the bung to move outward along the radius; your observations show otherwise. Here is the model your evidence supports.

Circular motion

When an object moves in a circle at a constant speed, the motion is called uniform circular motion.

The angular velocity ω is the angular displacement per unit time, in radians per second (rad/s). If the object turns through an angle θ (rad) in a time t (s),

$$\omega = \frac{\theta}{t}$$

The period T is the time for one revolution, and the frequency f is the number of revolutions per second (in Hz):

$$T = \frac{2\pi}{\omega}, \quad f = \frac{1}{T}$$

For a circle of radius r, the arc length l covered as the object turns through an angle θ is $l = r\theta$, so the linear speed is

$$v = \frac{l}{t} = \frac{r\theta}{t} = r\omega$$

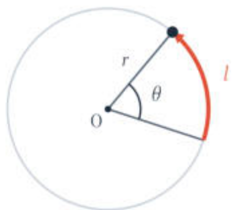


Figure 1-10-3 · The angle in radians is the arc length divided by the radius.

Although the speed is constant, the velocity continuously changes direction, so the object is accelerating. In a short time interval Δt the velocity vector keeps its magnitude v but rotates through the same small angle $\Delta\theta = \omega\Delta t$, the angle through which the object moves. The resulting change in velocity has magnitude $\Delta v = v\Delta\theta$ anc is directed toward the center of the circle. Dividing by Δt gives the acceleration, directe toward the center — the centripetal acceleration:

$$a_c = \frac{\Delta v}{\Delta t} = v\omega = r\omega^2 = \frac{v^2}{r}$$

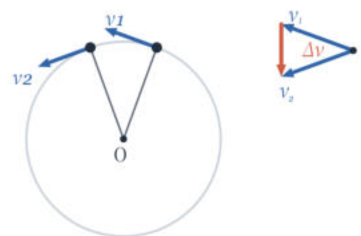


Figure 1-10-4 · The change in velocity points toward the center.

By Newton's second law, this acceleration requires a resultant force in the same direction — the centripetal force:

Key fact

The centripetal force is the resultant force on the object, always directed toward the center of the circle. It is not an additional outward force — there is no 'centrifugal force' on the object itself. Remove the inward force and the object continues in a straight line along the tangent.

Point

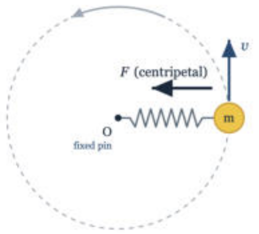
- ! When an object moves in a circle at a constant speed, this motion is called uniform circular motion. The angle through which it rotates per unit time is called the angular velocity: if it rotates through an angle θ [rad] in a time t [s], then $\omega = \frac{\theta}{t}$, in radians per second (rad/s).
- ! Period and frequency The time T [s] for one revolution is called the period: $T = \frac{2\pi}{\omega}$. The number of revolutions per unit time is the frequency f [Hz]: $f = \frac{1}{T}$. On a circle of radius r [m] the distance moved is $l = r\theta$, so the speed is $v = r\omega$
- ! The direction of motion changes even when the magnitude of the velocity does not, so the object has an acceleration $a = r\omega^2 = \frac{v^2}{r}$, always directed toward the center of the circle. The resultant force that produces it is called the centripetal force, and the equation of motion toward the center is $F = m\frac{v^2}{r}$.

It is the equation $F = ma$ written toward the center.

Equation box

Angular velocity ω	$\omega = \frac{v}{r}$
Centripetal acceleration a	$a = r\omega^2 = \frac{v^2}{r}$
Centripetal force F	$F = \frac{mv^2}{r}$

Worked example

Situation	A 0.50 kg ball is attached to a spring and moves in uniform circular motion on a smooth horizontal surface. The spring is stretched from its natural length, and its elastic force provides the centripetal force required for the circular motion.
Given and required	Given: mass m = 0.50 kg, natural length L_0 = 0.15 m. final length L = 0.25 m, spring constant k = 20 N/m Required: (a) the elastic (spring) force (b) the speed (c) the centripetal acceleration
Representation	Show the forces acting on the ball, then find the resultant radial force directed toward the center — this resultant is the centripetal force.  Top view — uniform circular motion on a smooth horizontal surface
Strategy	1. Find the magnitude of the elastic force from $F = kx$, where x is the extension. 2. Use the centripetal force equation $F = \frac{mv^2}{r}$ to find v, with the radius r equal to the stretched length. 3. Use $a_c = \frac{v^2}{r}$ to find the centripetal acceleration.
Annotated solution	(a) Spring extension and magnitude of the force $x = L - L_0 = 0.25 - 0.15 = 0.10\ \text{m}$ $F = kx = 20 \times 0.10 = 2.0\ \text{N}$ $\mathbf{F = 2.0\ N}$ (b) Speed from the centripetal force (r = 0.25 m) $\frac{mv^2}{r} = F \Rightarrow \frac{0.50v^2}{0.25} = 2.0$ $v^2 = 1.0 \Rightarrow v = 1.0\ \text{m/s}$ $\mathbf{v = 1.0\ m/s}$ (c) Centripetal acceleration $a_c = \frac{v^2}{r} = \frac{1.0}{0.25} = 4.0\text{ms}^{-2}$ a_c **= 4.0 m/s²**
Reasonableness check	a_c = 4.0 m/s² is consistent with $a = \frac{F}{m} = (\frac{2.0}{0.5})$ and is about 4.0 m/s² Thus, the spring force provides exactly the required centripetal force.
Explain it yourself	Why is the elastic force exerted by the spring the only force that contributes to the resultant force toward the center in this situation?

Quick application

A ball moves in a horizontal circle of radius 0.20 m with a period of 0.50 s. Calculate its linear speed.

Now you try: An object is undergoing uniform circular motion, making 20 revolutions in 5.0 s in a circle of radius 0.20 m. Determine the following

(a) Period **(b)** Frequency **(c)** Angular velocity **(d)** Speed.

Try

(a) An object undergoes uniform circular motion, making 10 revolutions in 2.0 s on a circle of radius 0.40 m. Determine its period in s.

(b) For the same motion, determine its angular velocity in rad/s and its speed in m/s.

(c) A small ball of mass 0.50 kg attached to a thread of length 2.0 m undergoes uniform circular motion on a smooth horizontal surface with an angular velocity of 3.0 rad/s. Determine its centripetal acceleration.

(d) A small ball of mass 8.0×10^{-2} kg is attached to one end of a spring with a natural length of 0.13 m and a spring constant of 60 N/m, and is made to undergo uniform circular motion on a smooth horizontal surface around the other end of the spring, which results in a spring length of 0.15 m.

Determine the following to 2 significant figures.

(i) Determine the magnitude of the elastic force of the spring in N.

(ii) Determine the speed of the small ball in m/s.

(iii) Determine the angular velocity of the small ball in rad/s.

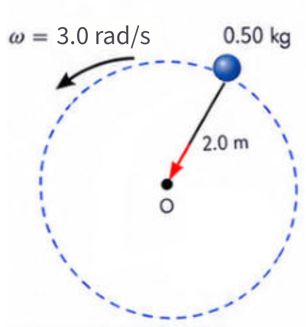


Figure 1-10-6 · A 0.50 kg body at 2.0 m turning at 3.0 rad/s.

In a new context — the washing-machine spin cycle

During the spin cycle, the wet clothes move with the rotating drum in a circular path. The normal force exerted by the inner surface of the drum provides most of the inward resultant force required for this motion.

At a perforation in the drum, there is no solid wall to exert the required inward contact force on the water. The water therefore continues with its instantaneous tangential velocity. Because the drum curves away from this straight-line path, the water passes through the perforation and is collected by the outer tub.

The water leaves the drum because it is no longer constrained to follow its circular path.

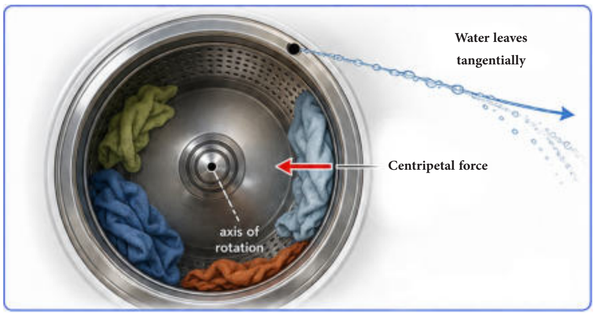


Figure 1-10-7 · Water leaves the drum once the wall stops pulling it inward.

Linking question

Why does the inward resultant force do no work on a body moving at constant speed in a circle?

Your turn: Why does water pass through the holes in the spinning drum while the clothes remain pressed against the inner surface?

Nature of Science: Theories — a relationship before a broader framework

In 1659, Christiaan Huygens derived a relationship for uniform circular motion. Expressed in modern notation, his result is equivalent in magnitude to $F_c = mv^2/r$.

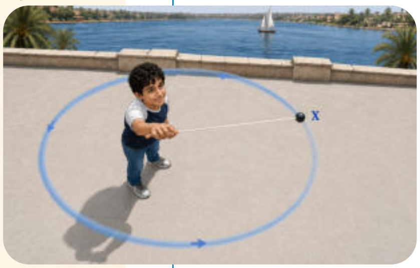
Huygens obtained this relationship geometrically, before Newton's laws provided a broader framework linking resultant force to acceleration. Newton's theory of universal gravitation later showed that gravity provides the inward force that keeps planets in orbit.

Your turn: Give one quantity that Huygens's relationship could calculate, and one question it could not answer.

Lesson question, answered

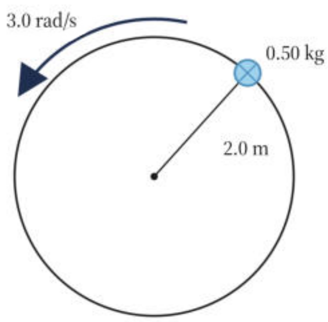
A ball is whirled in a circle on the end of a string. If the string is cut, in which direction does the ball move?

It moves along the tangent to the circle at the release point — the direction it was already moving — not outward, away from the center. While it moved in the circle, the string's inward tension provided the centripetal force that kept the object on its circular path. In an inertial frame, no outward (centrifugal) force acts on the object; the outward pull felt by your hand is the force exerted by the string on your hand. When the string is cut, the inward tension disappears, so no inward force remains to maintain the circular motion, and the object initially moves along the tangent.

Figure 1-10-8 · The whirled stone revisited.


Exercise

(a) A small ball of mass 0.50 kg was attached to one end of a thread with a length of 2.0 m and made to undergo uniform circular motion on a smooth horizontal surface around the other end of the thread, and the angular velocity became 3.0 rad/s.
(i) Determine the speed of the small ball in m/s.
(ii) Determine the acceleration of the small ball in m/s^2.
(iii) Determine the magnitude of the force required to maintain the circular motion, in N.

Figure 1-10-9 · A 0.50 kg body on a 2.0 m radius.


(b) As shown in Figure 1-10-10, an object of mass 3.0 kg moves in a circle of radius 2.0 m at a constant speed, making 1 revolution in 4.0 s. Assume π = 3.14.

(i) Determine the angular velocity of the object in circular motion in rad/s.

(ii) Determine the speed of the object in circular motion in m/s.

(iii) Determine the magnitude of the acceleration in m/s^2. Also, at point P in the figure, which direction from A to D is the direction of the acceleration? Answer with the letter.

(iv) Determine the magnitude of the centripetal force required for this circular motion in N.

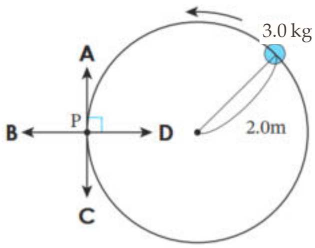


Figure 1-10-10 · A 3.0 kg body at P: in which direction does the force act?

(c) A small ball of mass 0.50 kg is attached to one end of a spring with a natural length of 0.10 m and a spring constant of 30 N/m, and is made to undergo uniform circular motion on a smooth horizontal surface around the other end of the spring. If the resulting angular velocity is 6.0 rad/s, determine the extension of the spring in m.

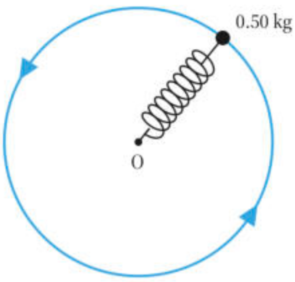


Figure 1-10-11 · A 0.50 kg body held by a spring.

Key fact

Before solving a circular-motion problem, first identify the force acting toward the center of the circle.

Lesson 1-11 — Circular Motion: Horizontal and Vertical

❓ **Lesson question**: Why can water remain inside a bucket at the top of a vertical circle?

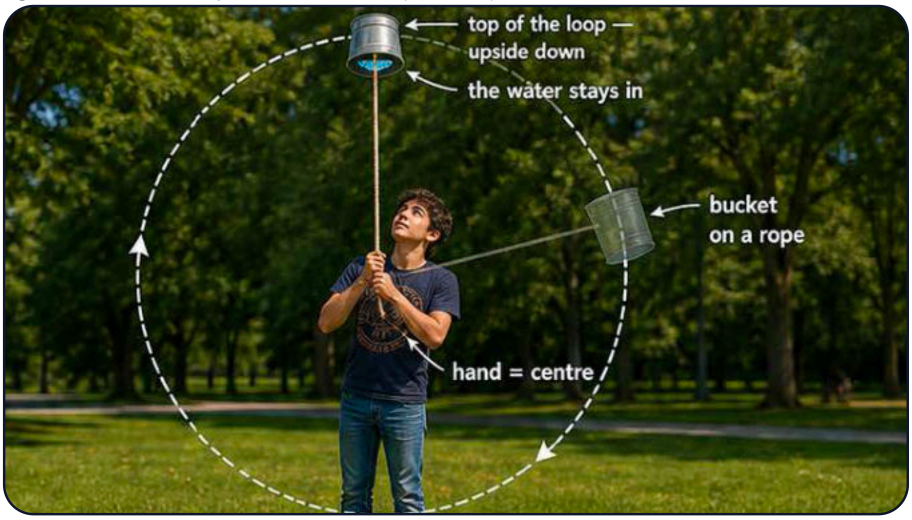
You will learn to

By the end of this lesson, you will be able to:

1. **Identify** the forces in horizontal and vertical circular motion.
2. **Analyze** horizontal and vertical circular motion.
3. **Determine** the tension and minimum speed in vertical circular motion.

🔍 The phenomenon — predict before you continue

Figure 1-11-1 · The water stays in the bucket at the top of the loop.



Tie a small bucket containing water to a rope and move it rapidly in a vertical circle above your head. At the highest point of the circle, the bucket is upside down, yet the water remains in the bucket and does not spill out. Now gradually reduce the speed of the bucket. Below a certain speed, the water begins to spill out near the top of the circle. Before reading on, imagine the bucket at the highest point of its motion and decide what keeps the water in the bucket.

Predict before you continue

(a) At the highest point, what keeps the water in the bucket: the pull of the rope, the push of the bucket, gravity, or nothing at all?

(b) Is there a minimum speed at the highest point below which the water spills out? If so, what determines this minimum speed?

🔍 Explore — Moving a bucket of water in a vertical circle

What you'll do: You will find how fast a bucket of water must be moved in a vertical circle to keep the water in, and where the water first leaves the bucket when the speed is too slow.

Time: 30 min

You will need:

- A small bucket with a handle, or a plastic cup tied securely with strong string.
- Water, enough to part-fill the bucket.
- A phone or camera with a slow-motion setting.
- A clear, open space outdoors.

Steps:

1. Part-fill the bucket with water. Hold the handle and move the bucket in a vertical circle at a steady speed at your side, fast enough that the water stays in throughout the full circle.
2. Ask a partner to film the circular motion in slow motion, with the whole circle in view.
3. Gradually reduce the speed, on each revolution, until the water first begins to leave the bucket.
4. From the slow-motion clip, identify where on the circle the water first leaves — at the bottom, at the side, or near the top.
5. Compare the bucket's speed at the top in the trial where the water first spills with its speed there in the faster trial that keeps the water in.

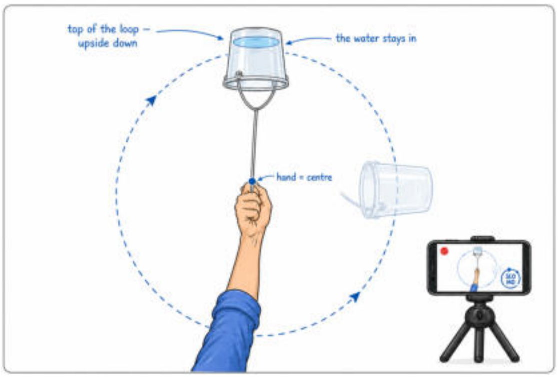


Figure 1-11-2 · Filming the bucket to see where the water stays in.

trial speed	Water spilled? (yes / no)	Where the water first left the bucket
Fast		
Medium		
Slow		

Analyze what you collected:

(a) At the first spill, where on the circle does the water begin to leave the bucket — at the bottom, at the side, or near the top?

(b) Explain why the bucket must be moved fast enough for the water to stay in at the top. What provides the force that keeps the water moving in a circle there, and what happens when the rotation is too slow?

Write, in two lines, what must be true about the speed at the top for the water to stay in — before reading the next page.

⭐ A safety note

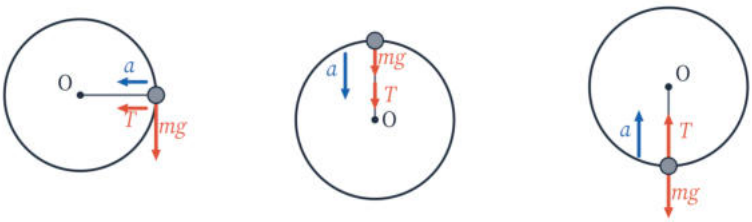
Carry out the activity in a clear, open space outdoors, well away from other people and from windows. Use only a small amount of water, and check that the handle or string and its attachment are secure before each trial.

From observation to model

In Explore, the water remained inside the bucket throughout its motion in a vertical circle when the bucket's speed was sufficiently high. As the speed was gradually reduced, the water first began to leave the bucket near the top of the circle. At first, it might seem that an upward force is needed to keep the water in the bucket at the top. The slow-motion video shows otherwise: the required resultant force is directed toward the center of the circle, which at the top is vertically downward. The evidence from the experiment supports the model of vertical circular motion, in which the speed changes around the circle.

Vertical circle

Consider an object of mass m, attached to a string of length r, being moved in a vertical circle. Its speed now changes around the circle — greatest at the bottom, least at the top. The figure shows the object at three positions and how the tension is found at each, where v is its speed at that position. The tension is least at the top and greatest at the bottom.



$$T = \frac{mv^2}{r}$$

$$T + mg = \frac{mv^2}{r} \qquad \therefore\ T = \frac{mv^2}{r} - mg$$

$$T - mg = \frac{mv^2}{r} \qquad \therefore\ T = \frac{mv^2}{r} + mg$$

Figure 1-11-3 · Tension at the side, at the top, and at the bottom of the loop.

Minimum speed at the top

At the top of the circle, both the tension and the weight act toward the center, so

$$\frac{mv^2}{r} = mg + T$$

A string can only pull, so its tension cannot be negative; the limiting case is when it just falls to zero. Setting $T = 0$:

$$\frac{mv^2}{r} = mg$$

which gives

$$v_{\min} = \sqrt{gr}$$

This is the minimum speed needed so that the object just maintains circular motion without the string going slack at the top of the circle.

Exam tip

Make sure you understand the diagrams and the method used to calculate the tension in the string when an object is moved in a vertical circle.

A conical pendulum

Consider an object of mass m attached to a string PQ of length l. P is fixed to a support, and the object moves in a horizontal circle of radius r, as shown below. When the object moves at constant speed, the string makes a constant angle θ with the vertical.

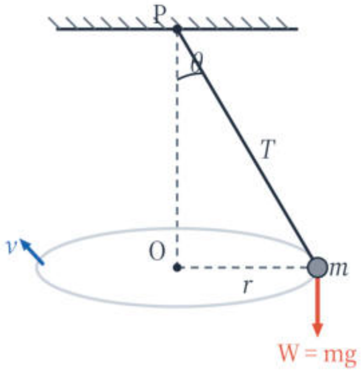


Figure 1-11-4 · A conical pendulum: the bob traces a horizontal circle.

Figure 1-11-5 shows the forces acting on the object: its weight W = mg and the tension T, resolved into a horizontal component $T\sin\theta$ and a vertical component $T\cos\theta$.

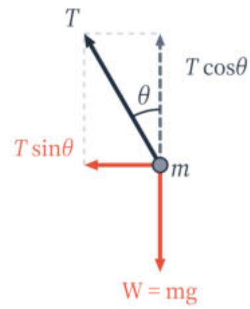


Figure 1-11-5 · The tension resolved into vertical and horizontal components.

Since the object moves in a circular path, a centripetal force must act toward the center of the circle, O. This is provided by the horizontal component of the tension, $T\sin\theta$. Using $F = ma_c$

$$(1) \qquad T\sin\theta = \frac{mv^2}{r}$$

The object does not move in the vertical direction, so the vertical forces are balanced:

$$(2) \qquad T\cos\theta = mg$$

Dividing equation (1) by equation (2):
$$\frac{T\sin\theta}{T\cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\tan\theta = \frac{v^2}{rg}$$

Figure 1-11-6 · The angle follows from the ratio of the two components.

Point

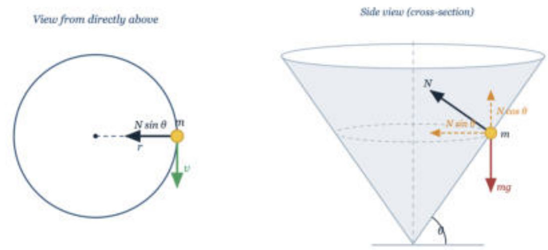
- ! For a ball moving on the smooth inner surface of a conical container, the resultant of the normal force and the weight points toward the center of the circle, and for a circle of radius r whose slant makes an angle θ with the horizontal (so the normal makes θ with the vertical), $v^2 = gr\tan\theta$.
- ! Motion in a vertical circle is not uniform circular motion: the speed is greatest at the bottom of the circle and least at the top, and the tension follows it. Set up the equation of motion toward the center at each moment.
- ! The object only just passes the highest point when the tension falls to $T = 0$ and the weight alone supplies the centripetal force. That gives $v^2_{\min} = gr$.

Equation box

Minimum tension T (top)	$T = \frac{mv^2}{r} - mg$
Maximum tension T (bottom)	$T = \frac{mv^2}{r} + mg$

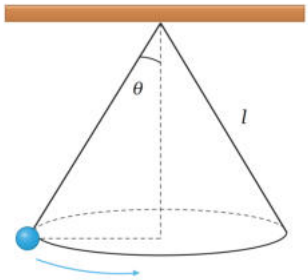
Minimum speed v_{min} (top)	$v_{min} = \sqrt{gr}$

Worked example

Situation	A small ball of mass m moves in uniform circular motion on the smooth inner surface of a conical container. It travels in a horizontal circle of radius r at speed v, and the slant side of the cone makes an angle θ with the horizontal. Determine the speed and the period of the motion.
Given and required	**Given:** mass m, radius r, angle θ, acceleration due to gravity g, smooth (frictionless) surface. **Required:** (a) the speed v, (b) the period T
Representation	Only two forces act on the ball: its weight mg (vertically down) and the normal force N. Resolve N into a vertical component $N\cos\theta$ and a horizontal (radial) component $N\sin\theta$; the horizontal component points toward the center and provides the centripetal force.  Figure 1-11-7 · A body on a conical surface, seen from above and in section.
Strategy	Decompose the forces acting on the object into horizontal and vertical directions, set up an equation of equilibrium for the vertical direction, and the equation of motion for uniform circular motion for the horizontal direction.
Annotated solution	Vertical equilibrium **(a) the speed v** $N\cos\theta - mg = 0 \Rightarrow N = \dfrac{mg}{\cos\theta}$ Radial direction (Newton's second law) $\dfrac{mv^2}{r} = N\sin\theta$ Substitute N: $\dfrac{mv^2}{r} = \dfrac{mg}{\cos\theta}\sin\theta = mg\tan\theta$ $\Rightarrow v^2 = gr\tan\theta$ $v = \sqrt{gr\tan\theta}$ **(b) the period T** $T = \dfrac{2\pi r}{v} = \dfrac{2\pi r}{\sqrt{gr\tan\theta}} \Rightarrow T = 2\pi\sqrt{\dfrac{r}{g\tan\theta}}$ $T = 2\pi\sqrt{\dfrac{r}{g\tan\theta}}$
Reasonableness check	The speed is independent of the mass, which is reasonable because the weight and the normal force are both proportional to m, so it cancels. Dimensions agree: $\sqrt{gr\tan\theta}$ has units √(m/s² · m) = m/s (a speed), and the period works out in s. A larger radius r increases both v and T, as expected for a wider circle.
Explain it yourself	Why does the ball stay at the same height even though the normal force acts at an angle, and why does only the horizontal component of the normal force contribute to the circular motion?

Now you try: On a smooth rail where a slope meets a vertical semicircle of radius r at the lowest point A, a small ball of mass m is released from rest at point P on the slope, at a height h above A, and travels around to the highest point B. Let the acceleration due to gravity be g. Determine the speed v of the ball as it passes the highest point B, in terms of g, h and r.

Figure 1-11-8 · A conical pendulum of slant length l.



Try

One end of a thread of length l [m] is fixed to the ceiling, and a small ball of mass m [kg] attached to the other end undergoes uniform circular motion in a horizontal plane. Let the angle between the thread and the vertical direction be θ, the acceleration due to gravity be g [m/s²], and the ratio of the circumference of a circle to its diameter be π.

(i) Determine the magnitude of the tension in the thread in N.

(ii) Determine the period of the circular motion in s.

In a new context — the roller-coaster loop

A roller-coaster loop is an example of motion in a vertical plane, like the water in the bucket. But the car's speed is determined by energy conservation, rather than being set by the rider. A circular loop has the same radius throughout. At the top, the car needs a minimum speed: $v^2 = gr$. By the bottom it is moving much faster, so the track pushes on the riders with a normal force $N = mg + mv^2/r$, where v is the car's speed at the bottom — about six times their weight. Real loops are clothoid (teardrop) curves instead. The small radius at the top lowers the speed needed there, and the large radius at the bottom lowers the force — which one fixed radius cannot do.

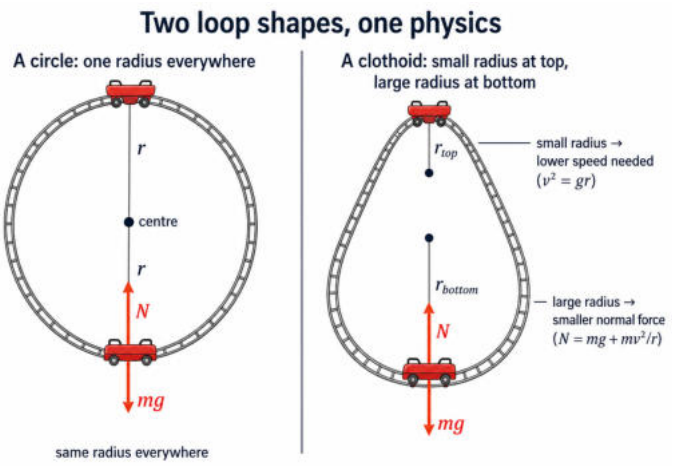


Figure 1-11-9 · A clothoid loop needs a smaller speed at the top than a circle.

Explain why a car that just maintains contact with the track at the top of a circular loop produces a large normal force on the riders at the bottom.

Nature of Science: Models — roller coasters

Early roller coasters at Coney Island were rapidly improved after unsafe designs. In 1895, the Flip-Flap Railway introduced a small circular loop that was exciting but uncomfortable and dangerous. In 1901, the Loop-the-Loop used a smoother oval loop to reduce forces on passengers. Later, the Drop-the-Dip added higher speeds and introduced lap bars for passenger safety.

Your turn: Which assumption of the circular-loop model is removed in a variable-radius loop?

Lesson question, answered

Why can water remain inside a bucket at the top of a vertical circle? At the top of the circle, the center of the circular path lies below the water, so the centripetal acceleration—and therefore the resultant radial force on the water—must be directed vertically downward, toward the center. The water's weight acts downward, and the bucket also exerts a downward normal force while contact is maintained. At the minimum speed required to maintain contact, the water is on the point of losing contact with the bucket, so the normal force is zero. The water's weight alone then provides the required resultant radial force, giving $v_{min} = \sqrt{gr}$.

If $v \geq \sqrt{gr}$, the water follows the circular path and remains inside the bucket; if $v < \sqrt{gr}$, it loses contact and falls out.

Key fact

In vertical circular motion, the speed of the body changes continuously as it moves around the circle.

Exercise

Answer the following questions.

(a) One end of a thread of length l [m] is fixed to the ceiling, and a small ball of mass m [kg] attached to the other end undergoes uniform circular motion in a horizontal plane. Let the angle between the thread and the vertical direction be θ, the acceleration due to gravity be g [m/s²], and the ratio of the circumference of a circle to its diameter be π.

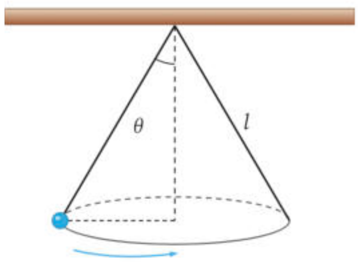


Figure 1-11-10 · The same pendulum at a wider angle.

(i) Determine the magnitude of the centripetal force in N.

(ii) Determine the speed of the small ball v in terms of l, g, and θ.

(b) One end of a thread of length l [m] is fixed to the ceiling, and a small ball of mass m [kg] attached to the other end undergoes uniform circular motion on a smooth horizontal plane while receiving a normal force from the surface. Let the angle between the thread and the vertical direction be θ, the angular velocity be ω [rad/s], and the magnitude of gravitational acceleration be g [m/s²].

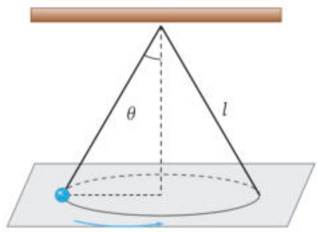


Figure 1-11-11 · The bob turns just above a horizontal surface.

(i) Determine the magnitude of the tension in the thread in N.

(ii) Determine the magnitude of the normal force the small ball receives from the horizontal plane in N.

Exam-style question

A small ball of mass 0.50 kg is tied to a string and moved in a vertical circle of radius 0.80 m. At the lowest point of the circle its speed is 4.0 m/s. Determine the tension in the string at the lowest point.

(c) As shown in Figure 1-11-12, a small ball of mass m undergoes uniform circular motion at a height h inside a conical container with a smooth inner surface. Let the angle between the axis of the cone and the slant side be θ, the magnitude of gravitational acceleration be g, and the ratio of the circumference of a circle to its diameter be π.

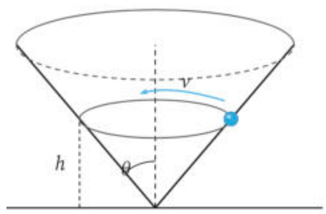


Figure 1-11-12 · A body circling inside a funnel.

(i) Determine the magnitude of the normal force N that the small ball receives from the conical surface in terms of m, θ, and g.

(ii) Determine the speed of the small ball v in terms of h and g.

(iii) Determine the period of the circular motion T using h, θ, and g.

Lesson 1-12 — Kepler's Laws and Universal Gravitation

❓ **Lesson question**: Why does one satellite appear fixed above Earth and what determines the time a satellite takes to complete an orbit?

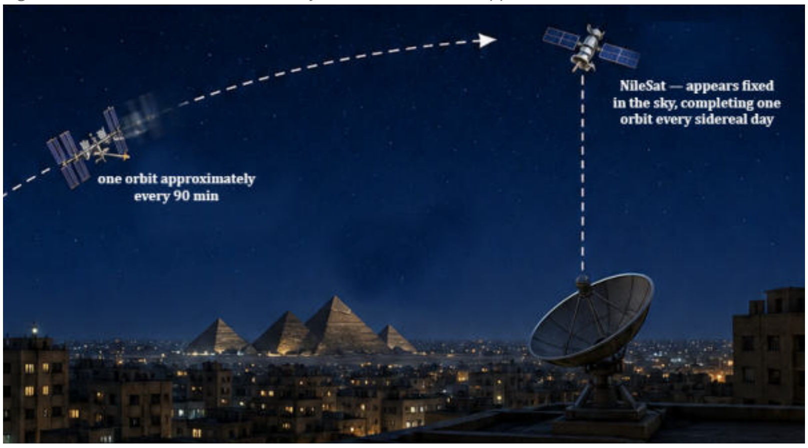
You will learn to

By the end of this lesson, you will be able to:

1. **Describe** and apply Kepler's laws of planetary motion.
2. **Explain** how the orbital period depends on orbital radius for planets and satellites.
3. **Determine** gravitational force and gravitational field strength.

🔍 The phenomenon — predict before you continue

Figure 1-12-1 · A low satellite circles every 90 min while NileSat appears fixed.



Consider the night sky: Some satellites appear fixed relative to Earth's surface, while others move rapidly across the sky. Planets also follow regular orbits around the Sun, and their motion has been studied for centuries. These motions are not random. Gravity continuously accelerates each object toward the body it orbits, while its tangential velocity carries it forward. As a result, the object remains in continuous free fall, following a curved orbital path rather than falling directly toward the central body.

Predict before you continue

(a) What determines whether a satellite appears fixed relative to Earth's surface or moves rapidly across the sky?

(b) Suppose a satellite's instantaneous velocity is unchanged when the gravitational force changes. How would a stronger or weaker gravitational force affect its orbit?

⭐ **Key fact**

To keep a body moving in a circle, a net inward force must point at the center.

Explore — Orbital Motion Investigation

What you'll do: Analyze the orbital radii and periods of Jupiter's four largest moons to identify a mathematical pattern.

Time: 12 min

You will need:

- The provided table of orbital radius (r) and orbital period (T).
- A calculator.

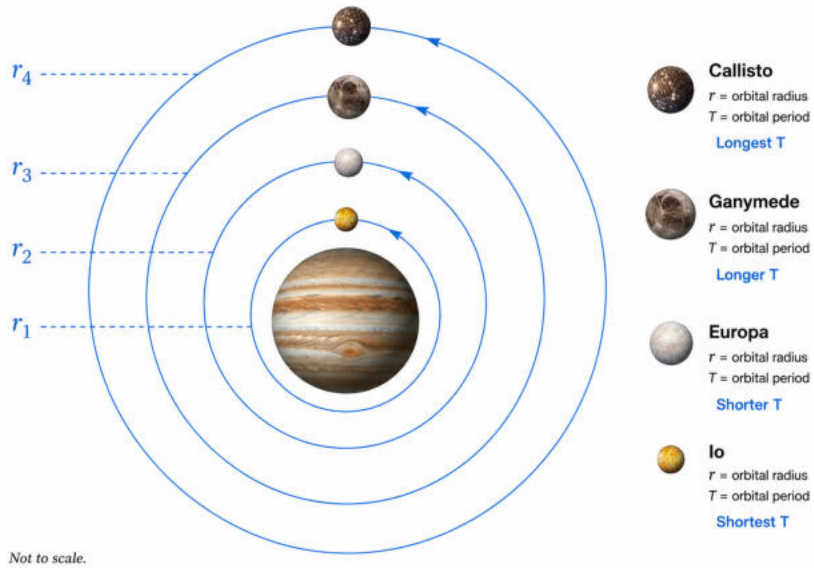


Figure 1-12-2 · The farther the moon, the longer its period.

Steps:

1. Use the values of orbital radius, r, and orbital period, T, given for each moon.
2. For each moon, calculate T^2, r^3, and the ratio $\frac{T^2}{r^3}$.
3. Record the calculated value of $\frac{T^2}{r^3}$ in the final column of the table.
4. Compare the values of $\frac{T^2}{r^3}$ for all four moons.

Moon	Orbit radius r (• 10^8 m)	Period T (days)	$\frac{T^2}{r^3}$ (day^2/m^3)
Io	4.22	1.77	
Europa	6.71	3.55	
Ganymede	10.7	7.15	
Callisto	18.8	16.7	

Analyze what you collected:

(a) Does the ratio remain approximately constant for the four moons, or does it vary significantly?

(b) What property of the central planet determines this constant value, and which physical laws explain it?

Write your explanation in two lines before you read the next page.

From observation to model

In Explore, you calculated $\frac{T^2}{r^3}$ for Jupiter's four largest moons. The similar values for the four moons reveal a common pattern. This nearly constant ratio indicates that the moons orbit the same central mass and is consistent with an inverse-square gravitational force. **Newton's law of universal gravitation**

Every particle attracts every other along the line joining them; this attraction is the gravitational force. For two masses m_1 and m_2 a distance r apart, it is proportional to the product of the masses and inversely proportional to the square of the distance:

$$F = \frac{Gm_1 m_2}{r^2}$$

Here $G = 6.67 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$ is the universal gravitational constant, the same for all bodies.

Building the model

A moon of mass m orbits Jupiter (mass M) in a circle of radius r at speed v. The gravitational force between them, GMm/r^2, provides the centripetal force that keeps the moon in its circular orbit: $\frac{GMm}{r^2} = \frac{mv^2}{r}$

The moon's mass cancels, leaving its orbital speed: $v^2 = \frac{GM}{r}$ (1)

In one period the moon travels one circumference, so $v = 2\pi r/T$. Substituting into (1) and rearranging: $\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r} \Rightarrow \frac{T^2}{r^3} = \frac{4\pi^2}{GM}$ (2)

Interpreting the result

Equation (2) is the pattern you measured. Its right-hand side contains no property of the moon — only G and the central mass M. That is why the four moons share one value: they all orbit Jupiter, so the constant is fixed by its mass.

Kepler's laws

The relation $T^2 \propto r^3$ is the third of Kepler's three laws, drawn from planetary data long before Newton explained them.

First law. Each planet moves in an ellipse with the Sun at one focus. The longest diameter is the major axis; half of it is the semi-major axis a. For the Earth's orbit, a is one astronomical unit ($1\ AU = 1.50 \times 10^{11}$ m).

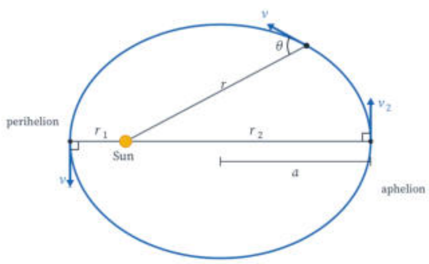


Figure 1-12-3 · An elliptical orbit: the Sun sits at one focus.

Second law. The radius vector from the Sun to a planet sweeps out equal areas in equal times, so the areal velocity stays constant. With θ the angle between the radius vector r and the velocity vector v,

$$\frac{1}{2} rv\sin\theta = \frac{1}{2} r_1 v_1 = \frac{1}{2} r_2 v_2 = \text{constant}$$

At perihelion (nearest, r_1) and aphelion (farthest, r_2) the velocity is perpendicular to the radius vector, so $\sin\theta = 1$ and $r_1 v_1 = r_2 v_2$: a planet moves faster when nearer the Sun.

Third law. The square of the period over the cube of the semi-major axis is the same constant for every body orbiting a given center:

$$\frac{T^2}{a^3} = k$$

> **Key fact**
> All objects orbiting the same central body have the same value of $\frac{T^2}{r^3}$. This value depends only on the mass of the central body.

This is the relation built above, with k = $4\pi^2/GM$; for a circular orbit the semi-major axis a is the radius r. Because k is shared, any two bodies orbiting the same center obey $T_1^2/a_1^3 = T_2^2/a_2^3$.

Point

Kepler's laws

First Law: Planets move in elliptical **orbits** with the Sun at one focus. The semi-major axis of the Earth's orbit is called **1 astronomical unit**.

Second Law: The area swept out per unit time (areal velocity) by the radius vector drawn from the Sun to a planet is constant: with radius vector r, velocity v, and angle θ between them, it is $\frac{1}{2}rv\sin\theta$. $\Delta ABC = \frac{1}{2}bc\sin A$ At the perihelion it is $\frac{1}{2}r_1v_1$ and at the aphelion $\frac{1}{2}r_2v_2$, so $\frac{1}{2}rv\sin\theta = \frac{1}{2}r_1v_1 = \frac{1}{2}r_2v_2$. $\sin 90° = 1$

Third Law: The ratio of the square of a planet's orbital period T to the cube of the semi-major axis a of its elliptical orbit is constant for all planets: $\frac{T^2}{a^3} = k$ (k is constant).

The attractive force between any two objects is the gravitational force, and for masses m_1, m_2 a distance r apart it is $F = G\frac{m_1 m_2}{r^2}$, with G [N·m²/kg²] the universal gravitational constant.

Equation box

Universal gravitation	$F = G\frac{m_1 m_2}{r^2}$
Orbital speed	$v^2 = \frac{GM}{r}$
Kepler's third law	$\frac{T^2}{a^3} = k = \frac{4\pi^2}{GM}$
Areal velocity (second law)	$\frac{1}{2}r_1v_1 = \frac{1}{2}r_2v_2$

Gravitational fields

A force field is a region in which an object experiences a non-contact force. A gravitational field can be represented by vectors, which show the direction and relative strength of the field at each point — the same direction as the force that would act on a mass placed there. It can also be represented by field lines: their direction shows the direction of the field, and the closer they are together, the stronger the field in that region.

A gravitational field is a force field in which any object with mass experiences a force.

Gravitational field strength

The gravitational field strength g at a point is the gravitational force per unit mass acting on a small test mass placed there:

$$g = \frac{F}{m}$$

It is a vector, directed along the field, and is measured in newtons per kilogram (N/kg, equivalently m/s²). For a spherical mass M, the field is radial, and its magnitude at a distance r from the center is

$$g = \frac{GM}{r^2}$$

so it obeys an inverse-square law: if the distance from the mass doubles, g falls to a

Key fact

Gravitational field lines always point toward the mass that produces the field, because gravity is only ever attractive. Where the lines are closer together, the field is stronger.

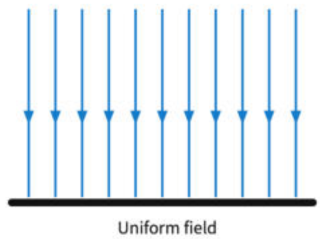


Figure 1-12-4 · A uniform field: parallel lines, evenly spaced.

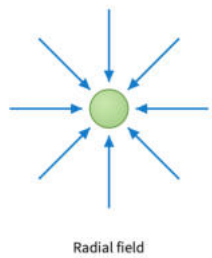


Figure 1-12-5 · A radial field: the lines converge on the mass.

quarter of its value. A gravitational field is usually shown in one of two ways — as a uniform field or as a radial field:

The arrows on the field lines give the direction of the field, which is the direction of the force on a mass. In a uniform field the field strength is the same at every point, so the field lines are parallel and equally spaced, and a given mass experiences the same force wherever it is placed. In a radial field the field strength depends on position: as r increases, the field lines spread apart and g decreases (following $g = GM/r^2$), so the force on a given mass decreases too. The Earth's gravitational field is radial; however, over a small region close to the surface it is very nearly uniform, which is why $g \approx 9.8\ \mathrm{N/kg}$ can be treated as constant there.

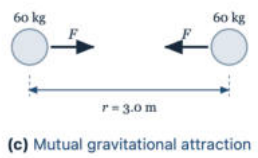
Worked example

Situation	Three independent problems on gravitation and orbital motion. **(a)** A comet moves in an elliptical orbit in which its distance from the Sun at aphelion is 5.0 times its distance at perihelion. **(b)** An asteroid's orbit around the Sun has a semi-major axis of 4.0 astronomical units (AU). **(c)** Two objects, each of mass 60 kg, are placed 3.0 m apart.
Given and required	Given: (a) aphelion distance = 5.0 × perihelion distance. (b) asteroid semi-major axis a = 4.0 AU; Earth's semi-major axis = 1.0 AU and orbital period = 1.0 year. (c) $m_1 = m_2$ = 60 kg, separation $r = 3.0\ \mathrm{m}$ · gravitational constant $G = 6.67 \times 10^{-11}\ N{\cdot}m^2/\mathrm{kg}^2$ Required: (a) the ratio of the comet's speed at aphelion to its speed at perihelion. (b) the orbital period T of the asteroid. (c) the magnitude of the gravitational force F between the two objects.
Representation	**(a)** The comet follows an elliptical orbit with the Sun at one focus. At perihelion (distance r) the speed v_1 is greatest; at aphelion (distance $5.0r$) the speed v_2 is least. At both points the velocity is perpendicular to the line joining the comet to the Sun. **(c)** Each object attracts the other with an equal and opposite gravitational force of magnitude F.
Strategy	**(a)** Apply Kepler's second law (equal areas in equal times). At perihelion and aphelion the velocity is perpendicular to the radius, so the quantity $\frac{1}{2}\ rv$ is the same at both points. **(b)** Apply Kepler's third law, $T^2 \propto a^3$. **(c)** Apply Newton's law of universal gravitation, $F = \frac{Gm_1m_2}{r^2}$.
Annotated solution	**(a)** Ratio of the speeds Let the perihelion distance be r and the speeds at perihelion and aphelion be v_1 and v_2; the aphelion distance is 5.0r. Kepler's second law gives: $\frac{1}{2} r v_1 = \frac{1}{2}(5.0r) v_2$ $\quad v_2 = \frac{v_1}{5.0} = 0.20 v_1$ $\quad \frac{v_2}{v_1} = 0.20$ **(b)** Orbital period of the asteroid By Kepler's third law, T^2/a^3 has the same value for the asteroid and the Earth: $\frac{T^2}{(4.0)^3} = \frac{(1.0)^2}{(1.0)^3}$ **$T^2 = (4.0)^3 = 64 \rightarrow T = 8.0$ years** $\quad$ **$T = 8.0$ years** (c) Gravitational force $F = \frac{Gm_1m_2}{r^2}$ $F = \frac{(6.67\times 10^{-11})(60)(60)}{(3.0)^2} = 2.67 \times 10^{-8} \approx 2.7 \times 10^{-8}\ \mathrm{N}$ ***$F \approx 2.7 \times 10^{-8}$ N***

Quick application

The Moon's mass is 7.3×10^{22} kg and its radius is 1.7×10^6 m. Calculate the field strength g at its surface using $g = GM/r^2$.

Figure 1-12-6 · Two 60 kg spheres 3.0 m apart attract each other equally.



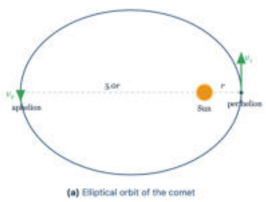


Figure 1-12-7 · A comet moves fastest at perihelion and slowest at aphelion.

Reasonableness check	The comet moves faster at perihelion and slower at aphelion, as Kepler's second law requires: the speed ratio $\frac{v_2}{v_1}$ = 0.20 is the inverse of the distance ratio. A larger orbit corresponds to a longer period: the asteroid's 8.0 years exceeds the Earth's 1.0 year, as required by Kepler's third law $T^2 \propto a^3$. The gravitational force between two objects of ordinary mass is extremely small, of order 10^{-8} N.
Explain it yourself	Explain, using Kepler's second law, why the comet moves fastest at perihelion and slowest at aphelion.

Now you try: Calculate the mass of the Sun, given that the period of the Earth's orbit around the Sun is 3.156 × 10^7 s and the semi-major axis of Earth's orbit is 1.496 × 10^{11} m.

Try

(a) When the distance of a comet from the Sun at aphelion is 2.5 astronomical units, and the distance from the Sun at perihelion is 1.5 astronomical units, determine the ratio of its speed at aphelion to its speed at perihelion.

(b) The semi-major axis of a certain asteroid's orbit is 16 astronomical units. Using the fact that the Earth's orbit has a semi-major axis of 1.0 astronomical unit and an orbital period of 1.0 year, determine the orbital period of this asteroid.

(c) Two objects, each with a mass of 60 kg, are 2.0 m apart. Determine the magnitude of the gravitational force acting between these two objects. Let the gravitational constant be 6.67 × 10^{-11} $N{\cdot}m^2/kg^2$.

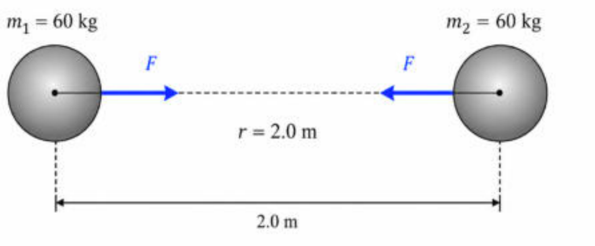


Figure 1-12-8 · Two 60 kg spheres 2.0 m apart.

Key fact

Kepler's second law states that equal areas are swept out in equal times. At perihelion and aphelion, the comet's velocity is perpendicular to the line joining the comet to the Sun.

In a new context — the geostationary orbit

A communications satellite is most useful when it stays above the same point on the equator, so that a dish on the ground can point in one fixed direction in the sky. For this, the satellite must orbit the Earth in the same time the Earth takes to rotate once on its axis, $T = 8.62 \times 10^4$ s (about one day). For a satellite of mass m in a circular orbit around Earth of mass M: the gravitational force provides the centripetal force, $\frac{GMm}{r^2} = \frac{mv^2}{r}$, so $v^2 = \frac{GM}{r}$. Substituting $v = \frac{2\pi r}{T}$ leads back to the relation derived earlier, $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$. Rearranging for the radius, $r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} \approx 4.2 \times 10^7$ m, which is about 3.6×10^7 m above the surface. At this radius the orbital speed is $v = \frac{2\pi r}{T} \approx 3.1 \times 10^3$ m/s. Only this one radius lets the satellite orbit at the same angular speed as the Earth, so it stays above the same point.

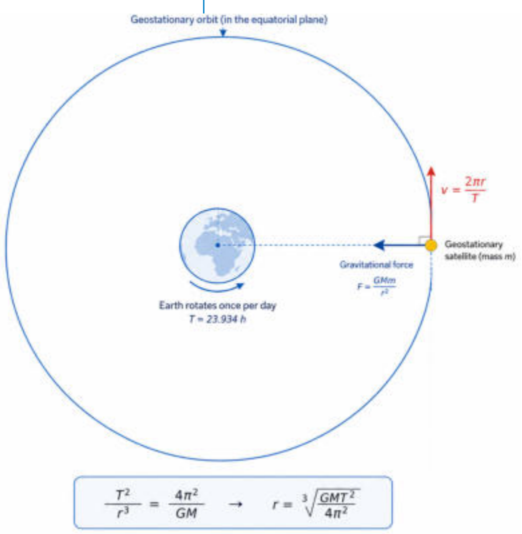


Figure 1-12-9 · The geostationary radius follows from one sidereal day.

Explain why a rocket launched eastward from near the equator requires less additional speed to reach an eastward orbit than a rocket launched westward from a high-latitude site.

Nature of Science: Patterns and trends — the laws that waited seventy years

Kepler did not derive his laws from a theory of force. He inferred them from patterns in Tycho Brahe's precise naked-eye observations of planetary motion. The first two laws were published in Astronomia Nova in 1609, and the third in Harmonices Mundi in 1619. For nearly seventy years, these laws remained reliable empirical descriptions without a generally accepted dynamical explanation. In Principia (1687), Newton showed how they could be explained using his laws of motion and universal gravitation. Kepler's second law follows from the central direction of the gravitational force, while the first and third depend on its inverse-square variation with distance.

Your turn: State one prediction that Kepler's third law allowed astronomers to make about a newly discovered planet's motion before Newton provided a gravitational explanation for the relationship.

Lesson question, answered

Why does one satellite appear fixed above Earth, and what determines the time required to complete an orbit?

Earth's gravitational force provides the centripetal force that keeps a satellite in orbit. The orbital period depends on the semi-major axis and Earth's mass. According to Kepler's third law:

$$T^2 = \frac{4\pi^2}{GM}a^3$$

where T is the orbital period and a is the semi-major axis (equal to the orbital radius r for a circular orbit).

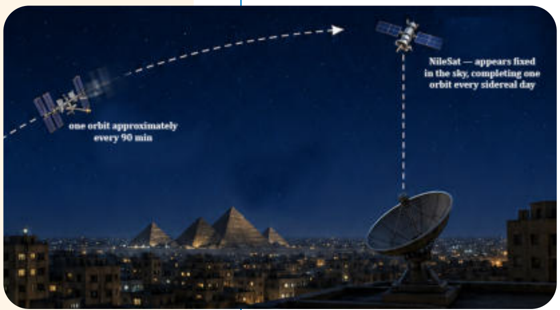


Figure 1-12-10 · The satellites revisited.

A geostationary satellite moves in a circular orbit above Earth's equator, in the same direction as Earth's rotation, and has an orbital period equal to Earth's rotational period. As a result, it remains above the same location and appears stationary in the sky.

A satellite in a lower orbit, such as the International Space Station, has a smaller orbit radius and so, by $T^2 \propto r^3$, a shorter period.

Exercise

Determine the following:

(a) When the distance of a comet from the Sun at aphelion is 4.0 times its distance from the Sun at perihelion, determine the ratio of its speed at aphelion to its speed at perihelion.

(b) The semi-major axis of a certain asteroid's orbit is 9.0 astronomical units. Using the fact that the Earth's orbit has a semi-major axis of 1.0 astronomical unit and an orbital period of 1.0 year, determine the orbital period of this asteroid.

(c) Two objects, each with a mass of 40 kg, are 3.0 m apart. Determine the magnitude of the gravitational force acting between these two objects. Let the gravitational constant be 6.67×10^{-11} N·m²/kg².

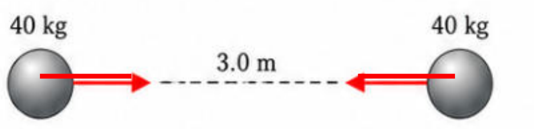
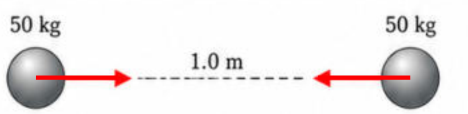


Figure 1-12-11 · Two 40 kg spheres 3.0 m apart.

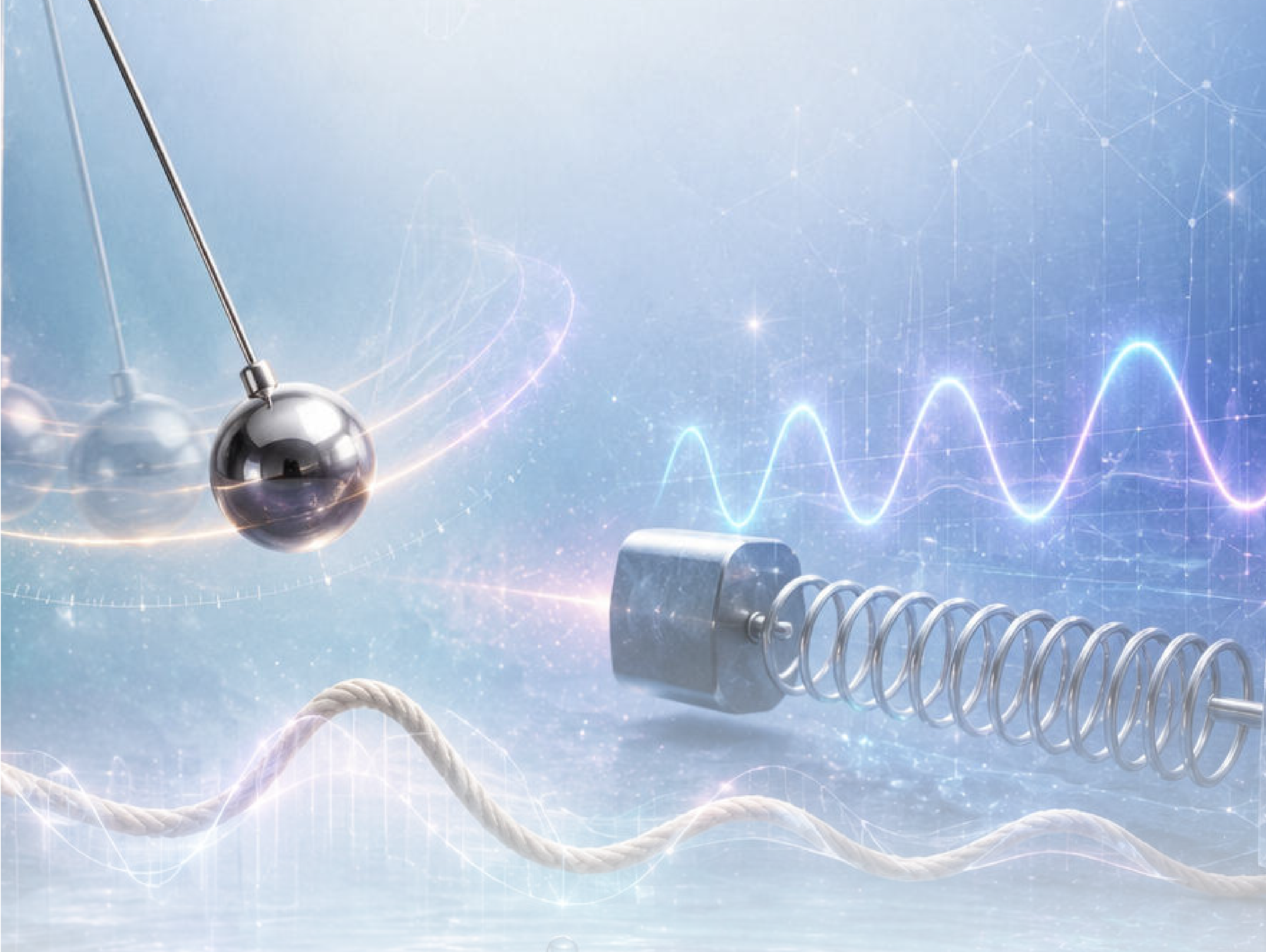
(d) Two objects, each with a mass of 50 kg, are 1.0 m apart. Determine the magnitude of the gravitational force acting between these two objects. Let the gravitational constant be 6.67×10^{-11} N·m²/kg².

Figure 1-12-12 · Two 50 kg spheres 1.0 m apart.



CHAPTER 2

Oscillations and Waves



Lesson questions

What makes a motion repeat, and what sets its rate?

How does a wave carry energy without carrying matter?

How do the same wave ideas explain sound and light?

Lesson 2-1 — Simple Harmonic Motion

❓ **Lesson question:** A ruler vibrates when its free end is displaced and released at the edge of a desk. What determines the frequency of this vibration?

You will learn to

By the end of this lesson, you will be able to:

1. **Identify** simple harmonic motion from its defining condition — a restoring force proportional to the displacement and directed toward the equilibrium position.
2. **Determine** the period of a simple harmonic motion, using $T = \frac{1}{f} = 2\frac{\pi}{\omega}$ and $T = 2\pi\sqrt{\frac{m}{k}}$

🔍 The phenomenon — predict before you continue

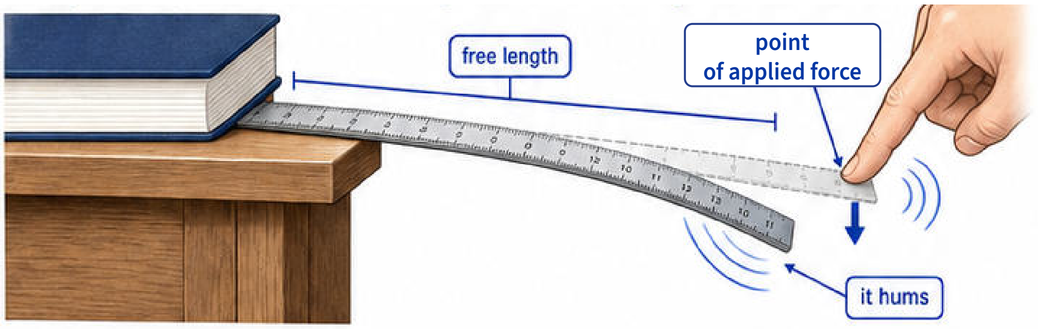


Figure 2-1-1 · A ruler clamped under books hums when its free end is released.

Clamp a steel ruler flat under a book so most of it extends beyond the desk edge. Press the free tip down and release it: it vibrates up and down and produces a steady note. Slide the ruler so **less** sticks out, press it again, and the pitch rises. The ruler is the same material throughout — only the free length has changed.

Predict before you continue:

(a) During the vibration, is the tip of the ruler moving fastest as it passes through the equilibrium position or at the turning points?

(b) At a turning point, the tip is momentarily at rest. Is the restoring force zero at this point? Explain.

⭐ Key fact

A restoring force is a force that acts toward the equilibrium position. In many systems, for small displacements, the restoring force is proportional to the displacement: $F = -kx$ where k is the spring constant and the negative sign shows that the force acts opposite to the displacement.

Explore — Set the ruler vibrating and measure its period

What you'll do: You will set a clamped ruler into vibration, use slow-motion video to measure its period, and observe how changing the free length affects the period.

Time: 12 min

You will need: • A steel ruler or hacksaw blade.

- A large eraser taped to its tip.
- A heavy book or clamp.
- A phone with slow-motion video.
- A desk with a clear edge.

Figure 2-1-2 · A slow-motion recording gives the period more precisely.

Steps:

1. Tape the eraser to the free end of the ruler. Clamp the ruler under a heavy book so that about 20 cm extends beyond the edge of the desk.
2. Displace the free end slightly downward and release it. Record the motion using slow-motion video.
3. Use the video timer to measure the time for ten complete oscillations. One complete oscillation is a motion from one extreme position back to the same extreme position.
4. Shorten the free length to about 12 cm while keeping the same eraser attached, then repeat the measurement.
5. Compare the time taken for ten complete oscillations in the two cases.

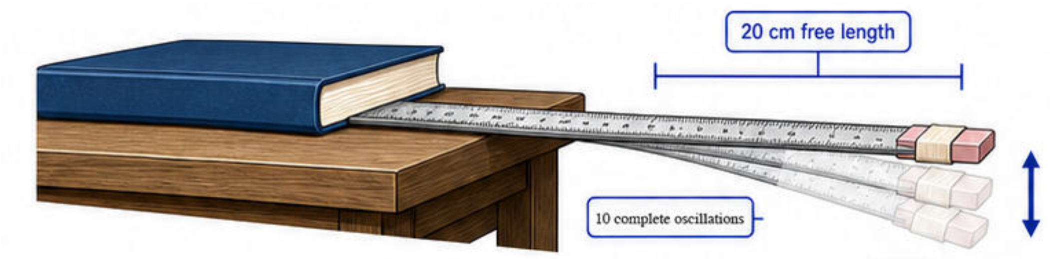


Figure 2-1-3 · Timing ten complete oscillations of a 20 cm free length.

Free length of ruler (cm)	Time for 10 complete oscillations (s)	Position of maximum speed (middle / turning points)

Analyze what you collected

(a) In the slow-motion video, where was the ruler tip moving fastest: at the **equilibrium position** or at the **turning points**? What was its speed at the instant it changed direction?

(b) Which free length had the shorter period? What does this suggest about the relation between spring constant, mass, and period?

Write your explanation in no more than two lines before you read the next page.

A measurement note

Time ten oscillations, not one, and read the time from the slow-motion clip's timer: a single oscillation is too quick for the eye, and ten together reduce the percentage uncertainty — the same reason experimental data tables usually contain multiple measurements.

From observation to model

In your clip, the free end reached its maximum speed as it passed through the equilibrium position and was instantaneously at rest at each turning point. And the shorter length finished its ten oscillations sooner. Here is the model your evidence supports.

A force that always acts toward equilibrium

A body performs simple harmonic motion when its acceleration is directly proportional to its displacement from the equilibrium position and directed toward the equilibrium position: $a = -\omega^2 x$

Equivalently, if the restoring force is proportional to displacement and directed toward the equilibrium position, then: $F = -kx$

where x is the displacement from equilibrium and k is the spring constant. The equilibrium position is the center of oscillation. The amplitude A is the maximum displacement from equilibrium. The period T is the time taken for one complete oscillation.

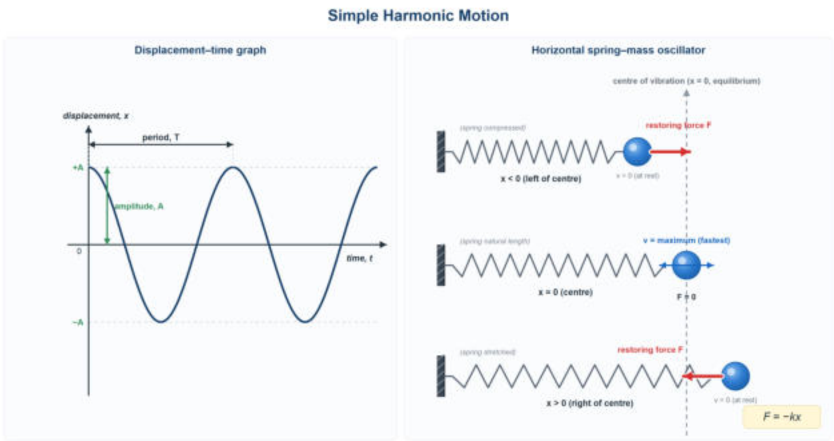


Figure 2-1-4 · Displacement against time, and the restoring force at each position.

Angular frequency

The angular frequency (ω) describes how rapidly the oscillation repeats. It is related to the period by: $\omega = \frac{2\pi}{T}$ so: $T = \frac{2\pi}{\omega}$

The SI unit of angular frequency is rad/s, often written as 1/s because the radian is dimensionless.

> ⭐ **Angular frequency**
>
> the rate at which the reference point moves around its circle, $\omega = \frac{2\pi}{T}$; it is measured in rad/s and indicates how rapidly the oscillation repeats.

Point

- ! A vibration whose acceleration is proportional to the displacement from the equilibrium position and directed toward it is called simple harmonic motion. The maximum displacement from the equilibrium position is called the amplitude A, and the time T [s] required to complete one oscillation is called the period.
- ! The rate at which the reference point moves around its circle is called the angular frequency: $\omega = \frac{2\pi}{T}$, so $T = \frac{2\pi}{\omega}$.
- ! The acceleration is $a = -\omega^2 x$ — proportional to the displacement and directed toward the equilibrium position.

Equation box

Defining acceleration	$a = -\omega^2 x$
Period from angular frequency	$T = \frac{1}{f} = \frac{2\pi}{\omega}$

From the defining condition to the period

The defining condition $a = -\omega^2 x$ also gives you the period. Apply Newton's second law to a mass m with restoring force $F = -kx$ (k = force per unit displacement): $ma = -kx$ gives $\omega^2 = \frac{k}{m}$, so $\omega = \sqrt{\frac{k}{m}}$; substitute that into $T = \frac{2\pi}{\omega}$:

Equation box

Period of spring–mass oscillator	$T = 2\pi\sqrt{\frac{m}{k}}$

Across each cycle the motion also exchanges energy: it is entirely stored as elastic potential energy at the turning points, and entirely as kinetic energy at the center. We treat that exchange **qualitatively** here — the quantities come later.

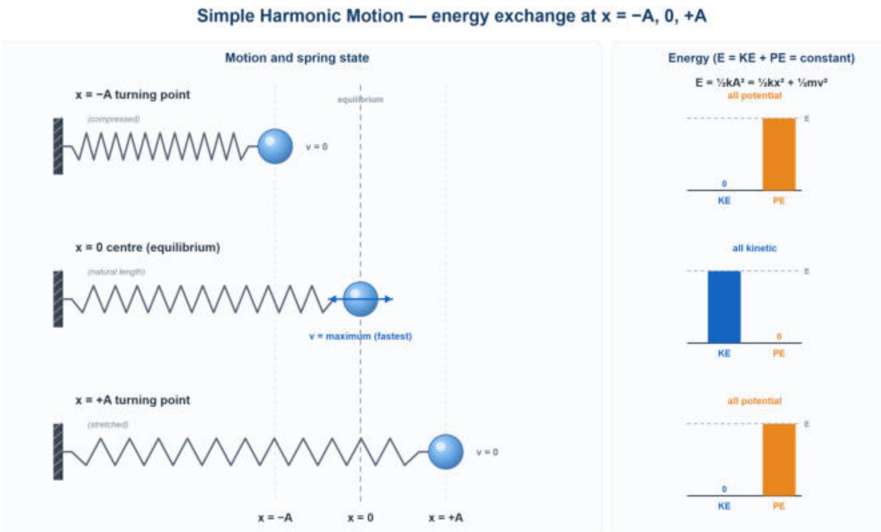


Figure 2-1-5 · Energy moves between kinetic and potential, and their sum stays fixed.

Quick application: An object of mass 0.30 kg oscillates under a restoring force $F = -30x$, with x the displacement in meters. **Determine** the period. (Take π = 3.14)

Put it to the test: on one displacement-time sketch, mark where speed is zero and where acceleration is zero, and **show** that they never coincide.

Reading the period directly from the motion

There are two ways to determine the period. The first you have already used: name the restoring force in the form $F = -kx$, then $\omega = \sqrt{\frac{k}{m}}$ $T = 2\pi\sqrt{\frac{m}{k}}$. The second method is more direct. Sometimes the motion is given already written out as a displacement that oscillates like a sine wave about the equilibrium position — in words, the position is the amplitude times the sine of (the angular frequency times the time). In symbols, for a body passing through the center at t = 0 with maximum speed,

$$x = A\sin\omega t.$$

Interpret the equation. The coefficient of the sine function is the amplitude A — the maximum displacement from the center. The coefficient of t inside the sine function is the angular frequency ω. Once you can identify ω, the period follows directly from $T = \frac{2\pi}{\omega}$ — no mass and no spring constant needed, because omega already encodes their ratio, $\omega^2 = k/m$.

Data booklet

$a = -\omega^2 x$,
$T = \frac{2\pi}{\omega}$ and
$T = 2\pi\sqrt{\frac{m}{k}}$
are all printed; read k from the given $F = -kx$.

Worked example

A small block of mass 0.20 kg slides on a smooth horizontal track. A restoring force pulls it back toward the equilibrium position, $F = -kx$, with $k = 80$ N/m and x the displacement in meters. Take π = 3.14. **Determine** the period of its oscillation.

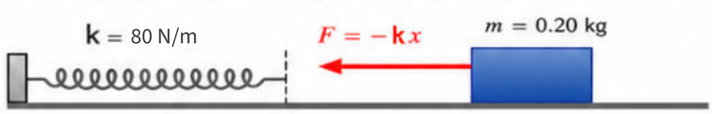


Figure 2-1-6 · A 0.20 kg mass on a spring of spring constant 80 N/m.

Situation	The block of mass $m = 0.20$ kg experiences a restoring force $F = -kx$ with $k = 80$ N/m. Find the period T of the simple harmonic motion.
Given and required	$m = 0.20$ kg, $k = 80$ N/m (the restoring force per unit displacement). $\pi = 3.14 \rightrightarrows T = ?$
Representation	The force is proportional to the displacement and points back toward the equilibrium position, so the motion is simple harmonic.
Strategy	Write the restoring force at displacement x as $F = -kx$; if it fits, the motion is simple harmonic, and $\omega = \sqrt{\frac{k}{m}}$ and $T = 2\pi\sqrt{\frac{m}{k}}$ — or, when the displacement equation is given to you directly, read ω directly from it and take $T = 2\frac{\pi}{\omega}$.
Annotated solution	1) Name the spring constant (read k from $F = -kx$): k = 80 N/m 2) Find the angular frequency ($\omega = \sqrt{\frac{k}{m}}$), substituting one value: $\omega = \sqrt{\frac{80}{0.20}} = \sqrt{400} = 20 \text{ rad s}^{-1}$ 3) Calculate the period last (one substitution into $T = \frac{2\pi}{\omega}$): $T = \frac{2\pi}{20} = \frac{2 \times 3.14}{20} \Rightarrow T = 0.31$ s
Reasonableness check	The period carries the unit of time (s), as it must; $\omega = 20$ rad/s is a rapid oscillation, and T = 0.31 s corresponds to a frequency f = 1/T = 3.2 Hz — a believable rate for a stiff, low-mass oscillator.
Explain it yourself	Why does a stiffer spring (a larger k), with the mass unchanged, give a shorter period?

Now you try: a block of mass 0.80 kg on a smooth track experiences a restoring force $F = -kx$ with $k = 20$ N/m (π = 3.14). **Determine** the period of its oscillation.

Try

Determine the following to two significant figures, taking π = 3.14.

(a) The displacement x (m) of a simple harmonic motion at time t (s) is $x = 0.50\sin(8.0\pi t)$. Determine its amplitude A (m) and its period T (s).

(b) An object of mass 0.50 kg undergoes simple harmonic motion under a restoring force F (N) given by $F = -2.0x$, where x (m) is the displacement. Determine the period of this motion in s.

Data booklet

Each item is one direct substitution — either a displacement written as a sine (identify ω, then

$T = \frac{2\pi}{\omega}$

or a restoring force

$F = -kx$

$\omega = \sqrt{\frac{k}{m}}$

$T = \frac{2\pi}{\omega}$

Decide which form the question gives you before you substitute numbers.

In a new context — the suspension under a metro car

Each metro car is supported by suspension springs. When the car body moves downward relative to the wheel assembly, it compresses the springs. The springs exert an upward restoring force toward the equilibrium position. For small vertical displacements, this motion can be approximated as simple harmonic motion.

The period depends on the mass of the car and the effective spring constant of the suspension:

$$T = 2\pi\sqrt{\frac{m}{k}}\ .$$

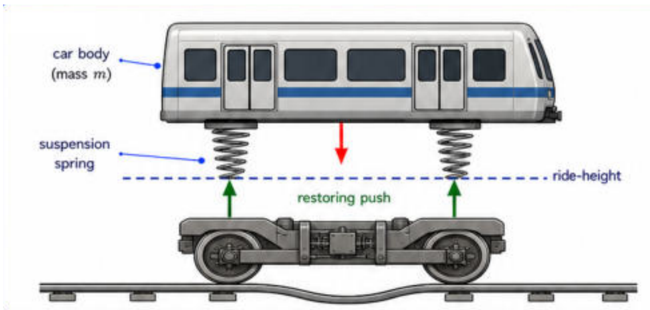


Figure 2-1-7 · A suspension spring restores the car body to its ride height.

Your turn: a fully loaded car has a larger mass m than an empty one. Using $T = 2\pi\sqrt{\frac{m}{k}}$, **predict** whether it oscillates with a longer or shorter period, and **explain** why a stiffer spring (larger k) gives the opposite effect.

Nature of Science: Science as a shared endeavor — the law Hooke hid in an anagram

In 1676, Robert Hooke stated his law in the form of a Latin anagram. He later revealed it as: ut tensio, sic vis, which means: “as the extension, so the force”. In modern notation, this idea is expressed as: $F = -kx$ for a restoring force acting opposite to the displacement.

Lesson question, answered

A ruler vibrates when its free end is displaced and released. For small displacements, the ruler exerts a restoring force approximately proportional to the displacement and directed toward the equilibrium position. This produces simple harmonic motion described by: $a = -\omega^2 x$
The frequency of vibration depends on the effective spring constant of the ruler and the effective moving mass. Shortening the free length makes the ruler effectively stiffer, so the angular frequency increases and the period decreases: $T = \frac{2\pi}{\omega}$
As a result, the pitch of the sound becomes higher.

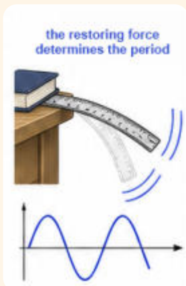


Figure 2-1-8 · The restoring force sets the period.

Exercise

(a) The displacement x (m) of a simple harmonic motion at time t (s) is $x = 0.20 \sin(4.0\pi t)$. Determine its amplitude A (m) and its period T (s).

(b) An object of mass 0.50 kg undergoes simple harmonic motion under a restoring force F (N) given by $F = -50x$, where x (m) is the displacement. Determine the period of this motion in s.

(c) An object of mass 7.5 kg undergoes simple harmonic motion under a restoring force F (N) given by $F = -30x$, where x (m) is the displacement. Determine the period of this motion in s.

Linking question

How can an understanding of simple harmonic motion be applied to the wave model?
The wave model builds on this: each particle in the medium performs this oscillation.

Exam tip

Show the substitution line in every answer — examiners award marks for the method, not only for the final answer.

Exam-style question

A body of mass 0.20 kg undergoes simple harmonic motion under a restoring force $F = -kx$.

Its acceleration is measured as $a = -\omega^2 x$ with $\omega = 5.0$ rad / s.

Determine the value of k, then determine the period of the motion. (Take π = 3.14).

Lesson 2-2 — The Mass–Spring Oscillator

❓ **Lesson question**: If the same mass is attached to a vertical spring instead of a horizontal spring, does gravity change the period of oscillation?

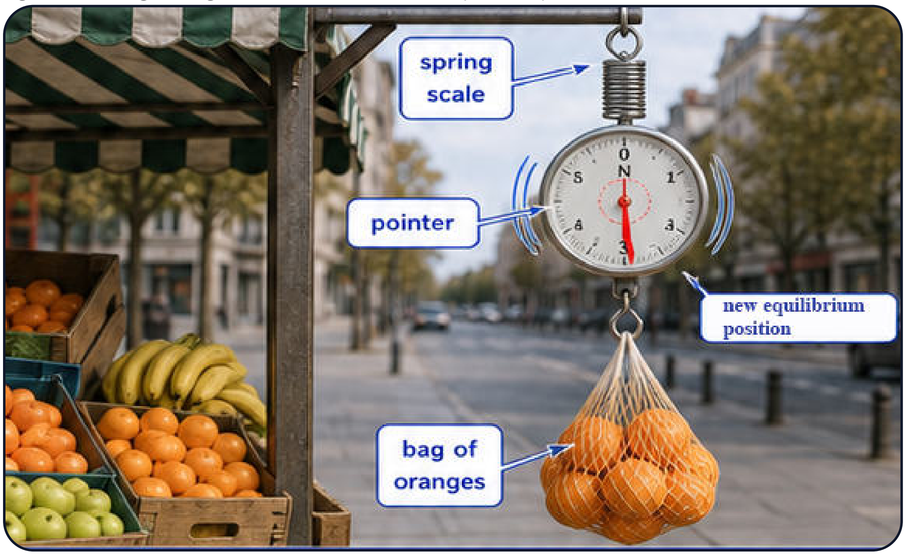
You will learn to

By the end of this lesson, you will be able to:

1. **Determine** the period of a mass–spring oscillator from $T = 2\pi\sqrt{\frac{m}{k}}$.
2. **Show** that a vertical hanging spring keeps the same period as a horizontal one — choosing the equilibrium position as the origin gives the restoring force $F = -kx$

The phenomenon — predict before you continue

Figure 2-2-1 · A bag of oranges stretches the scale to a new equilibrium position.



At a fruit stall the seller hooks a bag of oranges onto a hanging spring scale. The pointer drops, overshoots, then oscillates up and down about a new lower reading before settling there. The same bag on a flat counter would not oscillate at all — yet hanging, it oscillates several times before resting.

Predict before you continue

(a) The same mass and the same spring are placed on a smooth horizontal surface. Does the vertical oscillator oscillate with a slower, faster, or identical period?

(b) If you pulled the bag down further before releasing it — a larger amplitude — would one complete oscillation take longer?

Key fact

For a mass–spring oscillator, the period is:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where m is the mass and k is the spring constant. The spring constant can be determined from the restoring force:

$$F = -kx$$

and then used to determine the period.

Explore — Load the spring and measure the period

What you'll do: add different masses to a spring or rubber band, measure the extension and the period of the oscillations.

Time: 12 min

You will need:

- A soft spring or rubber band.
- A small bag or paper cup partly filled with coins.
- A paper clip for hanging the mass.
- A ruler.
- A stopwatch.
- A fixed support for hanging the spring.

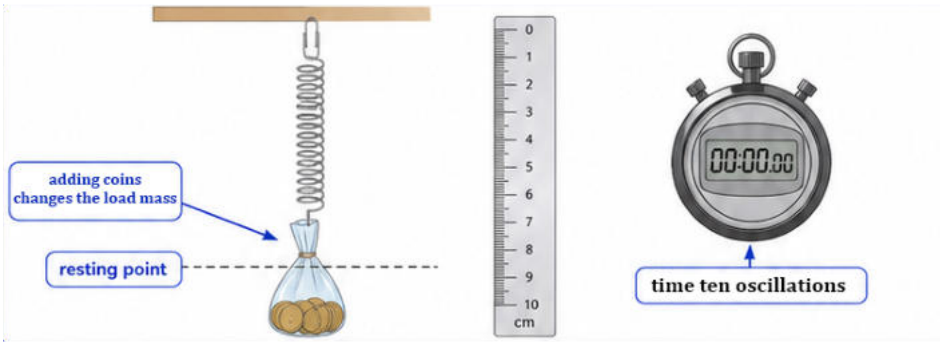


Figure 2-2-2 · Adding coins changes the load, and ten oscillations are timed.

Steps:

1. Hang the spring from a fixed support and attach the empty bag.
2. Add a few coins until the spring stretches by a measurable amount. Allow the system to come to rest. This position is the equilibrium position.
3. Pull the bag downward through a small distance and release it gently.
4. Measure the time for ten complete oscillations. One complete oscillation is motion from one extreme position back to the same extreme position.
5. Add more coins, allow the system to reach its new equilibrium position, and repeat the measurement.
6. Return to the first load. Repeat the measurement using a small initial displacement and then a larger initial displacement, keeping the mass unchanged.

(number of coins)	Initial displacement (cm)	Time for 10 complete oscillations (s)

Analyze what you collected:

(a) As the number of coins increased, did the time for ten oscillations increase or decrease? What does this suggest about the effect of mass on the period?

(b) For the same mass, did increasing the initial displacement significantly change the time for ten oscillations? What does this suggest about the dependence of period on amplitude for small oscillations?

Write your explanation in no more than two lines before you read the next page.

Measurement note

Time ten oscillations and divide, never one: a single oscillation is too quick to time accurately, and ten together reduce the percentage uncertainty in the timing.

From observation to model

In the Explore, one finding was left to one side: pulling the bag down further barely changed the timing. But an unchanged period does not mean an unchanged motion — a larger pull swept through the middle noticeably faster and snapped back harder. To read the whole swing — how fast, how hard, and how long each leg takes — we add three quantities to the model you already have.

Speed: fastest at the center

You met this shape once before: with the vibrating ruler in Lesson 2-1, the tip raced through the middle and stood still for an instant at each end. Here is that behavior with a value on it. The velocity varies as $v = A\omega \cos \omega t$ so the speed is greatest at the center O, where $v_{max} = A\omega$, and falls to zero at each end, where the bag turns around. This is why a bigger pull — a bigger amplitude A — is faster: the oscillator covers a longer path in the same period, so its maximum speed is greater.

Since $a = -\omega^2 x$, the acceleration is zero at the center, where the spring is at its resting stretch, and greatest at the ends, where the displacement from the equilibrium position is largest: $a_{max} = \omega^2 A$. So a bigger pull is not only faster through the middle but also more strongly accelerated at the turning points — the two effects that the period alone does not reveal.

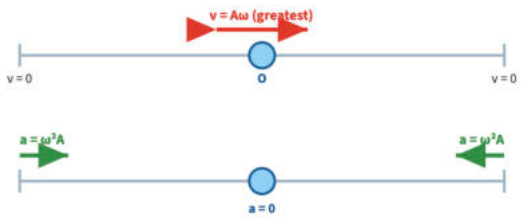


Figure 2-2-3 · Speed is greatest at the center, acceleration greatest at the ends.

Speed peaks at the center; the acceleration peaks at the ends. A larger amplitude raises both, but not the period.

Point

- The force that returns the object to the center is opposite to the displacement x, so it is written $F = -kx$ ($k > 0$), and the equation of motion for simple harmonic motion is $ma = -kx$. If the equation of motion is $ma = -kx$, the motion is simple harmonic.
- Speed and acceleration through the swing The speed is $v = A\omega \cos \omega t$: greatest, $A\omega$, at the **center** of the vibration and 0 at **both ends**. The acceleration is $a = -\omega^2 x$: 0 at the center and greatest, $\omega^2 A$, at both ends.
- The period does not depend on the amplitude: substituting $a = -\omega^2 x$ into $ma = -kx$ gives $T = 2\pi\sqrt{m/k}$. A weight hanging on a spring swings about its equilibrium position, not the natural length of the spring.

Equation box

Speed and acceleration in simple harmonic motion	$v_{max} = A\omega, \quad a_{max} = \omega^2 A$

Timing the swing

A whole cycle takes the period T. By the symmetry of the swing, each leg between an end and the center takes a quarter of it. So a bag released from an extreme first reaches the resting point O after

Time from an extreme to the center	$t = \frac{1}{4}T = \frac{\pi}{2}\sqrt{\frac{m}{k}}$

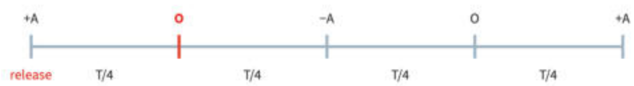


Figure 2-2-4 · One full cycle takes four quarter-periods.

Key fact

The size of the swing does not touch the period, but it does set the rest:

$v_{max} = A\omega$

$a_{max} = \omega^2 A$

and each leg from an end to the center takes $\frac{T}{4}$.

Data booklet:

The standard forms:

$v = \pm\omega\sqrt{(A^2 - x^2)}$

$a = -\omega^2 x$

At the center $v = v_{max}$, $a = 0$; at an end $v = 0$, $a = a_{max}$

Released at an end, the bag first passes O after one quarter of a period.

A hanging spring sets its own amplitude

The model told us to read a hanging spring at its resting point, a distance $x_0 = \frac{mg}{k}$ below the natural length. That one fact fixes the amplitude in a common set-up. Lift the bag until the spring returns to its natural length and let go from rest: it starts exactly x_0 above O, so the amplitude is $A = x_0$.

Its maximum speed at O is then $v_{max} = A\omega = g\sqrt{\frac{m}{k}}$, and the lowest point lies A below O — an extension of $x_0 + A = 2x_0$.

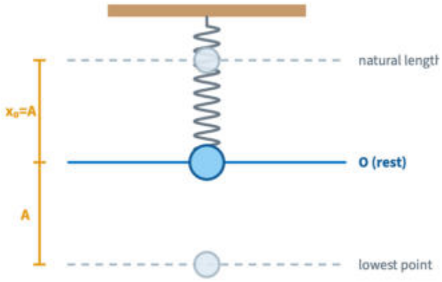


Figure 2-2-5

Quick application: A mass of 0.10 kg on a spring of spring constant k = 40 N/m is pulled 0.050 m from its resting point and released (π = 3.14). Determine its greatest speed and the time it takes to first reach the resting point.

Worked example

A light spring of constant k = 50 N/m carries a mass m = 2.0 kg. First it lies on a smooth horizontal surface; then the same spring hangs the same mass vertically. Each time the mass is pulled 0.08 m from its resting point and released gently. Take π = 3.14 and g = 9.8 m/s².

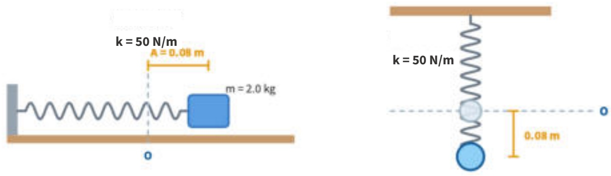


Figure 2-2-6 · The same spring and load, set horizontally and vertically.

Situation	It is used first on a smooth horizontal surface, then hung vertically from the same support; each time the mass is pulled A = 0.08 m from its resting point and released.
Given and required	m = 2.0 kg, k = 50 N/m, A = 0.08 m, π = 3.14, g = 9.8 m/s² **(a)** horizontal — the period, the speed at O, and the greatest acceleration. **(b)** the same spring hung vertically — the extension x_0 at O, the period, and the time to first reach O.
Representation	In both set-ups the spring gives a restoring force $F = -kx$, so the motion is simple harmonic and $T = 2\pi\sqrt{\frac{m}{k}}$ applies. Horizontally, k is read straight from the spring. Vertically, gravity first stretches the spring to a resting point O; measuring x from O, the weight cancels at the balance $kx_0 = mg$ and the force keeps the same form $F = -kx$.
Strategy	Build everything from $\omega = \sqrt{\frac{k}{m}}$: then $T = \frac{2\pi}{\omega}$, the speed at O is $v_{max} = A\omega$, and the greatest acceleration is $a_{max} = \omega^2 A$. The initial displacement is the amplitude A; it sets v and a, never T. For the vertical spring, find $x_0 = \frac{mg}{k}$; because the restoring force is identical, ω, T, v and a carry over, and the time from an extreme to O is a quarter-period, $\frac{T}{4}$.
Annotated solution	**(a) Horizontal** **1.** $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50}{2.0}} = \sqrt{25} = 5.0\ \text{rad s}^{-1}$ **2.** $T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{5.0} = 1.256 \Rightarrow T = 1.3\ \text{s}$ $\quad$ $\boldsymbol{T}$ **= 1.3** s **3.** speed at O: $v_{max} = A\omega = 0.08 \times 5.0 = 0.40$ m/s **4.** greatest acceleration: $a_{max} = \omega^2 A = 5.0^2 \times 0.08 =$ **2.0 m/s²** **(b) Same spring, vertical** **5.** extension at O: $kx_0 = mg \Rightarrow x_0 = \frac{mg}{k} = \frac{2.0 \times 9.8}{50}$ **= 0.39 m** **6.** measured from O: $F = mg - k(x + x_0) = -kx$ — the same form, so ω, T, v and a are unchanged: $\boldsymbol{T}$ **= 1.3** s, $\quad$ v_{max} **= 0.40 m/s**, $\quad$ a_{max} **= 2.0 m/s²** **7.** time to first reach O: $t = \frac{T}{4} = \frac{1.256}{4} = 0.314\ \text{s} = 0.31$ s

Key fact

The period needs only m and k — read k from the spring, or from the resting balance

$kx_0 = mg$.

Then

$$\omega = \sqrt{\frac{k}{m}}, \quad T = \frac{2\pi}{\omega}$$

The starting pull is the amplitude; it drives v_{max} and a_{max}, never T.

Reasonableness check	Units are right *(s, m, m/s, m/s²). T = 1.3 s means about 0.77 oscillations each second* — believable for a 2 kg mass on a fairly soft spring. Both set-ups share the same m, k and A, so they share the same period, the greatest speed and the greatest acceleration; gravity only lowered the center by $x_0 = 0.39$ m.
Explain it yourself	Why does hanging the spring move where the mass rests (by x_0) yet leave the period, the greatest speed and the greatest acceleration untouched?

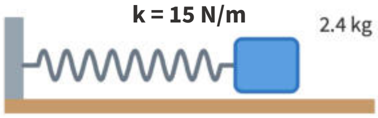
Now you try: a mass $m = 0.20$ kg hangs on a spring of spring constant $k = 80$ N/m and oscillates about its equilibrium position (π = 3.14). **Determine** the period of the oscillation.

Try

Determine the following to two significant figures, taking π = 3.14 and $g = 9.8$ m/s².

(a) One end of a light spring of spring constant $k = 15$ N/m is fixed to a wall on a smooth horizontal surface, and a block of mass 2.4 kg is attached to the other end. The block is pulled a little from its natural length and released gently. Determine the period of the oscillation in s.

Figure 2-2-7 · A 2.4 kg load on a spring of spring constant 15 N/m.



(b) The upper end of a light spring is fixed to the ceiling and a mass m is attached to the lower end; it balances at a point O where the spring is extended by x_0 from its natural length. The mass is then lifted until the spring reaches its natural length and released gently, so it undergoes simple harmonic motion.

(i) Determine the spring constant k (in terms of m, g, x_0).

(ii) Determine the speed v as the mass passes through O.

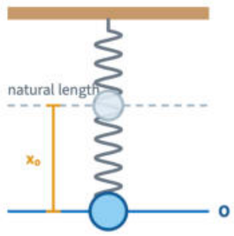


Figure 2-2-8 · The extension at rest fixes the equilibrium position.

In a new context — A fisherman's spring scale at Lake Nasser

A fisherman at Lake Nasser attaches a fish to a hand-held spring scale. The pointer moves downward and oscillates about the equilibrium position before coming to rest. The fish stretches the spring until the upward spring force balances its weight. Each small oscillation about the equilibrium position can be modeled as simple harmonic motion. The period is $T = 2\pi\sqrt{\frac{m}{k}}$. Gravity determines the equilibrium reading, while the period depends on m and k.

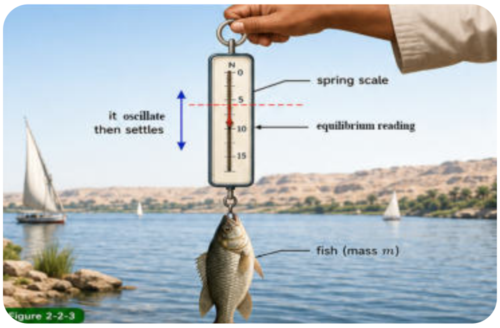


Figure 2-2-9 · The scale oscillates, then settles at the equilibrium reading.

Linking question

How does a particle's simple harmonic motion build up the wave model?

A heavier fish has a larger mass m. Using $T = 2\pi\sqrt{\frac{m}{k}}$, predict whether the fish oscillates with a longer or shorter period. Explain why a stiffer spring, with larger k, gives the opposite change in period.

Nature of Science — Science as a shared endeavor – the clock that kept accurate time at sea

A pendulum clock depends on gravity because the period of a simple pendulum depends on g. On a rolling ship, changes in the effective gravitational field can affect the motion of a pendulum clock. Marine chronometers developed by John Harrison used spring-controlled oscillators instead of pendulums, making them more suitable for use at sea.

Your turn: Suggest one reason a spring-regulated timekeeper was the better choice for a rolling ship. Link your reason to the result of this lesson.

Determine the spring constant k from $k x_0 = mg$, then **determine** the period of the oscillations.

Lesson question, answered

Attaching the same mass to a vertical spring instead of a horizontal spring does not change the period, provided the spring constant is the same and the oscillations are small. In the vertical case, gravity shifts the equilibrium position downward. At equilibrium, $kx_0 = mg$. When displacement is measured from this equilibrium position, the resultant force is $F = -kx$. Thus, $\omega = \sqrt{\frac{k}{m}}$ and $T = 2\pi\sqrt{\frac{m}{k}}$. Therefore, the vertical and horizontal systems have the same period for the same m and k.

The expression for the period contains no g. **Choose the equilibrium position as your origin and a hanging spring has the same period as a horizontal spring: same mass, same spring constant, same period.**

Figure 2-2-10 · The spring scale revisited.

Exam tip

Show the substitution line in every answer — examiners award marks for the working, not the answer alone.

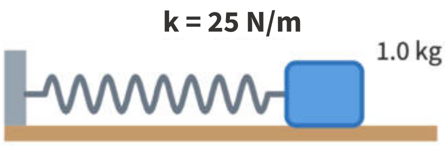
Exam-style question

A mass of 0.20 kg hangs on a light spring, stretching it by $x_0 = 0.049$ m at rest. Take $g = 9.8$ m/s^2, π = 3.14.

- State the spring constant k.
- Determine the period of small vertical oscillations.

Exercise

Determine the following to two significant figures, taking π = 3.14 and g = 9.8 m/s^2.



(a) A light spring of spring constant k = 25 N/m is fixed to a wall on a smooth horizontal surface, with a block of mass 1.0 kg on the other end. It is pulled a little from its natural length and released gently. Determine the period in s.

(b) A light spring of spring constant k = 25 N/m is fixed to a wall on a smooth horizontal surface, with a block of mass 0.80 kg on the other end. The block is pulled to a point 0.20 m from its equilibrium point O and released gently.

(i) Determine the period in s.

(ii) Determine the speed as it passes through O in m/s.

(iii) Determine the greatest magnitude of the acceleration in m/s^2.

(c) The upper end of a light spring of spring constant $k = 9.8$ N/m is fixed, and a mass 5.0×10^{-2} kg hangs from the lower end. The mass is pulled down 2.0×10^{-2} m from equilibrium and released. Determine the amplitude, the period, and the maximum speed.

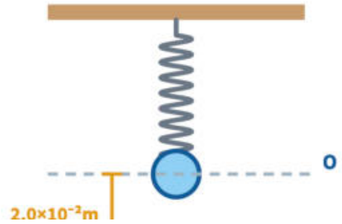


Figure 2-2-12 · The load released 2.0×10^{-2} m below the rest position.

(d) The upper end of a light spring of spring constant k is fixed, and a mass m hangs from the lower end. The mass is released from rest at the position where the spring is at its natural length, so it undergoes simple harmonic motion.

(i) Determine the amplitude, the period, and the maximum speed.

(ii) Determine the extension of the spring at the lowest point.

(iii) Determine the time from release until it first passes through the center of vibration.

(e) A light spring is fixed to the ceiling and a mass m hangs from it, in equilibrium at O where the spring is extended by x_0. It is pulled down d from O and released gently, vibrating vertically. Determine the speed as it passes through O (in terms of d, g, x_0).

Exam tip

The starting pull is the amplitude — it sets v_{max} and a_{max}, but never the period. Show every substitution line.

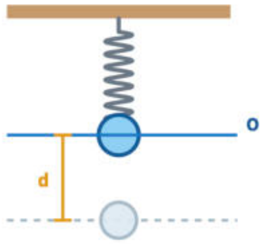


Figure 2-2-13 · A load released a distance d below the rest position.

Lesson 2-3 — The Simple Pendulum

❓ **Lesson question:** Does the mass of the bob or the amplitude of a small initial displacement affect the period of a simple pendulum?

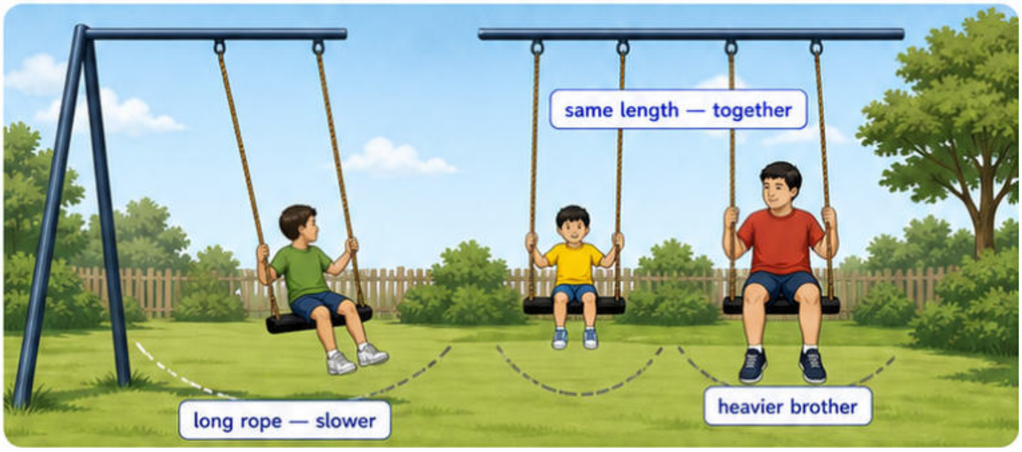
You will learn to

By the end of this lesson, you will be able to:

- Determine the period of a simple pendulum using $T = 2\pi\sqrt{\frac{L}{g}}$ for small angular displacements.
- Explain why the period of a simple pendulum depends on L and g, but not on the mass of the bob.
- Explain why the period is approximately independent of amplitude for small oscillations.

🔍 The phenomenon — predict before you continue

Figure 2-3-1 · Swings of the same length keep time together, whatever the rider's mass.



In a garden, two swings hang side by side from the same bar, with the same length of rope. A small child sits on one, his heavier brother on the other. They are released and — oscillation after oscillation — they stay together, side by side. At the club, a swing on a much longer rope keeps a longer period: it takes longer to complete one oscillation.

Predict before you continue — commit in writing:

(a) The brother is heavier. Will his swing take longer, shorter, or the same time per oscillation as the lighter child's swing, on equal ropes?

(b) Give one swing a bigger push so it sweeps a wider arc. Does one oscillation now take longer, shorter, or the same time?

⭐ **Key fact**

A motion is simple harmonic when the restoring force is directly proportional to the displacement from equilibrium and directed opposite to it: $F = -kx$. For a system of mass m, this gives $T = 2\pi\sqrt{(m/k)}$, where k is the effective force constant.

Explore — Measure the period while changing one variable

What you'll do: You will change one variable at a time: pendulum length, bob mass, or initial displacement, and measure the period of oscillation.

Time: 20 min

You will need:

- A thread of length 1–2 m.
- A small dense object to act as the pendulum bob.
- A second bob of different mass.
- A fixed support.
- A stopwatch or phone timer.
- A meter stick.

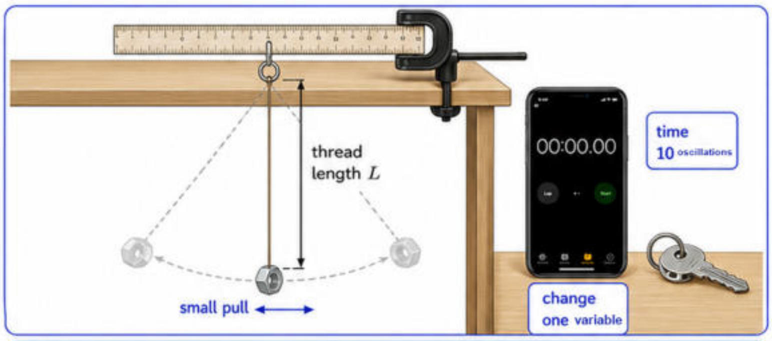


Figure 2-3-2 · The thread length is measured, and ten oscillations are timed.

Steps:

1. Tie the bob to the thread and suspend it from a fixed support. Measure the length (L) from the point of suspension to the center of the bob.
2. Displace the bob through a small angle and release it without pushing.
3. Measure the time for ten complete oscillations. One complete oscillation is the motion from one extreme position back to the same extreme position.
4. Divide the measured time by 10 to obtain the period (T).
5. Repeat the measurement after changing only the length.
6. Repeat the measurement after changing only the mass of the bob.
7. Repeat the measurement after changing only the initial displacement, while keeping the angular displacement within the small-angle approximation (typically less than about 10°).

Variable changed	Length L / m	Time for 10 oscillations t / s	Period T (t/10) / s
Initial pendulum			
Four times the length			
Heavier bob			
Larger initial angle (still less than 10°)			

Analyze what you collected:

(a) When the length was made four times larger, how did the period change? Compare your result with the prediction: $T \propto \sqrt{L}$

(b) Did changing the bob mass significantly affect the period?

(c) Did changing the initial displacement significantly affect the period when the angle remained small?

Write your explanation in no more than two lines before you read the next page.

Measurement note

Measure the time for ten complete oscillations, then calculate the period using $T = t/10$. Timing several oscillations reduces the percentage uncertainty caused by reaction time when starting and stopping the stopwatch.

From observation to model

In Explore, four times the length gave about double the period, while the heavier bob and the larger amplitude left it unchanged. You may have predicted that the heavier bob would lag or the wider push would increase the period — the timing disagrees. Here is the model your evidence points toward.

Resolving the weight into components

For a pendulum bob of mass m attached to a light string of length L, the weight mg acts vertically downward. The tension in the string acts along the string. The component of the weight tangential to the arc provides the restoring force: $F = -mg\sin\theta$, where θ is the angular displacement from the vertical. The negative sign indicates that the force acts toward the equilibrium position. **Only the along-arc part, mg sin θ, acts as the restoring force — always back toward O.**

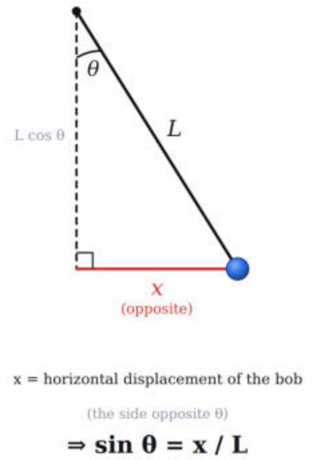


Figure 2-3-3 · The horizontal displacement of the bob is L sin θ.

> ⭐ **Simple pendulum**
>
> A simple pendulum consists of a small dense bob suspended from a fixed point by a light, inextensible string. For small angular displacements, its motion can be approximated as simple harmonic.

The small-angle approximation

When a pendulum bob is displaced through an angle θ, the tangential component of its weight acts as the restoring force:

$F = -mg\sin\theta.$

For small angular displacements, with θ measured in radians, $\sin\theta \approx \theta$. The displacement s of the bob along the circular arc is given by s = Lθ, so $\theta = s/L$. Therefore, $F \approx -mg\theta = -\dfrac{mg}{L}s.$

Hence, for small oscillations, the restoring force is directly proportional to the displacement from the equilibrium position and acts in the opposite direction. The pendulum therefore undergoes approximately simple harmonic motion, with an effective force constant $k = \dfrac{mg}{L}$.

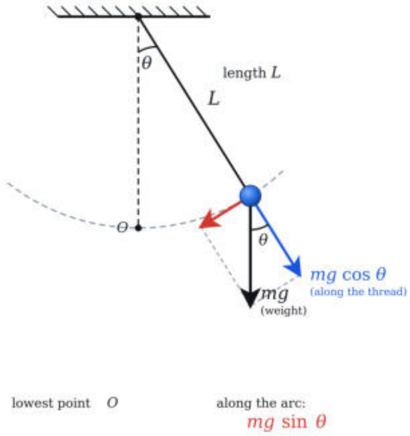


Figure 2-3-4 · Only the component along the arc restores the bob.

From the restoring force to the period — cancellation of the mass For small oscillations, the restoring force is $F = -\dfrac{mg}{L}s$. Comparing this with the standard form $F = -ks$ gives $k = \dfrac{mg}{L}$. Substituting into the general expression $T = 2\pi\sqrt{\dfrac{m}{k}}$ gives $T = 2\pi\sqrt{\dfrac{m}{mg/L}} = 2\pi\sqrt{\dfrac{L}{g}}$. Thus, the mass of the bob cancels, so the period is independent of the bob's mass and depends only on L and g, provided that the small-angle approximation applies.

Point

- ! For a bob of mass m [kg] on a thread of length L [m], the force acting in the tangential direction is the component of the weight in the direction of motion, $mg\ sin\ \theta$. Since its direction is opposite to the displacement s along the arc from the lowest point O, $F = -mg\sin\theta$.
- ! Where the swing angle θ is sufficiently small, $\sin\theta \approx \theta$, and the displacement along the arc is $s = L\theta$. Substituting gives $F = -\dfrac{mg}{L}s$: the restoring force is proportional to the displacement, so a simple pendulum with a small swing angle can be approximately regarded as simple harmonic motion.
- ! T = 2π√(L/g). It depends on the length L and on g, but not on the mass of the bob nor on the amplitude.

The mass m cancels between numerator and denominator.

Equation box

Period of a simple pendulum	$T = 2\pi\sqrt{\frac{L}{g}}$

In $\sqrt{\frac{m}{\left(\frac{mg}{L}\right)}}$ the mass m appears in both numerator and denominator and cancels, leaving $\sqrt{\frac{L}{g}}$. Thus, the period depends on the length L and the field strength g, but not on the mass of the bob. The amplitude never entered either: for a small angle the force stayed proportional to x. A small-amplitude oscillation has a period independent of the mass and amplitude — the **isochronism** that your Explore timings showed.

Put it to the test: hang two equal threads, a heavy nut on one and a light nut on the other; release them together from the same small angle and **show** they remain in phase for ten oscillations.

Determining the restoring force for a simple pendulum

For a spring, the restoring force is written directly as $F = -kx$. For a simple pendulum, the effective restoring force is obtained from the tangential component of the bob's weight. For small angles, $\sin\theta \approx \theta$ and $s = L\theta$, so $F = -mg\sin\theta \approx \frac{mg}{L}s$, giving $k = \frac{mg}{L}$. Substituting into $T = 2\pi\sqrt{\frac{m}{k}}$ gives $T = 2\pi\sqrt{\frac{L}{g}}$.

Worked example

A simple pendulum has a thread of length $L = 1.8$ m. Take $\pi = 3.14$ and the gravitational field strength $g = 9.8\ \mathrm{m/s^2}$. **Determine** the period of its oscillation, to two significant figures.

Situation	A small bob hangs on a light thread of length $L = 1.8$ m and swings through a small angle. Find the period T.
Given and required	$L = 1.8\ \mathrm{m} \cdot g = 9.8\ \mathrm{m/s^2} \cdot \pi = 3.14 \Longrightarrow T = ?$ (the mass is not given — and is not needed)
Representation	Small-angle swing $\Longrightarrow$ simple harmonic with effective force constant $k = mg/L$, so the period is $T = 2\pi\sqrt{\frac{L}{g}}$. Only L and g enter.
Strategy	Name the restoring force at displacement x: from the geometry $F = -(mg/L)x$, so $k = mg/L$; then $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{L}{g}}$. For a pendulum, substitute L and g directly.
Annotated solution	1) Name the restoring force (geometry gives k): $k = mg/L$, so the period is $T = 2\pi\sqrt{\frac{L}{g}}$ 2) Form the ratio L/g (one number): $L/g = 1.8 / 9.8 = 9/49$ 3) Substitute into $T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{9}{49}} = 2\pi \times \frac{3}{7} = \frac{2 \times 3.14 \times 3}{7} \Longrightarrow T \approx 2.7$ s
Reasonableness check	The answer carries the unit of time (s), as a period must; $\sqrt{\frac{9}{49}} = \frac{3}{7}$ is a useful check, and ~2.7 s is a reasonable period for a roughly 1.8 m swing — one oscillation about every three seconds.
Explain it yourself	The mass of the bob was never given. Why is the period still fully determined?

Measurement note

The gravitational field strength near Earth's surface is approximately $g = 9.8\ \mathrm{m/s^2}$. For pendulums at the same location, g is constant, so the period depends only on the length L.

Data booklet

for the pendulum, use $T = 2\pi\sqrt{\frac{L}{g}}$ directly; you only need L and g — the mass is not required and does not appear.

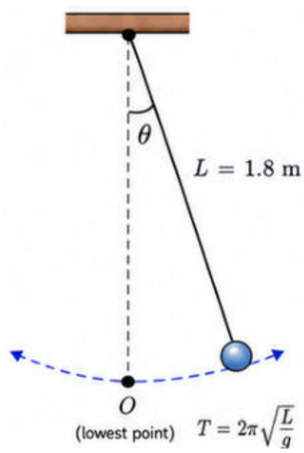


Figure 2-3-5 · A pendulum of length 1.8 m.

Linking question

How can the period of a simple pendulum be used to determine the gravitational field strength g?

Now you try: a simple pendulum has a thread of length $L = 1.8\ \mathrm{m}$ (π = 3.14, $g = 9.8\ \mathrm{m/s^2}$). **Determine** the length of thread that would make its period **half** as long. (Full reasoning in the unit answer section).

Try

(a) Determine the period, in s, of a simple pendulum whose thread length is 36 cm.

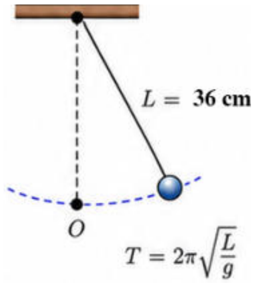


Figure 2-3-6 · A pendulum of length 36 cm.

(b) A simple pendulum has thread length L and period T. Determine the thread length that would make the period **half** as long.

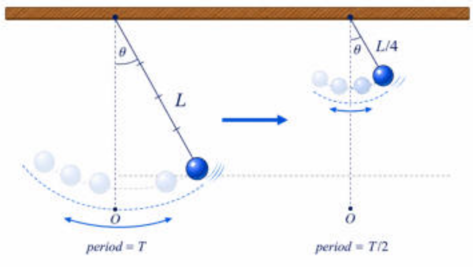


Figure 2-3-7 · Quartering the length halves the period.

In a new context — the Foucault pendulum in the Library of Alexandria

A long Foucault pendulum is suspended in the Planetarium Science Center of the Bibliotheca Alexandrina. Because its length is large, its period is longer than that of an ordinary garden swing. For small oscillations, its period is $T = 2\pi\sqrt{\frac{L}{g}}$. The Foucault pendulum shows that the Earth rotates. While the pendulum swings back and forth in the same direction, the Earth and the building turn underneath it. To us, it looks like the pendulum is slowly changing direction. However, this turning movement does not affect how fast the pendulum swings.

The Alexandria pendulum's cable is about 15 m long ($\pi = 3.14$, $g = 9.8\ \mathrm{m/s^2}$). Using $T = 2\pi\sqrt{\frac{L}{g}}$, estimate its period and comment on how it compares with the 2.7 s period of the 1.8 m pendulum.

Nature of Science: Evidence and patterns — the clock that revealed variation in g on Earth

In 1672, Jean Richer observed that a pendulum clock that kept time in Paris ran slow in Cayenne, near the equator. Since $T = 2\pi\sqrt{\frac{L}{g}}$, a longer period indicates a smaller value of g. This provided evidence that g varies slightly with location on Earth — being smaller near the equator.

Key fact

Each item here uses only L and g. Read $T = 2\pi\sqrt{\frac{L}{g}}$ as "two pi times the square root of length over g" substitute last, and answer to two significant figures.

Data booklet

Use π, g and $T = 2\pi\sqrt{\frac{L}{g}}$ from the booklet; your task is to choose the relation and substitute L and g into it, not recalling the formula.

Lesson question, answered

For small oscillations, neither the bob mass nor a small change in amplitude affects the period of a simple pendulum. The tangential component of the weight provides the restoring force $F = -mg \sin\theta$. For small angles $\sin\theta \approx x/L$, so $F = -(mg/L)x$. This is the form of simple harmonic motion with effective force constant k = $\mathbf{m}g/L$. Substituting into $T = 2\pi\sqrt{\frac{m}{k}}$ gives $T = 2\pi\sqrt{\frac{L}{g}}$.

Therefore, the period depends on L and g, but not on the bob mass. It is also approximately independent of amplitude for small angular displacements.

Exercise

(a) Determine the period, in s, of a simple pendulum whose thread length is **5.0 m**.

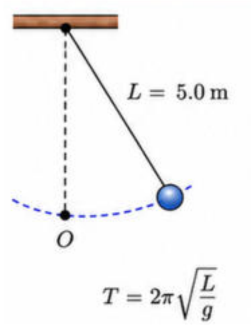


Figure 2-3-9 · A pendulum of length 5.0 m.

(b) Determine the period, in s, of a simple pendulum whose thread length is **7.2 m**.

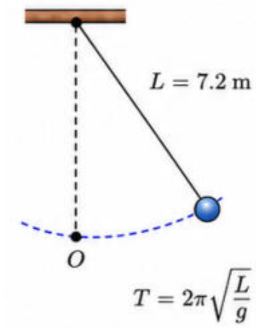


Figure 2-3-10 · A pendulum of length 7.2 m.

(c) Determine the factor by which the period of a simple pendulum changes, in each case (answers may be left in radical form):

(i) the amplitude is made half as large;

(ii) the thread length is doubled;

(iii) the mass of the bob is quadrupled.

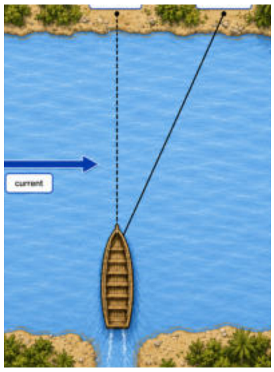
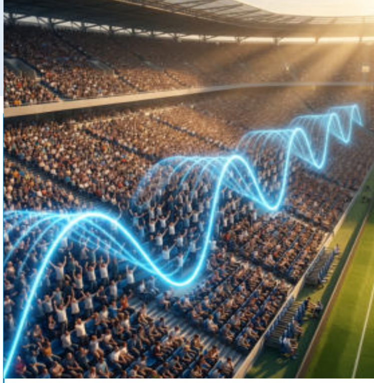


Figure 2-3-8 · A boat held by a rope swings like a pendulum in the current.

Exam-style question

A simple pendulum has a thread of length 2.0 m. Take π = 3.14 and g = 9.8 m/s^2. **Determine** the period of its small oscillation, and **state** what happens to that period if the bob is replaced by one of twice the mass.

Lesson 2-4 — Sine Waves



❓ **Lesson question**: In a stadium wave, the disturbance travels around the stadium while each spectator remains near their seat. What is transferred by the wave?

You will learn to

By the end of this lesson, you will be able to:

- Describe a wave as a traveling disturbance that transfers energy without net transfer of matter.
- Determine amplitude and wavelength λ from a displacement–position graph $(y-x)$, and period T from a displacement–time graph $(y-t)$.
- Calculate wave speed using $v = f\lambda$.
- Distinguish between transverse and longitudinal waves, and between mechanical and electromagnetic waves.

🔍 The phenomenon — predict before you continue

Figure 2-4-1 · The wave crosses the stand while each fan stays in their seat.

At a crowded football game, the crowd makes a “wave”: one block of fans stands and raises their arms, then sits as the next block stands. The disturbance travels all the way around the whole stadium in a few seconds — yet every single fan stays in their own seat. A disturbance clearly travels all the way round, but the people do not move along with it.

Predict before you continue

(a) In a stadium wave, do the spectators travel around the stadium with the wave, or do they remain near their positions?

(b) If the spectators move through a larger vertical displacement, does this necessarily increase the wave speed? Explain.

⭐ Key fact

In mechanical waves, each particle of the medium oscillates about its equilibrium position. The disturbance travels through the medium, but the particles of the medium do not travel with the wave.

🔍 Explore — Send a pulse along a rope and observe a marker

What you'll do: You will send a pulse along a stretched rope and observe the motion of a colored marker attached to the rope.

time = 12 min

You will need:

- A rope or washing line of length 3–4 m.
- A colored cloth marker.
- A fixed support or a partner to hold one end.

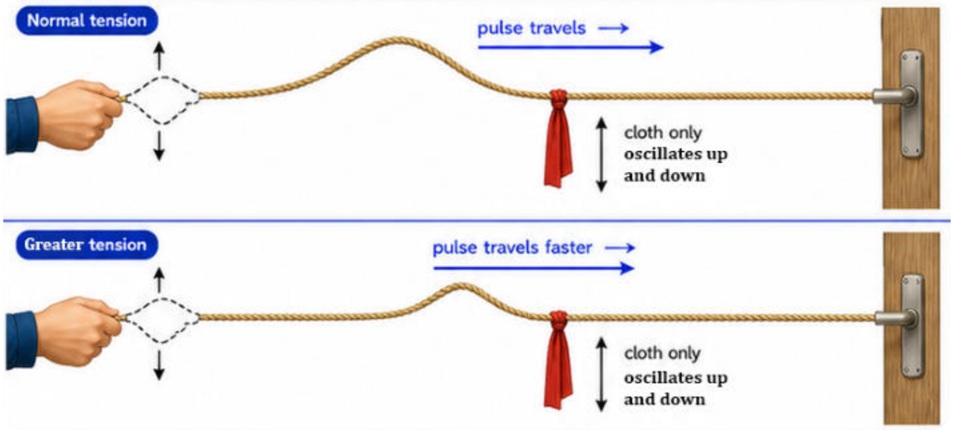


Figure 2-4-2 · Greater tension carries the pulse faster along the same rope.

Steps:

1. Stretch the rope and tie the colored marker near the middle.
2. Hold one end of the rope while the other end is fixed or held by a partner.
3. Move your end sharply up and down once to send a single pulse along the rope.
4. Observe whether the marker travels along the rope or oscillates about its equilibrium position as the pulse passes.
5. Increase the tension in the rope and repeat the pulse. Compare the speed of the pulse in the two cases.

Change made	Motion of the marker	Relative pulse speed
Normal tension		
Greater tension		

Analyze what you collected:

(a) As the pulse passed the marker, did the marker move along the rope or oscillate about its equilibrium position?

(b) How did increasing the rope tension affect the pulse speed? What does this suggest about the dependence of wave speed on the properties of the medium?

Write your explanation in two lines before you read the next page.

Measurement note

Send several pulses and watch the same strip each time: a single observation may not be sufficient for reliable conclusions, and repeating makes oscillation of the strip unmistakable.

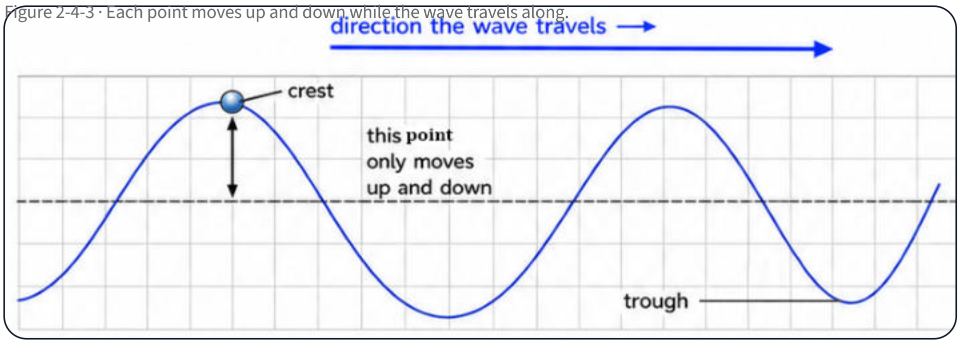
From observation to model

In Explore you saw the pulse travel along the rope while the cloth strip oscillated about its equilibrium position. You may have predicted that the cloth strip would be transported — it was not. And tightening the rope changed the pulse speed, even though the pulse was produced in the same way. Here is the model your evidence points toward.

A disturbance travels through a medium

A wave is a traveling disturbance that transfers energy from one place to another without net transfer of matter. In a mechanical wave, the disturbance travels through a material medium such as a rope, water, or air. Each small part of the medium oscillates about its equilibrium position as the wave passes.

Figure 2-4-3 · Each point moves up and down while the wave travels along.



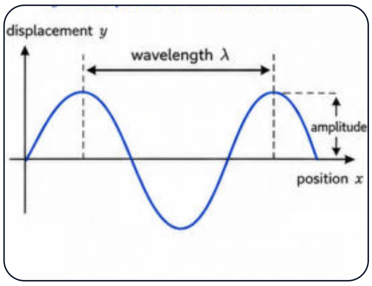
In the rope, the pulse travels along the rope, while each small section of the rope moves up and down about its equilibrium position. Therefore, the wave pattern and energy travel along the rope, but the material of the rope does not travel with the wave. If the source of a wave oscillates sinusoidally, the displacement of the medium can form a repeating sinusoidal wave. The highest points of the wave are called crests, and the lowest points are called troughs. The wavelength (λ) is the distance between two successive points that are in phase, such as crest to crest or trough to trough. It is measured in meters (m).

Reading displacement-position and displacement-time graphs

A displacement-position graph (y – x) shows the displacement (y) of points in the medium at different positions (x) at one instant. From this graph, the wavelength (λ) and amplitude (A) can be determined. It does not show the period directly.

A displacement-time graph (y – t) shows the displacement (y) of one fixed point in the medium as time (t) changes. From this graph, the period (T) and amplitude (A) can be determined. It does not show the wavelength directly.

Figure 2-4-4 · Displacement against position gives the wavelength.



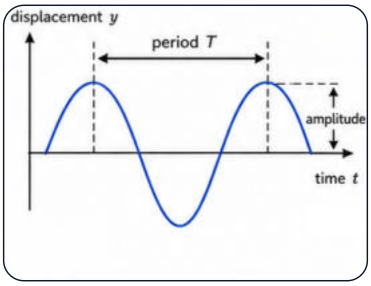


Figure 2-4-5 · Displacement against time gives the period.

The wave - speed relationship

A source produces f waves per second. During one second, the disturbance travels $f\lambda$, so the wave speed is $v = f\lambda$, and since frequency is one over the period, the speed is also the wavelength divided by the period.

wavelength

The distance between two successive points in phase, such as from one crest to the next crest or from one trough to the next trough, on a $y - x$ graph.

Exam tip

State which graph you read each quantity from: λ and amplitude from the $y - x$ (space) graph, period from the y – t (time) graph. Mixing the two is the most common error.

Point

- The phenomenon in which vibrations generated at a certain point propagate through the surrounding medium is called a wave. The point where the vibration starts is the wave source; the length of one complete wave is the wavelength λ [m]; and the number of vibrations per second is the frequency f [Hz].
- Letting the period be T [s] and the speed of the propagating wave be v [m/s]: $f = \frac{1}{T}$ and $v = f\lambda$. A wave whose displacement follows a sine curve is called a sine wave.
- Pay attention to the horizontal axis. A y–x graph represents the waveform at a certain time — it is equivalent to a photograph of the entire wave. A y–t graph represents one point of the medium over time — it is equivalent to a video of that point's movement.

Equation box

Wave equations	$v = f\lambda = \frac{\lambda}{T}$
Frequency and period	$f = \frac{1}{T}$

Put it to the test: Send a small-amplitude pulse, then a larger-amplitude pulse, along the same rope and watch the cloth strip: show that each pulse reaches it in the same short time — only the amplitude differs.

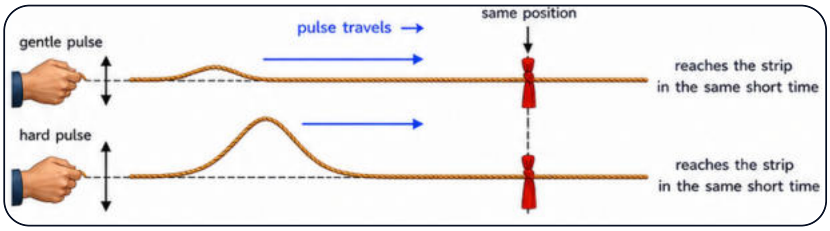


Figure 2-4-6 · Amplitude does not change the speed of the pulse.

Types of waves: transverse, longitudinal, mechanical, and electromagnetic

In a transverse mechanical wave, the particles of the medium oscillate perpendicular to the direction of wave travel. A wave on a rope is an example.

In a longitudinal wave, the particles of the medium oscillate parallel to the direction of wave travel. Sound in air is an example; it consists of compressions and rarefactions.

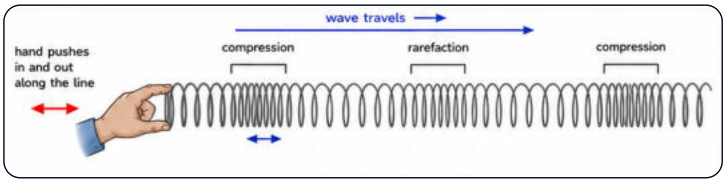
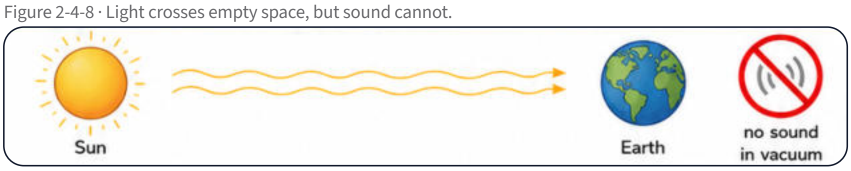


Figure 2-4-7 · A longitudinal wave: compressions and rarefactions along the line.

Electromagnetic waves

Sound is a mechanical longitudinal wave, so it requires a material medium and cannot travel through a vacuum.

Light is an electromagnetic wave. It can travel through a vacuum at $c = 3 \times 10^8\ \mathrm{m/s}$.

Figure 2-4-8 · Light crosses empty space, but sound cannot.


Key fact

For waves on the same rope under the same tension, increasing the amplitude increases the energy carried by the wave, but does not change the wave speed. The speed depends on the properties of the medium, such as tension and mass per unit length for a stretched string.

Linking question

How does simple harmonic motion help describe a sine wave?

In an ideal sine wave, each point of the medium oscillates about its equilibrium position, while the disturbance propagates through the medium.

Worked example

A sine wave travels in the positive x-direction. A displacement-position graph $(y-x)$ shows the waveform at time $t=0$ s. A displacement-time graph $(y-t)$ shows the motion of the point at $x=3.0$ m. Determine the amplitude, wavelength, and period.

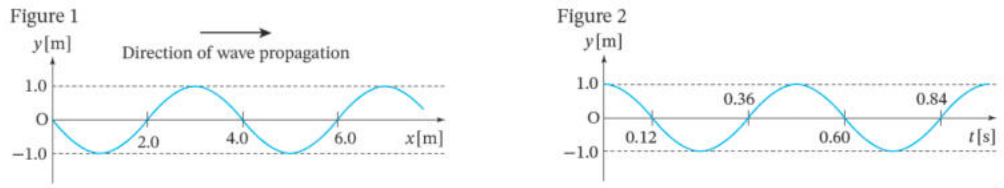


Figure 2-4-9 · The same wave read against position and against time.

Situation	The $y-x$ graph gives the waveform at t = 0 s; the $y-t$ graph gives the motion of the point at $x=3.0$ m.
Given and required	**Given:** y–x graph: crest height and one full cycle along x · $y-t$ graph: one full cycle along t. **Required:** amplitude = ? · wavelength = ? · period = ?
Representation	The two graphs give different quantities: take the wavelength and amplitude from the snapshot $(y-x)$, and the period from the time trace $(y-t)$. Mark one full cycle on each before reading off.
Strategy	From the y–x snapshot read the wavelength and amplitude; from the y–t trace of one point read the period; if a speed is then wanted, combine them with $v=f\lambda$.
Annotated solution	**1)** Amplitude from the height (it can be read from either graph — the crest reaches the same height): the crest is at $y=1.0$ m, so the amplitude is 1.0 m. **2)** Wavelength from the $y-x$ graph (one full crest-to-crest span along x): one cycle extends from 0 to 4.0 m, so the wavelength is 4.0 m. **3)** Period from the y–t graph (one full cycle along t): one cycle extends from 0.12 s to 0.60 s, so the period is $0.60-0.12=0.48$ s.
Reasonableness check	The amplitude and wavelength are read from the $(y-x)$ graph, so their units are meters. The period is read from the $(y-t)$ graph, so its unit is the second. This is consistent with the definitions of these quantities.
Explain it yourself	Why can the period be determined from the $y-t$ graph but not from the $y-x$ graph?

Now you try: a sine wave travels in the $+x$ direction; its y–x graph shows one complete cycle from $x=0$ to $x=2.0$ m with a crest height of 1.5 m, and its $y-t$ graph (for the point at $x=2.0$ m) shows one complete cycle over 0.40 s. **Determine** the amplitude, wavelength, and period.

Try

(a) An object in simple harmonic motion has a period of 2.0 s. Determine its frequency in Hz.

(b) A sine wave has a wavelength of 2.0 m and a frequency of 3.0 Hz. Determine the speed at which it travels in m/s.

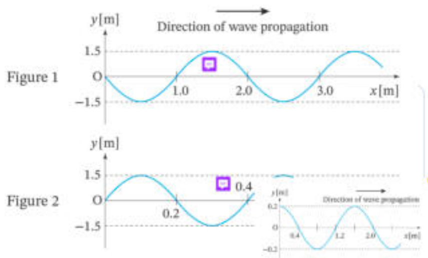


Figure 2-4-10 · Two graphs of one wave, for the worked example.

(c) For the sine wave of Figure 2-4-3 (y–x at t = 0 s; y–t for the point at x = 2.0 m), determine (i) the amplitude in m, **(ii)** the wavelength in m, **(iii)** the period in s.

(d) A sine wave moving in the +x direction at 4.0 m/s has the waveform of the graph at $t = 0$ s. Draw it at $t = 0.50$ s.

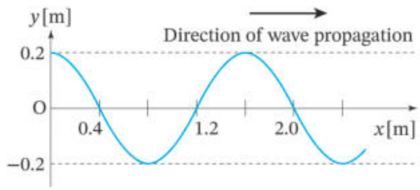


Figure 2-4-11 · A wave of amplitude 0.2 m.

In a new context — the two waves of an earthquake

When an earthquake occurs, it produces different types of seismic waves. Primary waves, or **P-waves**, are longitudinal and generally travel faster. Secondary waves, or **S-waves**, are transverse and travel more slowly. Both waves leave the source at nearly the same time, but the P-wave reaches a monitoring station first.

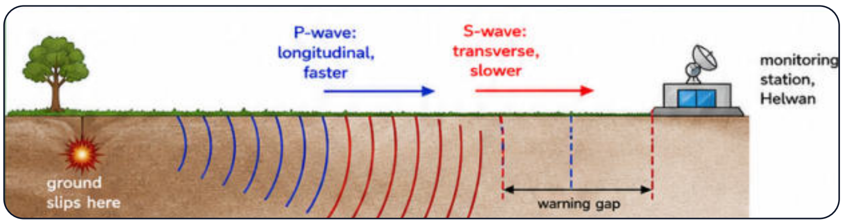


Figure 2-4-12 · The faster P-wave arrives first, leaving a warning gap.

Your turn: The P-wave and S-wave leave the source at nearly the same time, but the P-wave travels faster. Explain why the time gap between their arrivals increases for a station farther from the source.

Nature of Science: Models and evidence — Hertz detects Maxwell's predicted waves

Maxwell's electromagnetic theory predicted that light is an electromagnetic wave and that electromagnetic waves could be generated electrically. In 1887, Heinrich Hertz generated and detected electromagnetic waves in the laboratory using a spark transmitter and a receiver loop. This provided experimental evidence for Maxwell's prediction.

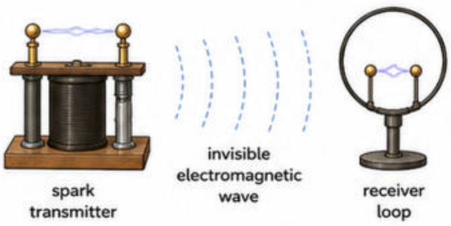


Figure 2-4-13 · Hertz's spark transmitter and receiver loop.

Your turn: Why was Hertz's experimental evidence essential for confirming Maxwell's prediction?

Exam tip

Show the substitution step — write $v = f\lambda$ with the numbers in before the result. For a moving-waveform sketch, shift the wave by $d = v \times \Delta t$ in the travel direction.

Exam tip

Quote $f = \frac{1}{T}$ and $v = f\lambda$, then show the substitution step — the marks are awarded for correct working.

Lesson question, answered

In a stadium wave, the disturbance and its energy travel around the stadium, but the spectators do not travel with the wave. Each spectator moves up and down about their position, then returns to its equilibrium position. Similarly, in a mechanical wave, the particles of the medium oscillate about their equilibrium positions while the wave transfers energy through the medium.

Figure 2-4-14 · The stadium wave revisited.

Exercise

(a) A simple harmonic motion has a period of 0.10 s. Determine its frequency in Hz.

(b) A simple harmonic motion has a period of 4.0 s. Determine its frequency in Hz.

(c) A sine wave has a period of 0.20 s and a wavelength of 3.0 m. Determine its speed in $\mathrm{m/s}$.

(d) A sine wave has a wavelength of 1.5 m and a frequency of 4.0 Hz. Determine its speed in $\mathrm{m/s}$.

(e) For the sine wave of the graphs (y–x at $t = 0$ s; $y - t$ for the point at $x = 2.0$ m), determine

(i) the amplitude in m

(ii) the wavelength in m

(iii) the period in s.

Figure 1

Figure 2

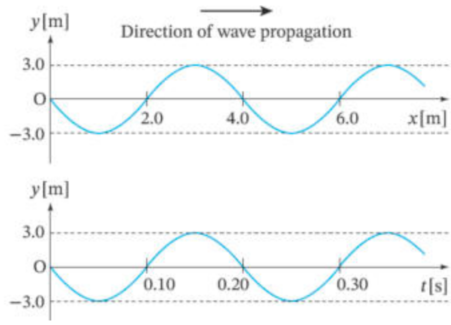


Figure 2-4-15 · Two graphs of one wave, for the exercise.

(f) A sine wave moving in the $+x$ direction at 1.0 m/s has the waveform of 2-4-8 at $t = 0$ s. Draw it at $t = 1.0$ s.

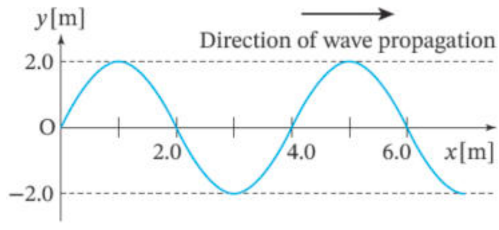


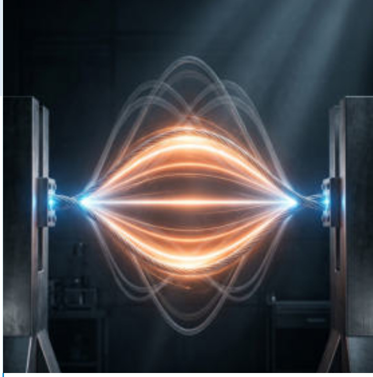
Figure 2-4-16 · A wave of amplitude 2.0 m.

Exam-style question

A sine wave on a string has a $y - x$ graph showing a wavelength of 0.60 m and a $y - t$ graph showing a period of 0.20 s.

Determine the frequency, then determine the wave speed in m/s

Lesson 2-5 — Wave Interference and Standing Waves



❓ **Lesson question**: When two speakers emit coherent sound waves of the same frequency, the sound may be loud at some positions and quiet at others. Under what conditions do the waves reinforce or cancel each other?

You will learn to

By the end of this lesson, you will be able to:

1. Apply the principle of superposition to determine the resultant displacement when two waves overlap.
2. Use path difference to predict constructive and destructive interference for coherent sources.
3. Describe how two waves of the same frequency and amplitude traveling in opposite directions can form a standing wave.

🔍 The phenomenon — predict before you continue

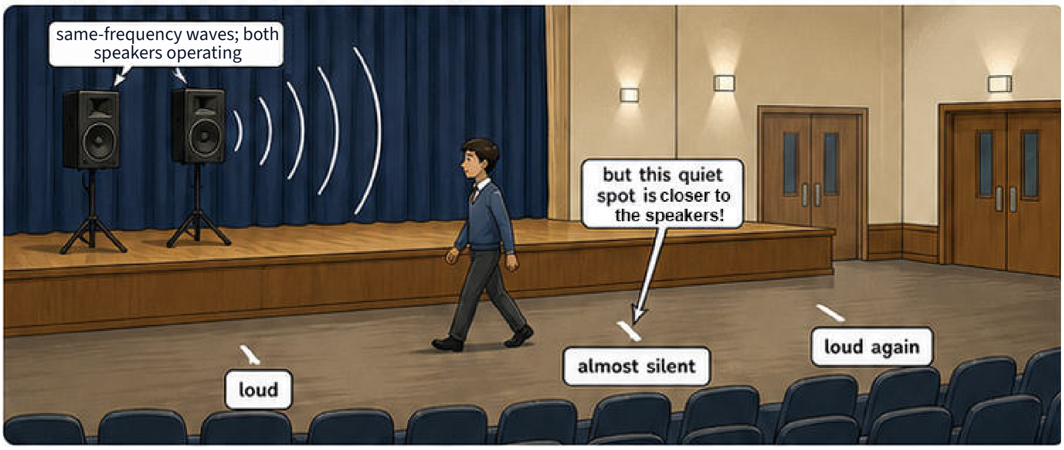


Figure 2-5-1 · Two speakers give loud and quiet spots across the hall.

Before a school concert, a steady test tone plays through the two stage speakers. As you walk along the front, the note is loud, then almost gone, then loud again — a repeating pattern. What is surprising is that this quiet spot is closer to the speakers than the next loud spot farther along your path.

Predict before you continue

(a) Is a point farther from the speakers always quieter? Explain why this may not be true when two waves interfere.

(b) If two identical speakers are used instead of one, must every point in the hall be louder? Explain.

⭐ **Key fact**

When two waves overlap at a point, the resultant displacement is the algebraic sum of the displacements due to the individual waves: $y = y_1 + y_2$ This is the principle of superposition. After overlapping, the waves continue to travel as before, provided the medium behaves linearly.

🔍 Explore — Observe the interference of water waves

What you'll do: You will create two sets of water ripples and observe their interference pattern.

Time: 12 min

You will need: • A wide shallow tray.

• Water about 1 cm deep.

• Two fingers or two pencil tips fixed a few centimeters apart.

• A bright lamp or phone flashlight.

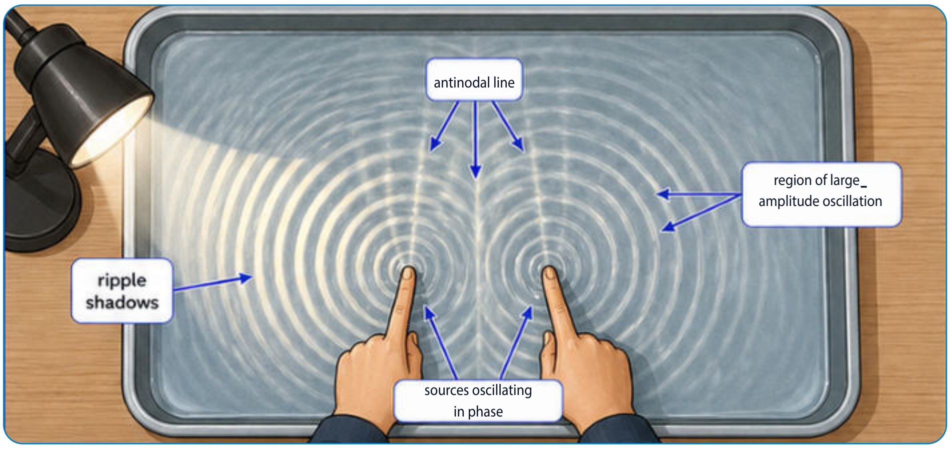


Figure 2-5-2 · Two sources in phase on a ripple tank make lines of large and small oscillation.

Steps:

1. Fill the tray with water and allow the surface to become still.
2. Dip both pencil tips or fingers into the water and tap the surface together at a steady rate.
3. Observe the ripple pattern and the shadows on the bottom of the tray.
4. Identify lines where the water remains nearly still and nearby regions where the water oscillates strongly.
5. Increase the tapping frequency and observe how the spacing between adjacent still lines changes.

Tapping rate	Number of still lines observed	Spacing between adjacent still lines
Slow and steady		
Faster and steady		

Analyze what you collected:

(a) Are the still lines approximately regularly spaced? What happens to their spacing when the tapping frequency is increased?

(b) Both sources tap in phase. Why do some points between the sources remain nearly still?

✏️ **Write your explanation in two lines before you read the next page.**

From observation to model

In Explore, lines along which the water stayed nearly still (nodal lines) formed in the interference pattern produced by the two in-phase sources, and the spacing between them decreased as the frequency increased. You may well have predicted "two sources, larger amplitude everywhere"; most people do. The sources continued to vibrate at the same rate, so the observed pattern results from the superposition of the two waves where they overlap.

Two waves at one point — the displacements add

When two waves meet at a point in a linear medium, the resultant displacement equals the algebraic sum of the displacements produced by the two waves: $y = y_1 + y_2$.

If two crests arrive together, the resultant displacement is larger; this is constructive interference. If a crest and an equal trough arrive together, the resultant displacement is zero; this is destructive interference.

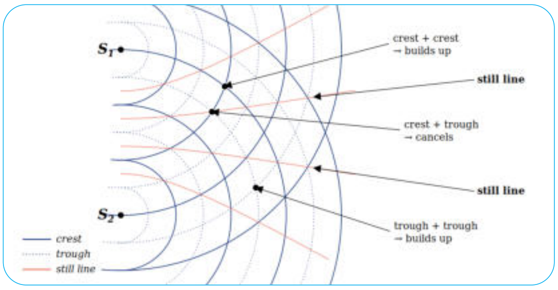


Figure 2-5-3 · Crest on crest builds up; crest on trough cancels.

The path difference decides — expressed in half-wavelengths

For two coherent sources S_1 and S_2, the path difference at a point P is: $\Delta d = |S_1P - S_2P|$ where S_1P and S_2P are the distances from the two sources to point P.

For two coherent sources vibrating in phase, constructive interference occurs when $\Delta d = n\lambda$, where $n = 0, 1, 2, 3, \ldots$. Destructive interference occurs when $\Delta d = \left(n + \frac{1}{2}\right)\lambda$, where $n = 0, 1, 2, 3, \ldots$.

If the two coherent sources vibrate in opposite phase, the conditions are reversed: path differences that produced constructive interference for in-phase sources produce destructive interference, and vice versa.

Point

- The quantity that represents the state of vibration of a medium — its displacement and its velocity — is called the **phase**. Two points in the same state of vibration are said to be in phase; two points whose displacements are equal and opposite at every instant, so that their directions of motion are opposite too, are in **opposite phase**.
- The phenomenon in which waves from coherent sources overlap and produce different effects from place to place is called the **interference** of waves. Where a crest meets a crest, or a trough meets a trough, the waves reinforce: this is constructive interference, and for two coherent in-phase sources the path difference is $n\lambda$.Where a crest meets a trough the waves cancel: this is destructive interference, and the path difference is $(n + ½)\lambda$.
- When two coherent waves of the same frequency and amplitude travel in **opposite directions** and interfere, the pattern no longer travels: a standing wave is formed. The points that never move are nodes, and the points that oscillate with the greatest amplitude are antinodes.

Equation box

Constructive interference (coherent, in-phase sources)	**path difference =** $n\lambda$
Destructive interference	**path difference =** $\left(n + \frac{1}{2}\right)\lambda$

Quick application: Two coherent in-phase sources send waves of wavelength 4.0 cm; the paths to a point differ by 6.0 cm. State the count of half-wavelengths, and whether the point undergoes constructive or destructive interference.

Key fact

The path difference at a point (P) is the difference between the distances from the two sources to that point:

$\Delta d = |S_1P - S_2P|$.

It determines whether the waves arrive in phase or out of phase.

Data booklet

$n\lambda$ and $\left(n + \frac{1}{2}\right)\lambda$ are printed in the wave section, where n is a whole number (0, 1, 2, …). Use the path difference and its number of half-wavelengths to choose the interference type.

Standing waves produced by waves traveling in opposite directions

A **standing wave** is formed when two coherent waves of the same frequency and amplitude travel in opposite directions and interfere. The resulting pattern does not travel along the medium. Instead, fixed points called nodes remain at zero displacement, while points called antinodes oscillate with maximum amplitude.

- **A node** is a point on a standing wave that remains at rest.
- **An antinode** is a point on a standing wave where the amplitude is maximum.

Resonance and standing waves. Resonance occurs when a system is driven at or near one of its natural frequencies, causing a large-amplitude oscillation. In a string fixed at both ends, only certain frequencies produce stable standing-wave patterns.

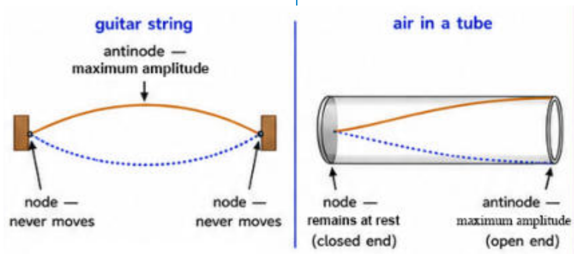
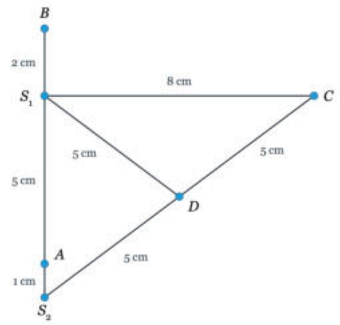


Figure 2-5-4 · Nodes and antinodes on a string and in a tube.

Worked example

Two-Source Interference at a Water Surface

Figure 2-5-5 · Path differences from the two sources to four points.



Situation	Two in-phase water-wave sources, S_1 and S_2, produce an interference pattern. The points A, B, C, and D are shown with their distances from the two sources, in centimeters. At any point P, the type of interference is determined by the path difference: $\Delta d = \lvert S_1P - S_2P \rvert$.
Given and required	**Given**: amplitude of each wave = 0.30 cm, wavelength $\lambda = 4.0$ cm, so $\lambda/2 = 2.0$ cm. **Required**: identify the constructive points among A, B, C, and D; find the number of constructive points P between S_1 and S_2; find the resultant amplitudes at C and D; and find the resultant amplitude at D when the sources emit in opposite phase.
Representation	Use the path difference $\Delta d = \lvert S_1P - S_2P \rvert$. For sources in phase, even multiples of $\frac{\lambda}{2}$ give constructive interference, and odd multiples of $\frac{\lambda}{2}$ give destructive interference.
Strategy	Since $\lambda/2 = 2.0$ cm, compare each path difference with multiples of 2.0 cm: 0, 4, 8, … cm are constructive; 2, 6, 10, … cm are destructive.
Annotated solution	**(a)** Path difference at each point: A: $\Delta d = 4\text{ cm} = 2(\frac{\lambda}{2}) \rightarrow$ constructive. B: $\Delta d = 6\text{ cm} = 3(\frac{\lambda}{2}) \rightarrow$ destructive. C: $\Delta d = 2\text{ cm} = 1(\frac{\lambda}{2}) \rightarrow$ destructive. D: $\Delta d = 0 = 0(\frac{\lambda}{2}) \rightarrow$ constructive. So, the constructive points are A and D. **(b)** Between S_1 and S_2, constructive interference occurs when $\Delta d = 0$ or 4 cm, giving three points: $(S_1P, S_2P) = (3\text{ cm}, 3\text{ cm}), (1\text{ cm}, 5\text{ cm}), (5\text{ cm}, 1\text{ cm})$. **(c)** At C, the interference is destructive, so the resultant amplitude is 0 cm. At D, the interference is constructive, so the resultant amplitude is $0.30 \times 2 = 0.60$ cm. **(d)** If the sources emit in opposite phase, D becomes destructive, so the resultant amplitude is 0 cm.

Reasonableness check	The results follow the rule for in-phase sources: equal paths or even half-wavelength differences give constructive interference; odd half-wavelength differences give destructive interference.
Explain it yourself	Find the path difference from S_1 and S_2, then compare it with $\frac{\lambda}{2}$. If the number of half-wavelengths is even, the interference is constructive; if it is odd, the interference is destructive.

Now you try: two coherent in-phase sources 10 cm apart, with wavelength 6.0 cm and amplitude 0.50 cm. At point P the two distances are $S_1P = 28$ cm and $S_2P = 37$ cm. **Determine** the path difference, its count of half-wavelengths, and the amplitude at P — first with the sources in phase, then in opposite phase.

Try

Determine the following.

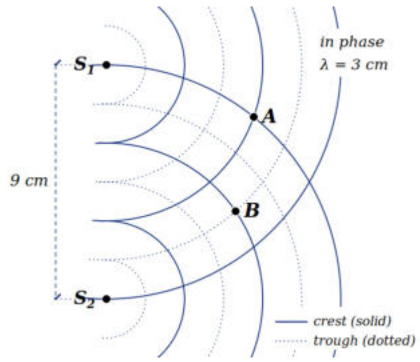


Figure 2-5-6 · Two sources 9 cm apart emitting waves of wavelength 3 cm.

(a) As shown in the figure, two sources S_1 and S_2, **9 cm apart**, vibrate in phase and emit waves of **wavelength 3 cm**; the figure shows crests as solid lines and troughs as dotted lines at one instant.

(i) Determine whether points A and B are points of constructive or destructive interference.

(ii) Determine how many still lines (lines of destructive interference) lie between S_1 and S_2.

(b) Two waves of **wavelength 5.0 cm** and **amplitude 1.0 cm** are emitted from two points S_1 and S_2. At point P the distances are $\mathbf{S_1P = 30}$ **cm** and $\mathbf{S_2P = 37.5}$ **cm**. Determine the amplitude at P:

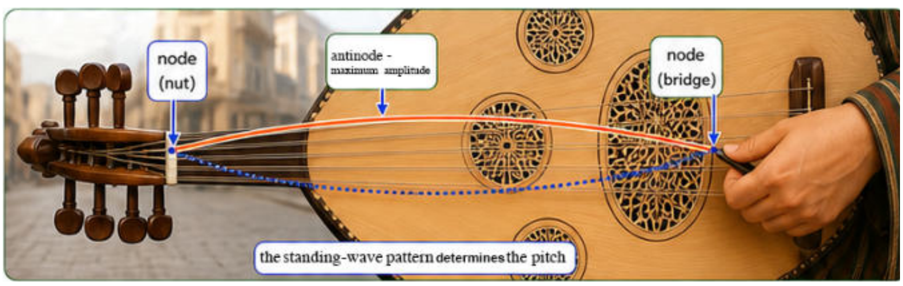
(i) when S_1 and S_2 vibrate in phase _______________

(ii) when S_1 and S_2 vibrate in opposite phase _______________

In a new context — the vibrating string of an oud

When an oud string is plucked, waves travel along the string and reflect at the fixed ends. The superposition of the incident and reflected waves can form a standing wave. Since the ends of the string are fixed, they must be nodes. In the fundamental mode, the midpoint of the string is an antinode.

Increasing the tension in the string increases the wave speed on the string. For the same vibrating length, this increases the natural frequencies, so the pitch becomes higher.



Exam tip

Show the path-difference line in every answer: write $|S_1P - S_2P|$ and its count of half-wavelengths before you state the result. Showing this step makes your reasoning clear and may earn method marks, depending on the mark scheme.

Linking question

How are the boundary conditions of a system linked to the standing waves it can support? A fixed end requires a node; the fundamental mode is the simplest standing wave those ends allow.

➡ The two ends of the plucked string are fixed and cannot move. **Explain** why these fixed ends must be nodes of the standing wave.

Nature of Science: Patterns and trends — a single principle governing water, sound and light

The same interference rules apply to water waves, sound waves, and light waves. Thomas Young's double-slit experiment showed bright and dark bands produced by constructive and destructive interference of light. This provided strong evidence that light has wave properties.

➡ **Suggest** why the same interference (reinforcement-and-cancellation) pattern occurs in water, sound, and light.

Lesson question, answered

When two coherent waves from sources vibrating in phase meet, their displacements add algebraically. At points where the path difference is a whole number of wavelengths, $\Delta d = n\lambda$, the waves arrive in phase and reinforce each other. At points where the path difference is an odd number of half-wavelengths, $\Delta d = (n + \frac{1}{2})\lambda$, the waves arrive in opposite phase and cancel each other. This explains why sound from two speakers can be loud at some positions and quiet at others.

Exam-style question

Two coherent sources emit identical water waves in phase. At point M the surface stays almost still.

Outline why M stays almost still.

Exercise

Determine the following.

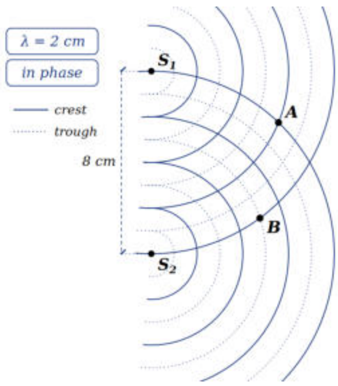


Figure 2-5-7 · Two sources 8 cm apart emitting waves of wavelength 2 cm.

(a) As shown in the figure, two sources S_1 and S_2, 8 cm apart, vibrate in phase and emit waves of wavelength 2 cm.

(i) Determine whether points A and B are points of constructive or destructive interference.

(ii) Determine how many very low intensity points (lines of destructive interference) lie between S_1 and S_2.

(b) Two points S_1 and S_2 on a water surface emit waves in phase, with amplitude 0.30 cm and wavelength 4.0 cm (attenuation ignored). At points A, B and C the path differences read off the figure are $|\Delta A| = 4$ cm, $|\Delta B| = 0$ cm and $|\Delta C| = 6$ cm. Determine the amplitude at A, B and C, in cm.

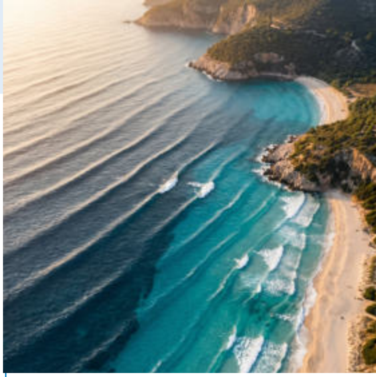
(c) Two waves of wavelength 6.0 cm and amplitude 0.40 cm are emitted from S_1 and S_2. At point P the distances are $S_1P = 20$ cm and $S_2P = 35$ cm. Determine the amplitude at P:

(i) when S_1 and S_2 vibrate in phase

(ii) when S_1 and S_2 vibrate in opposite phase

Key fact

For item (b), $\frac{\lambda}{2} = 2.0$ cm. Express each path difference as a multiple of $\frac{\lambda}{2}$, then determine whether the interference is constructive or destructive.



Lesson 2-6 — Refraction of Waves

❓ **Lesson question**: When a wave enters a medium in which its speed is different, why does its direction change at the boundary?

You will learn to

By the end of this lesson, you will be able to:

- **Determine** an unknown wave speed, wavelength, or angle at a boundary using Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$.
- **Explain** that refraction occurs because wave speed changes at a boundary, while frequency remains constant.
- **Use** $v = f\lambda$ to explain why the wavelength changes when the wave speed changes.

🔍 The phenomenon — predict before you continue

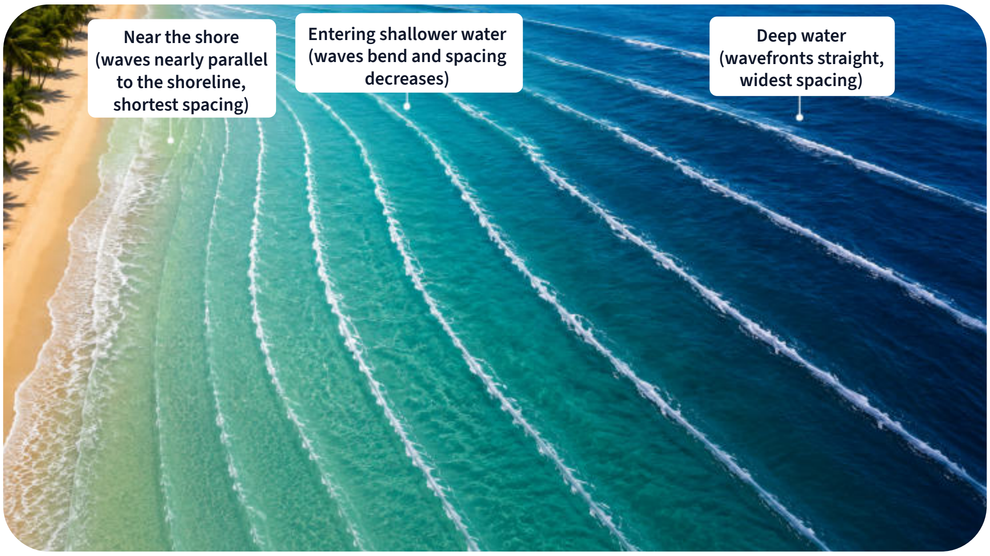


FIGURE 2-6-1 · Waves bend and their spacing shrinks in shallower water.

Stand on a North Coast beach and watch the sea. Far out, the wave crests approach at an oblique angle, angled across the water. But by the time they reach the sand, those same crests have refracted and arrive almost parallel to the shore — every time, whatever angle they started from in deep water offshore.

Predict before you continue

(a) Why do sea waves usually reach the shore with their crests nearly parallel to the shoreline? Does their direction remain unchanged, or does it change as the water becomes shallower?

(b) Water waves travel more slowly in shallow water. How does this change in speed affect the direction of the wavefronts as they approach the shore?

Key fact

For a wave, $v = f\lambda$, where v is wave speed, f is frequency, and λ is wavelength. A wavefront is a line joining points that are in phase, such as points on the same crest. A ray is a line drawn perpendicular to the wavefront, showing the direction of wave travel.

🔍 Explore — Observe refraction of plane water waves

What you'll do: You will observe how straight water waves change direction as they pass between regions of different depths.

Time: 12 min

You will need:

- A wide shallow tray.
- Water about 1 cm deep.
- A flat plate or tile to create a shallower region.
- A ruler to generate straight wavefronts.
- A flashlight or lamp.

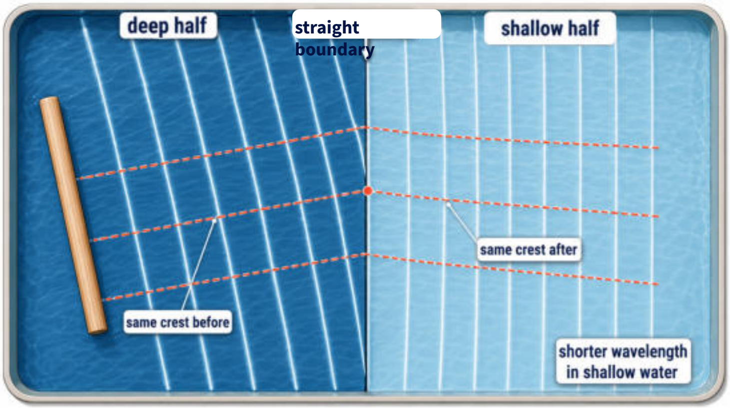


FIGURE 2-6-2 · THE WAVELENGTH SHORTENS IN THE SHALLOW HALF OF THE TANK.

Steps:

1. Place the plate in the tray so that one region is deeper and the other is shallower, with a straight boundary between them.
2. Illuminate the water from the side so that the wavefronts can be seen clearly.
3. Use the ruler to generate straight wavefronts in the deeper region, directed toward the boundary at an angle.
4. Sketch a wavefront just before it reaches the boundary and another just after it enters the shallower region.
5. Repeat using a smaller angle of incidence and compare the change in direction.

Incident wavefront	Angle to boundary before crossing	Angle to boundary after crossing	Direction of refraction
Steeper angle			
Smaller angle			

Analyze what you collected:

(a) When the waves entered the shallower region, did the wavefronts become more nearly parallel to the boundary?

(b) As the wave entered the shallower region, which quantities changed: speed, wavelength, or frequency? Which quantity remained constant?

✏️ **Write your explanation in no more than two lines before you read the next page.**

⭐ A measurement note

The bend is small and the crests move fast, so judge the angle from your sketch, not by direct observation in real time: record (sketch) one crest before the step and one after, then measure both angles from the same step line

From observation to model

In Explore, a crest crossing into the shallow, slower half rotated to become more nearly parallel to the boundary, while the rate at which crests arrived remained constant. While it might be predicted that wave crests keep traveling in their original direction or are deflected by the wind. Observations show that refraction occurs because the wave speed changes at the depth step. This evidence points toward the following model.

Wavefronts and rays

A wavefront is a line or surface joining points of a wave that are in phase. For water waves, a wavefront may be represented by a crest line. A ray is drawn perpendicular to the wavefront and indicates the direction of wave propagation.

At a boundary, the angle of incidence and the angle of refraction are measured from the normal, which is a line perpendicular to the boundary.

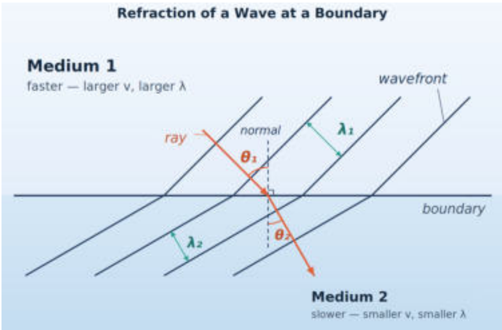


FIGURE 2-6-3 · AT THE BOUNDARY THE WAVEFRONTS CHANGE DIRECTION AND SPACING.

Why a change of speed refracts the wave

When a wavefront reaches a boundary at an angle, one side of the wavefront enters the new medium before the other side. If the wave speed is different in the new medium, that side changes speed before the other, causing the wavefront to rotate. This change in direction is called refraction.

If the wave enters a slower medium, the ray bends toward the normal. If it enters a faster medium, the ray bends away from the normal.

The frequency of the wave is determined by the source and remains constant as the wave crosses the boundary. Since $v = f\lambda$, a decrease in wave speed (v) at constant frequency (f) causes the wavelength (λ) to decrease.

The refraction relation

For waves crossing a boundary, $\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{v_1}{v_2} = \dfrac{\lambda_1}{\lambda_2}$

For light, Snell's law is often written as: $n_1 \sin\theta_1 = n_2 \sin\theta_2$ where n is the refractive index.

Refraction

the bending of a wave's direction of propagation at a boundary, caused by the change in its speed from one medium to the other

Point

- When a wave propagates through space, the locus of all points which oscillate in phase is called a wavefront. A wavefront moves with time in a direction **perpendicular to itself**. An arrow drawn perpendicular to a wavefront in the direction of propagation of the wave is called a ray.
- The angle i between the normal and the incident ray is the angle of incidence; the angle r **between the normal** and the refracted ray is the angle of refraction.
- When a wave travels from medium 1 (speed v_1, wavelength λ_1) to medium 2 (speed v_2, wavelength λ_2), the frequency is unchanged, so: $\dfrac{\sin i}{\sin r} = \dfrac{v_1}{v_2} = \dfrac{\lambda_1}{\lambda_2} = n_{12}$, and n_{12} is the refractive index **of medium 2 relative to medium 1**.

The booklet form $n1 \sin\theta 1 = n2 \sin\theta 2$ says the same thing.

Equation box

Refraction of waves	$n_1 \sin\theta_1 = n_2 \sin\theta_2$ For waves: $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

Quick application: A wave crosses a boundary with $\theta_1 = 45°$ and $\theta_2 = 30°$ (take $\sqrt{2} = 1.4$). **Determine** the ratio $\frac{v_1}{v_2}$ of the speeds.

Worked example

The dashed lines in the figure are the wavefronts as water waves pass from medium 1 into medium 2. The wavelength of the incident wave in medium 1 is **1.4 cm** and the frequency is **50 Hz**. The ray makes **45°** with the normal in medium 1 and **30°** in medium 2. Take $\sqrt{2} = 1.4$. **Determine** (a) the refractive index of medium 2 relative to medium 1, (b) the wave speeds in both media, and (c) the wavelength in medium 2 — each to two significant figures.

Problem	The dashed lines are wavefronts as water waves pass from medium 1 into medium 2. In medium 1, $\lambda_1 = 1.4$ cm and $f = 50$ Hz; the ray meets the normal at 45° in medium 1 and 30° in medium 2. Take $\sqrt{2} = 1.4$. Find, to 2 s.f.: (a) the refractive index n_{12}; (b) the speeds v_1 and v_2; (c) the wavelength λ_2.
Situation	Water waves cross a boundary. Medium 1: $\lambda_1 = 1.4$ cm, $f = 50$ Hz, ray at $\theta_1 = 45°$ to the normal. Medium 2: *ray at* $\theta_2 = 30°$. Find the relative index, both wave speeds, and λ_2.
Given and required	**Given:** $\lambda_1 = 1.4$ cm · $f = 50$ Hz · $\theta_1 = 45°$ · $\theta_2 = 30°$ · $\sqrt{2} = 1.4$ **Required:** $n_{12} = ?$ · $v_1 = ?$ · $v_2 = ?$ · $\lambda_2 = ?$
Representation	In medium 2 the wavefronts are spaced more closely and lie nearer the boundary (a smaller angle), so the ray bends toward the normal ($\theta_2 < \theta_1$): medium 2 is the slower medium. The frequency is unchanged across the boundary — only v and λ change.
Strategy	Draw the normal, read θ_1 and θ_2, then use one constant ratio $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$ and read off each unknown.
Annotated solution	The angles give the index — take the ratio of the sines. $n_{12} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1}{\sqrt{2}} \times 2 = \sqrt{2} = 1.4$ First speed *from* $v = f\lambda$ (one product). $v_1 = f\lambda_1 = 50 \times 1.4 = 70$ cm/s Second speed from the same ratio (one division). $v_2 = \frac{v_1}{\sqrt{2}} = \frac{70}{\sqrt{2}} \approx 50\ \mathrm{cm\,s^{-1}}$ Wavelength from the same ratio (one division). $\lambda_2 = \frac{\lambda_1}{\sqrt{2}} = \frac{1.4}{\sqrt{2}} \approx 1.0$ cm $n_{12} = 1.4$ $v_1 = 70$ cm/s $v_2 = 50$ cm/s $\lambda_2 = 1.0$ cm
Reasonableness check	The ray bends toward the normal in medium 2, so medium 2 is the slower medium. Therefore $v_2 < v_1$ and $\lambda_2 < \lambda_1$, while the frequency remains unchanged.
Explain it yourself	The frequency, 50 Hz, was used only once — to find v_1. Why was it not needed again to find v_2? Idea: v_2 comes from the fixed ratio $v_1 : v_2 = \sqrt{2} : 1$, which contains no frequency. f is unchanged across the boundary, so once $v_1 = f\lambda_1$ is known, the ratio gives v_2 directly.

Exam tip

If n_{12} denotes the refractive index of medium 2 relative to medium 1, then: $n_{12} = \frac{\sin i}{\sin r} = \frac{n_2}{n_1}$ Therefore, the sine of the angle of incidence is in the numerator. The booklet form $n_1 \sin\theta_1 = n_2 \sin\theta_2$ expresses the same relationship. Use the form specified in the question.

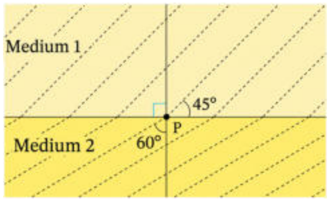


FIGURE 2-6-4 · WAVEFRONTS AT 45° IN MEDIUM 1 AND 60° IN MEDIUM 2.

Data booklet

Quote $n_1 \sin\theta_1 = n_2 \sin\theta_2$, and remember that for waves it is the constant ratio $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$. Pick the relation, read the angles from the wavefront geometry, then the speeds or wavelengths follow

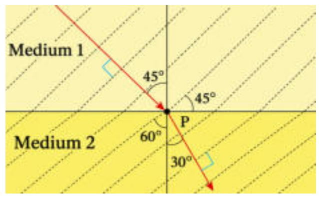


FIGURE 2-6-5 · THE RAY BENDS TOWARD THE NORMAL ON ENTERING THE SLOWER MEDIUM.

Now you try: water waves cross from medium 1 into medium 2 with the ray at 45° to the normal in medium 1 and 60° in medium 2; the speed in medium 1 is 60 cm/s and the frequency is 5.0 Hz (take $\sqrt{2} = 1.4, \quad \sqrt{3} = 1.7$). Determine the relative index and the wave speed in medium 2, and state which medium is faster.

Put it to the test: in your water tray, count crests passing per second on the deep side and the shallow side, and show the frequencies are equal.

Try

The dashed lines in the figure are the wavefronts as water waves pass from medium 1 into medium 2. The ray is at 60° to the normal in medium 1 and 30° in medium 2. The speed of the incident wave in medium 1 is 30 cm/s and the frequency is 4.0 Hz. Take $\sqrt{3} = 1.7$. Determine the following to two significant figures.

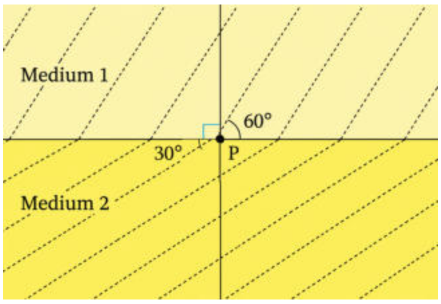


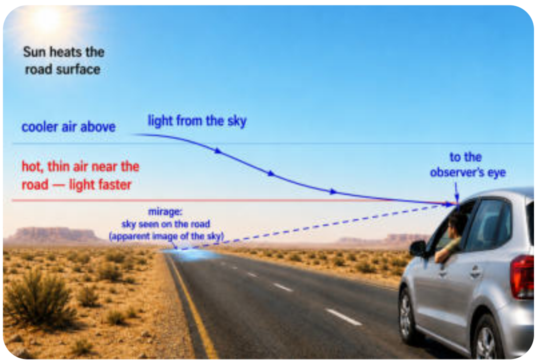
FIGURE 2-6-6 · WAVEFRONTS AT 60° IN MEDIUM 1 AND 30° IN MEDIUM 2.

(a) Draw, on the figure, the ray showing the direction the wave travels in medium 1 and in medium 2.
(b) Determine the refractive index of medium 2 relative to medium 1.
(c) Determine the wavelengths of the waves in medium 1 and in medium 2, in cm.
(d) Determine the wave speed in medium 2, in cm/s.

In a new context— Mirage over a desert road

On a hot desert road, the air just above the road surface is hotter and less dense than the air above it. Light travels faster in this hot air. As light passes through layers of air with gradually changing refractive index, its path bends gradually. Light from the sky can reach the observer's eye from a direction close to the road surface, so the road may appear to reflect the sky like water.

FIGURE 2-6-7 · HOT AIR NEAR THE ROAD BENDS LIGHT UPWARD, SHOWING A MIRAGE.



Your turn: Each thin layer of air near the road changes the speed of light slightly. Explain why the light ray bends near the road. State whether the ray bends toward or away from the normal as it enters a faster layer.

Nature of Science: Science as a shared endeavor — one law, two names, two countries

The law of refraction is often called Snell's law, after Willebrord Snellius, who derived it around 1621. René Descartes later published the same relation in 1637 in La Dioptrique. Publishing scientific results allows other scientists to test, use, and develop them.

Key fact

Read the law as "n-one sine theta-one equals n-two sine theta-two." This is equivalent to the constant ratio $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$. Draw the normal first, read both angles from it, substitute the numerical values last, and answer to two significant figures.

Linking question

How are the wave and ray models of refraction related?

Your turn: suggest one reason why a discovery that is written down but remains unpublished, like Snellius's, contributes less to science than one that is published, like Descartes's.

Lesson question, answered

When a wave crosses into a medium where it travels at a different speed, why does its direction bend at the boundary? Because a wavefront meeting the boundary at a slant has one end change speed before the other, so the crest line pivots. The frequency is set by the source and cannot change, so by $v = f\lambda$ a smaller speed corresponds to a smaller wavelength. The two angles are then related to the two speeds by $n_1 \sin\theta_1 = n_2 \sin\theta_2$ — the constant ratio $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$. The ray bends toward the normal when the wave enters a slower medium and away from the normal when it enters a faster medium.

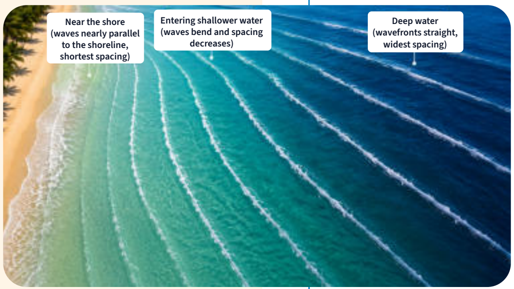
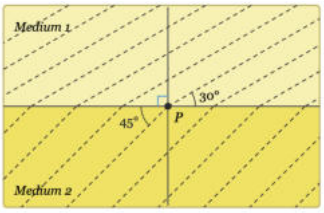


FIGURE 2-6-8 · THE SHORELINE WAVES REVISITED.

Exercise

(a) The dashed lines in Figure 2-6-9 represent the wavefronts when water waves travel from medium 1 to medium 2. The speed of the incident wave in medium 1 is 35 cm/s, and the frequency is 50 Hz.

FIGURE 2-6-9 · WAVEFRONTS AT 30° AND 45° ACROSS THE BOUNDARY.



(i) Draw, in the figure, the direction in which the wave passing through point P travels in media 1 and 2, respectively.

(ii) Determine the refractive index of medium 2 relative to medium 1.

(iii) Determine the wave speed in medium 2, in cm/s.

(iv) Determine the wavelength of the wave in medium 2, in cm.

(b) The dashed lines in Figure 2-6-10 represent the wavefronts when water waves travel from medium 1 to medium 2. The speed of the incident wave in medium 1 is 2.0 m/s, and the frequency is 5.0 Hz.

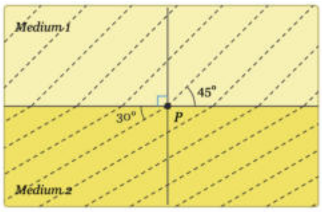


FIGURE 2-6-10 · WAVEFRONTS AT 45° AND 30° ACROSS THE BOUNDARY.

(i) Draw, in the figure, the direction in which the wave passing through point P travels in media 1 and 2, respectively.

(ii) Determine the refractive index of medium 2 relative to medium 1.

(iii) Determine the wave speed in medium 2, in m/s.

(iv) Determine the wavelengths of the waves in media 1 and 2, respectively, in m.

(c) The dashed lines in Figure 2-6-11 represent the wavefronts when water waves travel from medium 1 to medium 2. The wavelength of the incident wave in medium 1 is 2.0 cm, and the frequency is 5.0 Hz.

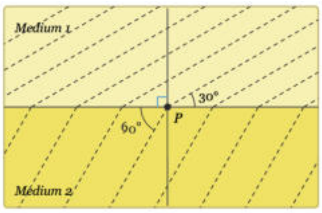


FIGURE 2-6-11 · WAVEFRONTS AT 30° AND 60° ACROSS THE BOUNDARY.

(i) Draw, in the figure, the direction in which the wave passing through point P travels in media 1 and 2, respectively.

(ii) Determine the refractive index of medium 2 relative to medium 1.

(iii) Determine the wave speeds in media 1 and 2, respectively, in cm/s.

(iv) Determine the wavelength of the wave in medium 2, in cm.

Data booklet

Quote $n_1 \sin\theta_1 = n_2 \sin\theta_2$ from the booklet; for waves write it as $\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$. Your job is choosing the relation and substituting the angles and speeds into it, not recalling the formula.

Exam tip

Show the sine-ratio relation — $\frac{\sin\theta_1}{\sin\theta_2}$ — and the substitution that follows it in every answer; examiners credit the working, not the final number alone

Exam-style question

A water wave passes from medium 1 into medium 2. The ray makes 60° with the normal in medium 1 and 45° in medium 2 (take $\sqrt{2} = 1.4$, $\sqrt{3} = 1.7$).

Determine the refractive index of medium 2 relative to medium 1, and state whether the wave speed increases or decreases.

Lesson 2-7 — Interference of Sound

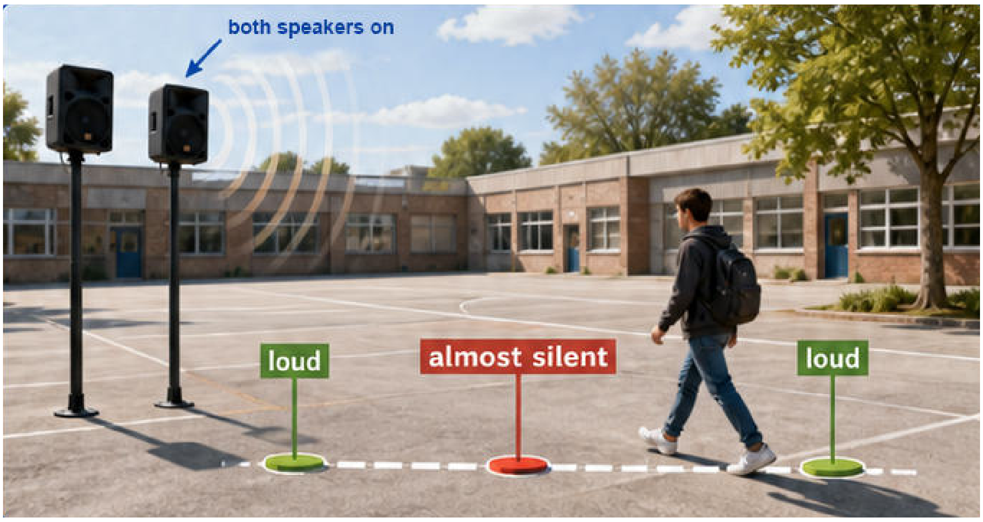
❓ **Lesson question:** When two loudspeakers emit the same steady tone, why is the sound loud at some positions and nearly silent at others?

By the end of this lesson, you will be able to:

1. **Explain** how two coherent sound sources produce positions of constructive and destructive interference.
2. **Predict** whether constructive or destructive interference occurs at a point from its path difference.
3. **Determine** the wavelength and frequency of sound using interference measurements.

🔍 The phenomenon — predict before you continue

Figure 2-7-1 · Walking past two speakers gives loud and almost silent points.



Before the morning announcements, the custodian tests the two courtyard loudspeakers with one steady tone. Crossing the yard, you walk through a region where the tone becomes almost inaudible — two steps later it returns to maximum intensity. Your friend, crossing elsewhere, detects no such variation. Both loudspeakers remain on throughout.

Predict before you continue

(a) You walk slowly across the yard while the tone plays. What will you hear: the same loudness everywhere, a gradual decrease in intensity with distance, or a repeating pattern of loud and quiet?

(b) A friend claims that with two loudspeakers on, every point in the yard must be louder than with one. Do you agree?

⭐ Key fact

A pure tone has a single frequency (f). In air, its wavelength is related to the speed of sound by: $v = f\lambda$. When two sound waves overlap, the resultant displacement of air particles is the algebraic sum of the individual displacements. This is the principle of superposition.

Locate positions of constructive and destructive sound interference

What you'll do: You will use two coherent sound sources to locate positions of constructive and destructive interference.

Time: 12 min

You will need:

- A phone with a tone-generator app.
- Sticky tape.
- Paper markers.
- Wired earphones.
- A meter stick or tape measure.
- A quiet open space.

Sound safety: Keep the volume low enough to be comfortable and safe.

Steps:

1. Tape the two earbuds to the edge of a table, facing the observer and separated by about 0.50 m.
2. Set the tone-generator app to a steady tone of 1700 Hz at a safe, low volume.
3. Walk slowly along a line about 2 m from the table, parallel to the line joining the earbuds.
4. Keep one ear facing the earbuds and cover the other ear.
5. Place a paper marker at each position where the sound intensity is at or near a minimum.
6. Repeat the walk and keep only positions that are reproducible.
7. Repeat using 3400 Hz and compare the spacing between adjacent quiet positions.

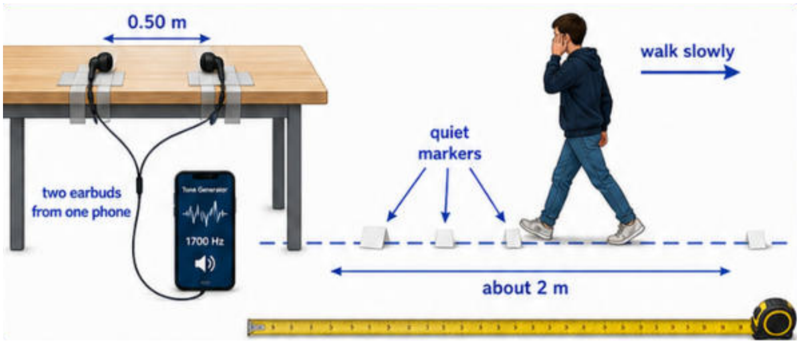


Figure 2-7-2 · Two earbuds 0.50 m apart, with the quiet points marked along the walk.

Trial	Frequency (Hz)	Number of quiet positions	Spacing between adjacent quiet positions (m)
1	1700		
2	1700		
3	1700		
4	3400		

Analyze what you collected:

(a) Near the middle of the path, are the quiet positions approximately equally spaced? What happens to this spacing when the frequency is doubled?

(b) The same signal is sent to both earbuds. Why must the quiet positions be due to interference in the air between the sources and the ear?

Write your explanation in no more than two lines before you read the next page.

From observation to model

In Explore, you found that the quiet positions near the center were approximately equally spaced. Doubling the frequency reduced the spacing between adjacent quiet positions. During each trial, the signal remained steady as the listening position changed, so the variation in sound intensity was caused by the superposition of the two sound waves in the air. The following model explains this pattern.

Superposition of two sound waves

Sound is a longitudinal wave. When two sound waves overlap at a point, the resultant displacement of the air particles is the algebraic sum of the displacements due to the two waves. If the waves arrive in phase, constructive interference occurs and, for comparable amplitudes, the sound is louder. If they arrive in opposite phase with equal amplitudes, destructive interference occurs and the sound is greatly reduced.

The two earbuds act as coherent sources because they are driven by the same signal. They have the same frequency and a constant phase relationship, so the interference pattern remains fixed in space.

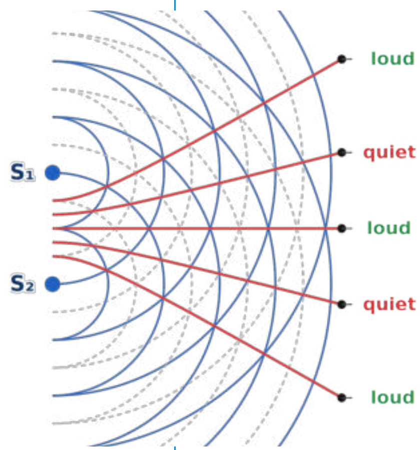


Figure 2-7-3 · Loud and quiet directions alternate away from the two sources.

The path-difference rule and how to measure it

At a point (P), the path difference is: $\Delta d = |S_1P - S_2P|$, where (S_1P) and (S_2P) are the distances from the two sound sources to the point.

For two coherent sources vibrating in phase: Constructive interference occurs when $\Delta d = n\lambda$. Destructive interference occurs when $\Delta d = (n + \frac{1}{2})\lambda$, where n = 0, 1, 2, 3, ….

Point

- Since sound waves are also waves, they exhibit the same phenomena as other waves. Sound is a longitudinal wave; where two sounds meet, each air particle takes the algebraic sum of the two displacements.
- When the wave sources S_1 and S_2 vibrate in phase and emit waves of the same wavelength λ, then at an observation point P:

  Constructive interference: $|PS_1 - PS_2| = \lambda/2 \times$ (even number).

  Destructive interference: $|PS_1 - PS_2| = \lambda/2 \times$ (odd number).
- When the wave sources are out of phase, the positions of constructive interference and destructive interference are reversed compared with those for sources in phase.

Equation box

Constructive interference (coherent, in-phase sources)	path difference = $n\lambda$
Destructive interference	path difference $= (n + \frac{1}{2})\lambda$
Wave speed	$v = f\lambda$

Counting in half-wavelengths

The path difference may be counted in multiples of ($\lambda/2$). An even number of half-wavelengths corresponds to constructive interference for in-phase sources, while an odd number corresponds to destructive interference.

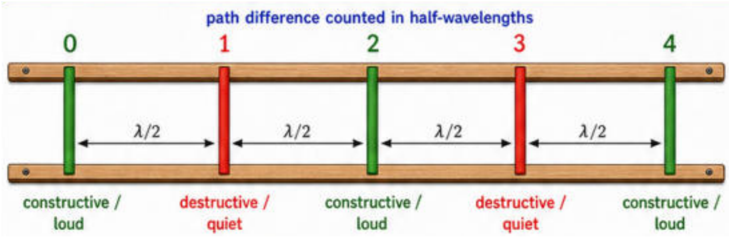


Figure 2-7-4 · Path difference in half-wavelengths decides loud or quiet.

Quincke's tube for measuring the wavelength of sound: a Quincke tube measures the wavelength of sound by interference. Sound from a single source is divided into two tubes and then recombined at the outlet. By pulling out one of the tubes (U-tube) and changing the path length, the "path difference" of the two waves changes. Taking advantage of the phenomenon where the sound intensity reaches a minimum because of destructive interference when the path difference is an odd multiple of half a wavelength, the wavelength of the sound wave can be accurately calculated from the pull-out distance of the tube.

Worked example

Situation	In the Quincke tube shown, sound entering at P is divided into two paths and recombines at Q. When tube B is fully inserted, the two path lengths are equal. As tube B is pulled out slowly, the first minimum in sound intensity occurs when the tube is pulled out by x = 8.5 cm. The speed of sound is 3.4×10^2 m/s. Determine: **(i)** the wavelength λ of the sound, in m; **(ii)** the frequency f of the sound, in Hz.
Given and required	pull-out $x = 8.5$ cm · $v = 3.4 \times 10^2$ m/s · first minimum at $Q \Longrightarrow \lambda = ?, f = ?$
Representation	Before/after sketch: route A fixed; route B gains x up one leg and x down the other — every centimeter of pull-out increases the path length by two centimeters.
Strategy	Find the path difference between the two routes first; express it as a number of half-wavelengths — an even count ($n\lambda$) gives constructive interference, an odd count $(n+\frac{1}{2})\lambda$ gives destructive interference — then determine λ.
Annotated solution	**1)** Path difference first (the slide increases the length of both arms of the U-bend): $\Delta d = 2 \times 8.5\ \text{cm} = 17\ \text{cm} = 0.17\ \text{m}$ **2)** First minimum = lowest odd half-wavelength (the smallest odd count of half-wavelengths is one): $0.17\ \text{m} = \frac{\lambda}{2} \Rightarrow \lambda = 0.34\ \text{m}$ — **3)** Calculate the frequency: $f = \frac{v}{\lambda} = \frac{3.4 \times 10^2}{0.34} \Rightarrow f = 1.0 \times 10^3\ \text{Hz}$
Reasonableness check	1.0×10^3 Hz lies within the audible range; a pull-out of a few centimeters giving a wavelength of a few tens of centimeters is of the expected order of magnitude for audible sound; and from the diagram, 17 cm is exactly half of 34 cm.
Explain it yourself	Why does pulling the slide out by x increase the path difference by $2x$ rather than by x?

Now you try: the same instrument, a different frequency. With tube B pushed back in, a new constant-frequency sound enters at P; the first minimum now occurs when the slide is out by only 2.5 cm. Calculate the wavelength and the frequency of this sound.

Quick application:

Two loudspeakers emit one tone in phase; at your seat the paths differ by 0.51 m and the wavelength is 0.34 m. State whether the sound is loud or quiet, and give the number of half-wavelengths in the path difference.

Try

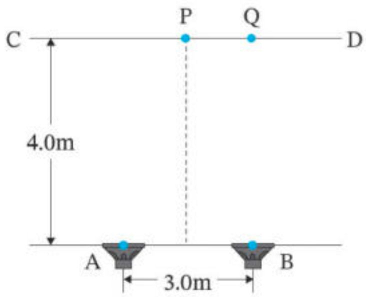


Figure 2-7-5 · Two speakers 3.0 m apart, with the line CD 4.0 m away.

(a) As shown in Figure 2-7-5, two speakers emit sound of the same frequency. As an observer moves along line CD, which is 4.0 m from line AB, the sound intensity is a minimum at point P, the intersection of line CD and the perpendicular bisector of AB. As the observer moves away from that point along CD, the sound intensity gradually increases and reaches a maximum at point Q, which is 1.5 m from point P.

(i) Determine whether the vibrations of sound sources A and B are in phase or out of phase.

(ii) Determine the wavelength of the sound emitted from the speakers, in m.

(b) In the device shown in Figure 2-7-6 (a Quincke tube), sound entering at P is divided into two paths, tube A on the left and tube B on the right, and recombines at Q, where interference occurs. When tube B is fully inserted, the lengths of the two paths are equal. While sound of a constant frequency is introduced at P, tube B is slowly pulled out, and the sound intensity becomes a minimum for the first time when it is pulled out by 5.0 cm. Let the speed of sound v be $3.4 \times 10^2\ m/s$.

(i) Determine the wavelength λ of the sound, in m. ______________

(ii) Determine the frequency f of the sound, in Hz. ______________

Key fact

Item (a) contains a 3-4-5 triangle: AQ is found using the Pythagorean theorem; BQ is simply the 4.0 m between the lines — before applying any wave analysis.

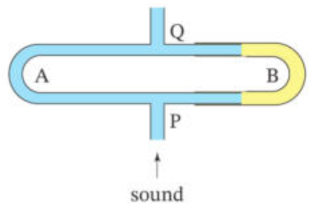


Figure 2-7-6 · A sound splits along two paths and recombines at Q.

In a new context — Noise-canceling earphones

Noise-canceling earphones use microphones to detect incoming sound. The electronics then produce a sound wave of the same frequency and nearly equal amplitude, but in opposite phase. At the eardrum, the two waves superpose and reduce the resultant pressure variation. This is destructive interference.

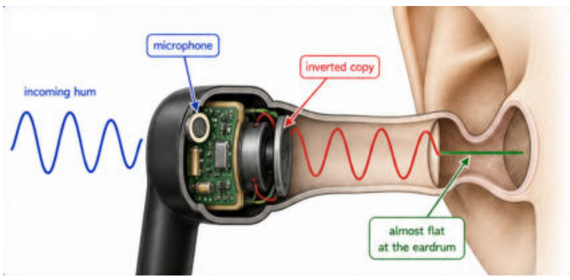


Figure 2-7-7 · The earphone adds an inverted copy, leaving the eardrum almost undisturbed.

Explain why noise-canceling earphones reduce a steady engine hum more effectively than a sudden, impulsive sound such as a door slam.

Nature of Science: Evidence — interference as evidence for the wave behavior of light

A quiet position in the figure of a two-source interference pattern is strong evidence for wave behavior. At such a point, two disturbances combine to produce nearly zero resultant effect, which cannot be explained by a simple particle-stream model. Young's double-slit experiment used similar interference evidence to support the wave model of light.

Your turn: suggest why the positions of minimum intensity, not the positions of maximum intensity, are the stronger evidence that sound is a wave.

Lesson question, answered

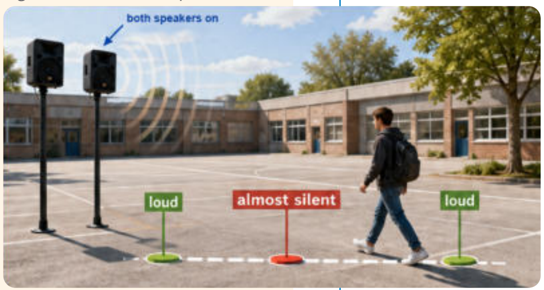


Figure 2-7-8 · The two speakers revisited.

When two coherent, in-phase loudspeakers emit the same steady tone, each point receives two sound waves that may have traveled different distances. If the path difference is a whole number of wavelengths, the waves arrive in phase and interfere constructively, producing a loud sound. If the path difference is an odd number of half-wavelengths, the waves arrive in opposite phase and interfere destructively, producing a quiet position.

Exercise

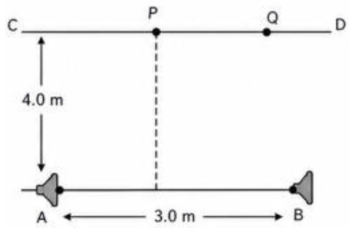


Figure 2-7-9 · The same arrangement, for the exercise.

(a) As shown in Figure 2-7-9, two speakers emit sound in phase at the same frequency, 1.7×10^2 Hz. Let P be the intersection of line CD, which is 4.0 m away from line AB, and the perpendicular bisector of AB, and let Q be a point 1.5 m from point P. Let the speed of sound be 3.4×10^2 m/s.

(i) Determine the wavelength of the sound emitted from the speakers, in m.

(ii) Determine whether point Q is a point of constructive or destructive interference.

(b) In the same device shown in Figure 2-7-6, with tub*e* B again fully inserted so the two paths are equal, sound of a constant frequency is introduced at P. Tube B is slowly pulled out, and the sound intensity becomes a minimum for the first time when it is pulled out by 17 cm. Let the speed of sound v be 3.4×10^2 m/s.

(i) Determine the wavelength λ of the sound, in m. ______________

(ii) Determine the frequency f of the sound, in Hz. ______________

(c) In the same device, with tube B again fully inserted, sound of a constant frequency is introduced at P. Tube B is slowly pulled out, and the sound intensity is a minimum for the first time when it is pulled out by 10 cm. Let the speed of sound v be 3.4×10^2 m/s.

(i) Determine the wavelength λ of the sound, in m. ______________

(ii) Determine the frequency f of the sound, in Hz. ______________

(iii) If the same experiment is performed using sound with twice the frequency, determine the distance by which tube B is pulled out when the sound intensity is a minimum for the first time, in m. ______________

Exam tip

Show each step of your working clearly. In interference questions, write the path difference explicitly before completing the calculation.

Exam-style question

Two loudspeakers, connected to the same signal generator, emit sound waves in phase. At point M the sound intensity is almost zero. Explain, in terms of path difference and phase difference, why the sound intensity is very small at M.

Lesson 2-8 — The Doppler Effect

❓ **Lesson question:** Why does the observed pitch of a siren change as an ambulance passes an observer?

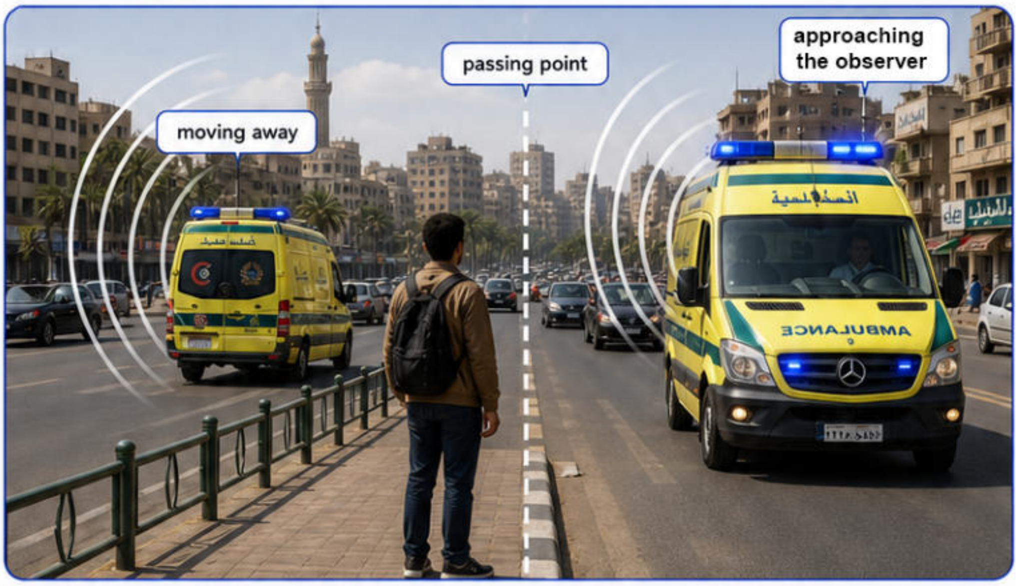
You will learn to

By the end of this lesson, you will be able to:

1. Explain qualitatively why relative motion between a wave source and an observer changes the observed frequency.
2. **Sketch** wavefront diagrams for a moving source and identify regions of shorter and longer wavelength.
3. **Use** the approximate Doppler relation for light: $\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$.

🔍 The phenomenon — predict before you continue

Figure 2-8-1 · The pitch is higher as the ambulance approaches and lower as it recedes.



Stand on a busy street as an ambulance passes, and listen. Everyone agrees the siren sounds different as it passes. Many listeners are unsure about exactly what changes, or when.

Predict before you continue

(a) As an ambulance approaches at constant speed, does the observed pitch increase continuously, or does it remain nearly constant until the ambulance passes?

(b) While the ambulance is approaching at constant speed, is the observed pitch higher than, lower than, or equal to the emitted pitch?

⭐ **Key fact**

For sound in still air, the wave speed is determined by the properties of the air. The quantities $v, f,$ and λ are related by: $v = f\lambda$. A higher observed frequency corresponds to a higher pitch.

🔍 Explore — Observe the Doppler shift of a moving sound source

What you'll do: You will move a sound source relative to a listener and listen for changes in the observed pitch.

Time: 12 min

You will need:

- A phone with a tone-generator app set to 600 Hz.
- A clear open space.
- A second phone or recorder for observations.

Steps:

1. One student stands still as the listener.
2. A second student carries the phone while it emits a steady (600 Hz) tone. The listener records this as the reference pitch.
3. The student then carries the phone directly toward the listener at a steady walking speed, then passes and moves away.
4. The listener records whether the pitch is higher, lower, or unchanged during approach and during recession.
5. Repeat the observation with different listeners.

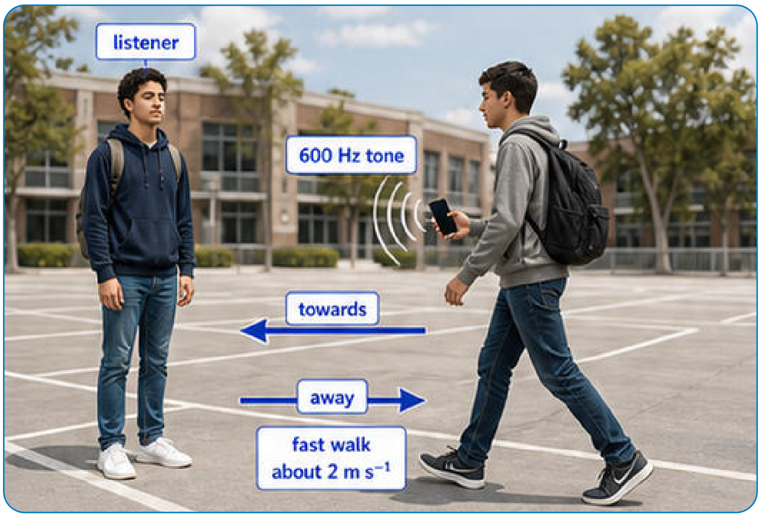


Figure 2-8-2 · A 600 Hz tone carried past a listener at walking speed.

Motion of source relative to listener	Observed pitch	Notes
Moving toward the listener		
Moving away from the listener		
Stationary source		

Analyze what you collected:

(a) During approach at steady speed, was the observed pitch continuously rising or approximately steady and higher than the reference pitch?

(b) Since the frequency setting of the phone did not change, what changed between the source and the listener?

Write your explanation in two lines before you read the next page.

Measurement note

Different listeners may judge changes in pitch differently. Repeat observations with several listeners to improve reliability.

From observation to model

In Explore, the observed pitch was higher while the source was moving toward the listener and lower while it was moving away. For motion directly toward or directly away from the listener at constant speed, the observed frequency remains constant during each stage: higher during approach and lower during recession. A gradual rise is a common prediction, but it is not what is observed. Since the emitted frequency remained constant, the change in observed frequency was caused by the motion of the source relative to the listener and the surrounding air. The following model is consistent with this evidence.

Moving source and changing wavelength

A sound source emits wavefronts that spread through air at the speed of sound. When the source moves toward an observer, successive wavefronts are emitted from positions closer to the observer. The wavefronts ahead of the source are closer together, so the observed wavelength is shorter, and the observed frequency is higher.

Behind the moving source, successive wavefronts are farther apart, so the observed wavelength is longer, and the observed frequency is lower.

For sound in still air, the speed of sound is unchanged by the motion of the source. The motion of the source changes the spacing of the wavefronts, not the speed of sound in the medium.

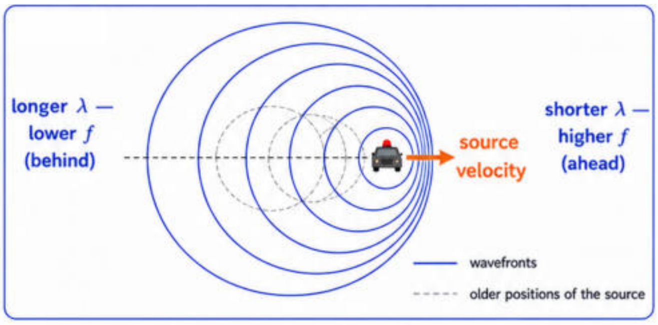


Figure 2-8-3 · Wavefronts crowd ahead of the moving source and spread out behind it.

Observer moving toward a stationary source

If the source is stationary and the observer moves toward it, the wavelength in the air is unchanged, but the observer meets wavefronts more frequently. Therefore, the observed frequency is higher. If the observer moves away from the source, the observer meets wavefronts less frequently, so the observed frequency is lower.

The Doppler effect for light

For light, the Doppler effect is observed as a shift in the wavelengths of spectral lines. If a star or galaxy is moving away from the observer, its spectral lines are shifted toward longer wavelengths; this is called redshift. If it is moving toward the observer, the lines are shifted toward shorter wavelengths; this is called blueshift.

For speeds much smaller than the speed of light: $\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$ *where* v is the radial speed along the line of sight and c is the speed of light.

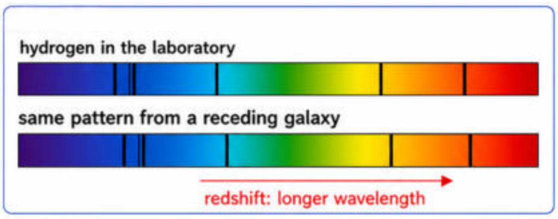


Figure 2-8-4 · The pattern from a receding galaxy is shifted toward longer wavelengths.

Doppler effect

The Doppler effect is the change in observed frequency or wavelength of a wave due to relative motion between the source and the observer.

Point

- The phenomenon in which the observed frequency differs from the frequency of the source because of relative motion is called the Doppler effect.

  Approaching → higher pitch. Receding → lower pitch.
  The same holds whether the source or the observer moves.

- Let v be the speed of sound, v_s the velocity of the source and v_o the velocity of the observer. The frequency f' heard by the observer and the frequency f_s of the source are related by $f' = f_s \frac{v + v_o}{v - v_s}$, where v_o is positive when the observer moves toward the source and v_s is positive when the source moves toward the observer.

- Ahead of an approaching source the wavefronts are closer together, so the observed wavelength is shorter. The motion of the source changes the spacing of the wavefronts, not the speed of sound in the medium.

Equation box

Doppler shift for light (v ≪ c)	$\frac{\Delta f}{f} = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$

Quick application: A hydrogen line in a galaxy's spectrum is shifted by $\Delta\lambda / \lambda = 0.0030$ toward longer wavelengths. Determine the galaxy's speed along the line of sight ($c = 3.00 \times 10^8$ m/s).

Put it to the test: two students stand on opposite sides of a road as a siren passes. Predict what each hears at the same instant, and what timing successive horn pulses would show — then sketch one wavefront diagram that explains both.

Worked example

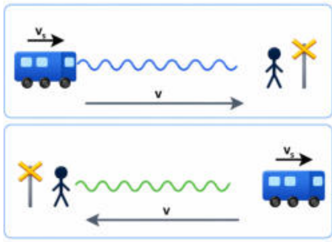
Problem	Let the speed of sound be 340 m/s. **(a)** Near a railroad crossing, a person hears a train approaching at 20 m/s while sounding a horn of frequency 720 Hz. **(i)** Find the frequency heard as the train approaches the crossing. **(ii)** Find the frequency heard as the train moves away. **(b)** A car moving east at constant speed while emitting a sound of frequency 660 Hz and a motorcycle moving west at 17 m/s pass each other. The frequency heard by the motorcycle rider before they pass is 770 Hz. **(i)** Find the speed of the car. **(ii)** Find the frequency heard by the rider after they pass.
Situation	A source and a listener move along one straight line. (a) the train (source, 720 Hz) passes a person standing still — first approaching, then receding. (b) a car (source, 660 Hz, east) and a motorcycle (listener, west, 17 m/s) meet, then pass.
Given and required	**Given** — $v = 340$ m/s · (a) $f_s = 720$ Hz, train 20 m/s, $v_o = 0$ · (b) $f_s = 660$ Hz, rider 17 m/s, f′before = 770 Hz **Required** — (a) f′ approaching & receding · (b) car speed & f′ after passing
Representation	Sign each velocity by whether it points toward the other body: v_o is positive when the observer moves toward the source, and vs is positive when the source moves toward the observer. (So the same eastward train counts as +20 when the listener is ahead of it, but −20 when the listener is behind it.)
Strategy	Substitute the signed velocities into the Doppler relation $f' = f_s(\frac{v + v_o}{v - v_s})$ ($v = 340$ m/s), then solve for the unknown.

Data booklet

quote $c = 3.00 \times 10^8\ m/s$ and $\frac{\Delta f}{f} = \frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$ from the booklet; the relation holds only when $v \ll c$. Read the fractional shift first, then the speed follows — the relation is provided, not memorized.

Exam tip

Quote the booklet relation as "delta-f over f equals delta-lambda over lambda, approximately v over c." Show the fractional line first, then the speed; examiners credit the working, not a bare number.

Annotated solution	**(a)** Train sounds 720 Hz · listener at rest ($v_o = 0$) **(i)** approaching: positive points toward the listener, so $v_s = +20$ m/s. $f = \frac{340}{340 - 20} \times 720 = 765 \text{ Hz}$ (ii) receding: the listener is now behind, so positive points the other way and $v_s = -20$ m/s. $f = \frac{340}{340 + 20} \times 720 = 680 \text{ Hz}$  Figure 2-8-5 · A source moving toward the observer, and away from them. (b) Car is the source (660 Hz) · the rider is the moving listener **(i)** before — both approach: $v_o = 17$ m/s and f' = 770 Hz, so $770 = \frac{340 + 17}{340 \quad v_s} \times 660\backslash$ $v_s = 34$ m/s **(ii)** after — both recede: now $v_o = -17$ m/s and $v_s = -34$ m/s, *so* $f = \frac{340 - 17}{340 + 34} \times 660 = 570 \text{ Hz}$ *(a·i)* $f' = 765$ Hz *(a·ii)* $f' = 680$ Hz *(b·i)* $v_{car} = 34$ m/s *(b·ii)* $f' = 570$ Hz
Reasonableness check	Approaching raises the pitch (765 > 720) and receding lowers it (680 < 720); the rider hears higher while approaching (770 > 660) and lower while receding (570 < 660). The *car's* 34 m/s (≈ 122 km/h) is fast but realistic.
Explain it yourself	In both pictures of part (a) the train moves the same way (to the right), yet v_s = +20 in (i) and −20 in (ii). Why? Because "positive" is the direction from source to observer. The listener is ahead in (i), so that direction points right and the train's rightward motion is toward it (+); the listener is behind in (ii), so the positive direction points left and the same rightward motion is now away (−).

Now you try: a hydrogen line from a distant galaxy is shifted toward longer wavelengths by $\Delta\lambda/\lambda = 4.0 \times 10^{-4}$. Take $c = 3.00 \times 10^8$ m/s. Determine the galaxy's speed along the line of sight, and state whether it is approaching or receding.

Try

Determine the following, assuming the speed of sound is 340 m/s.

(**a**) For each of the following cases, determine the frequency of the sound heard by the observer.

(i) A sound source moving at 20 m/s approaches a stationary observer while emitting a sound of frequency 720 Hz.

(ii) A sound source moving at 20 m/s moves away from a stationary observer while emitting a sound of frequency 640 Hz.

(iii) An observer moving at 34 m/s approaches a sound source emitting a sound of frequency 850 Hz.

(iv) An observer moving at 20 m/s moves away from a sound source emitting a sound of frequency 510 Hz.

(**b**) A car moving east at 20 m/s while emitting a sound of frequency 720 Hz and a motorcycle moving west at 20 m/s pass each other.

(i) Determine the frequency of the sound heard by the person riding the motorcycle before they pass, in Hz.

(ii) Determine the frequency of the sound heard by the person riding the motorcycle after they pass, in Hz.

(**c**) Near a railroad crossing, a person hears the sound of a train approaching at a constant speed while sounding a horn. The frequency of the sound heard when the train approaches the crossing is 800 Hz, and the frequency of the sound heard when the train moves away from the crossing is 600 Hz. Determine the speed of the train, in m/s.

(**d**) The sound source S and the observer O are moving along a straight line. For each of the following cases, determine the frequency of the sound heard by observer O.

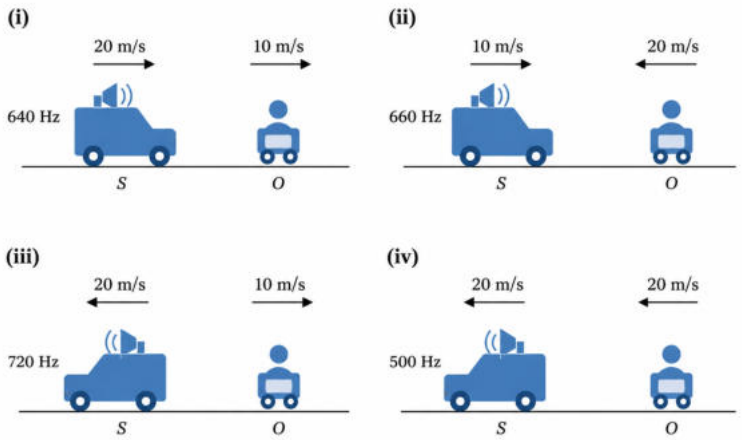


Figure 2-8-6 · Four cases of source and observer speeds.

In a new context — Doppler ultrasound measurement of blood flow

In Doppler ultrasound, a probe emits ultrasound of known frequency into a blood vessel. Moving blood cells receive the ultrasound as moving observers and then scatter it back while moving. As a result, the reflected ultrasound is Doppler-shifted. The measured frequency shift is used to determine the component of blood velocity along the direction of the ultrasound beam.

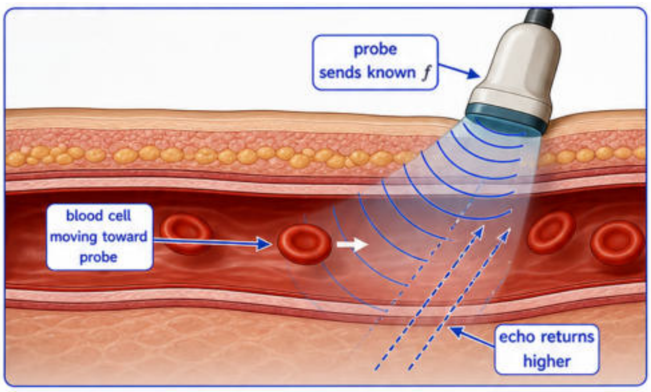


Figure 2-8-7 · The echo from a moving blood cell returns at a higher frequency.

The ultrasound echo from blood moving **toward** the probe has a higher frequency than the emitted ultrasound. Explain why the Doppler shift occurs **twice**: once as the moving blood cell receives the wave and again as it scatters the wave back toward the probe.

Nature of Science: Evidence — redshift as evidence for the expansion of the universe

The Doppler effect also applies to light. When light from distant galaxies is observed, spectral lines are often shifted toward longer wavelengths. Comparing these shifted lines with the same lines measured in the laboratory allows astronomers to determine the radial motion of galaxies. The widespread **redshift** of distant galaxies is evidence for the **expansion of the universe**.

Suggest: why evidence of redshift comes from comparing the observed spectral lines with the corresponding laboratory wavelengths, rather than from examining the galaxy's spectrum alone.

Linking question

What gives rise to emission spectra and how can they be used to determine astronomical distances? This question will be explored further in the topics on energy levels and stars.

Data booklet

$\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} \approx \frac{v}{c}$ is given in the booklet (with $c = 3.00 \times 10^8$ m/s) in the exam it is selected from the booklet and the fractional shift substituted into it, not memorized. The qualitative answers here need no formula at all.

Lesson question, answered

The emitted frequency of the siren remains constant. As the ambulance approaches, the wavefronts in front of it are closer together, so a stationary observer receives a higher frequency and hears a higher pitch. After the ambulance passes and moves away, the wavefronts reaching the observer are farther apart, so the observed frequency is lower. The speed of sound is determined by the air; the motion of the source changes the wavelength reaching the observer.

Figure 2-8-8 · The ambulance revisited.

Exercise

Determine the following, assuming the speed of sound is 340 m/s.

(a) For each of the following cases, determine the frequency of the sound heard by the observer.

(i) A sound source moving at 20 m/s approaches a stationary observer while emitting a sound of frequency 800 Hz.

(ii) A sound source moving at 20 m/s moves away from a stationary observer while emitting a sound of frequency 720 Hz.

(iii) An observer moving at 20 m/s approaches a sound source emitting a sound of frequency 510 Hz.

(iv) An observer moving at 34 m/s moves away from a sound source emitting a sound of frequency 850 Hz.

(b) A car moving east at a constant speed while emitting a sound of frequency 576 Hz and a motorcycle moving west at 10 m/s pass each other. The frequency of the sound heard by the person riding the motorcycle after they pass is 528 Hz.

(i) Determine the speed of the car.

(ii) Determine the frequency of the sound heard by the person riding the motorcycle before they pass.

(c) Near a railroad crossing, a person hears the sound of a train approaching at a constant speed while sounding a horn. The frequency of the sound heard when the train approaches the crossing is 918 Hz, and the frequency of the sound heard when the train moves away from the crossing is 816 Hz.

(i) Determine the speed of the train, in m/s.

(ii) Determine the frequency of the horn, in Hz.

(d) The sound source S and the observer O are moving along a straight line. For each of the following cases, determine the frequency of the sound heard by observer O.

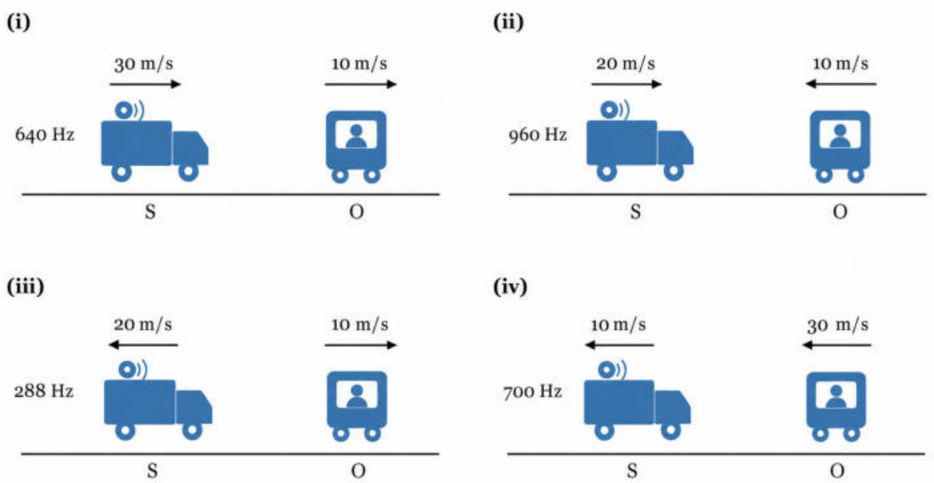


Figure 2-8-9 · Four more cases, for the exercise.

Exam-style question

A siren emitting sound of constant frequency approaches a stationary observer at constant speed.

Outline why the frequency heard by the observer is higher than the emitted frequency.

Lesson 2-9 — Refraction and Total Internal Reflection of Light

❓ **Lesson question:** When light travels from water to air, does it always cross the boundary, or can it be totally reflected back into the water?

You will learn to

By the end of this lesson, you will be able to:

- **Use** the absolute refractive index: $n = \frac{c}{v}$ and Snell's law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$ *to* determine the speed, wavelength, or angle of light at a boundary.
- **Explain** the conditions for total internal reflection when light travels from an optically denser medium to an optically rarer medium.
- **Determine** the critical angle using: $\sin \theta_c = \frac{n_2}{n_1}$, *where* $n_1 > n_2$.

The phenomenon — predict before you continue

Figure 2-9-1 · Light from inside the tank escapes only within a cone.

When light travels from water to air at a small angle of incidence, part of it is refracted into the air and part is reflected back into the water. At sufficiently large angles of incidence, no refracted ray emerges into the air; all the light is reflected back into the water. This is total internal reflection.

Predict before you continue

(a) Does light always cross a clear surface, bending a little but passing into the second medium — or can a surface send all of it back? If so, what property of the angle of incidence decides this?

(b) The light starts in the water (denser) and travels toward the air (rarer). Would total internal reflection appear if it came down from the air into the water instead?

Key fact

At a boundary between two transparent media, an obliquely incident ray may change direction because its speed changes. The angles are related by Snell's law:

$n_1 \sin \theta_1 = n_2 \sin \theta_2$

The frequency of light remains unchanged at the boundary, while its speed and wavelength change. All angles are measured from the normal to the boundary.

Explore — Observe total internal reflection at a water-air boundary

What you'll do: You will vary the angle of incidence of a narrow light beam traveling from water to air and observe refraction, partial reflection, and total internal reflection.
Time: 12 min

You will need:

- A tall transparent cup of water.
- A phone flashlight masked to a narrow beam.
- A darkened room.
- A sheet of white paper.

Safety: Use a low-power ray box or a masked flashlight. If a laser is used, it must be low power and used only under teacher supervision. Never direct the beam toward anyone's eyes or toward a shiny reflecting surface.

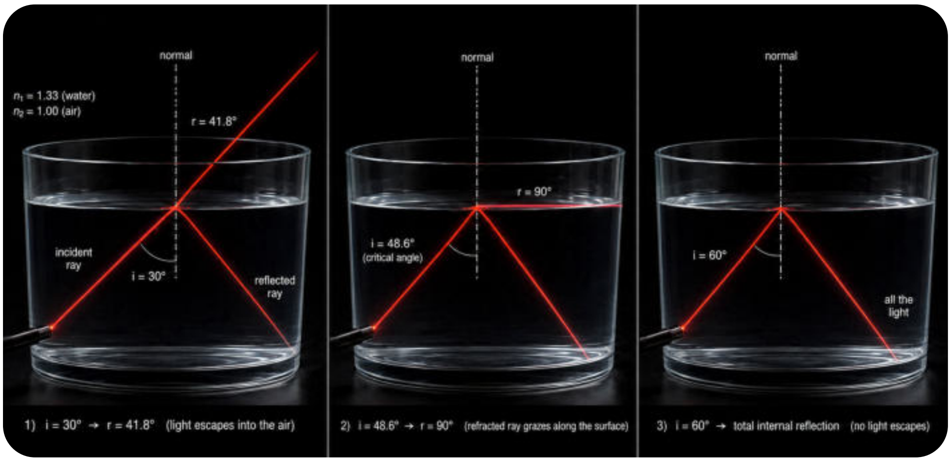


Figure 2-9-2 · Beyond the critical angle no light escapes into the air.

Steps:

1. In a darkened room, direct a narrow light beam upward through the side of a transparent cup of water so that it reaches the water-air surface from below.
2. Place a sheet of white paper above the surface to observe any refracted beam emerging into the air.
3. Begin with a small angle of incidence, close to the normal. Observe the refracted ray in air and the reflected ray in water.
4. Increase the angle of incidence gradually. Observe that the refracted ray moves closer to the surface and that the reflected ray becomes brighter.
5. Continue increasing the angle until the refracted ray just disappears. Beyond this angle, the light is totally internally reflected.

Angle of incidence	Refracted beam in air	Reflected beam in water
Small angle		
Near the critical angle		
Greater than the critical angle		

Analyze what you collected:

(a) As the angle of incidence increased, what happened to the refracted ray and the reflected ray?

(b) When the refracted ray disappeared, was the light absorbed, or was it totally reflected back into the water?

Write your explanation in no more than two lines before you read the next page.

Measurement note

Increase the angle of incidence in small steps near the transition. The critical angle is the limiting angle at which the refracted ray travels along the surface. Total internal reflection occurs when the angle of incidence is greater than this value.

From observation to model

In Explore, the escaping beam did not disappear gradually. As the angle of incidence increased, the refracted ray moved closer to the surface. At one particular angle, it traveled along the surface. When the angle of incidence increased further, no refracted ray emerged into the air, and the light was reflected back into the water. Recall Snell's law and add the absolute refractive index, which characterizes each medium on its own.

Each medium has its own characteristic value — the absolute refractive index

Light travels at its greatest speed in a vacuum: $c = 3.0 \times 10^8$ m/s. In a transparent medium, light travels at a lower speed (v). The absolute refractive index (n) of the medium is defined by: $n = \frac{c}{v}$

A medium with a larger refractive index is called optically denser. This means that light travels more slowly in it; it does not necessarily mean that the material has greater mass density.

For light passing between two media: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

where θ_1 and θ_2 are measured from the normal to the boundary.

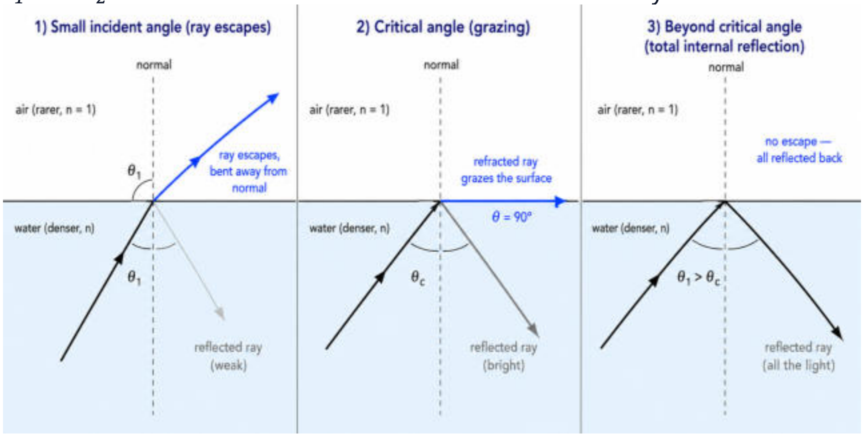


Figure 2-9-3 · Escape, grazing, and total reflection as the angle grows.

Critical angle and total internal reflection

When light travels from an optically denser medium (n_1) to an optically rarer medium (n_2), where $n_1 > n_2$, the refracted ray bends away from the normal. The critical angle (θ_c) is the angle of incidence in the denser medium for which the angle of refraction is 90°.

Using Snell's law:

$n_1 \sin\theta_c = n_2 \sin 90°$ Since $\sin 90° = 1$ then: $\sin\theta_c = \frac{n_2}{n_1}$

Total internal reflection occurs when:

light travels from an optically denser medium to an optically rarer medium;

the angle of incidence in the denser medium is greater than the critical angle.

At angles greater than θ_c, no refracted ray is formed, and the light is reflected back into the denser medium.

Critical angle (θc)

The angle of incidence in the denser medium at which the refracted ray has an angle of refraction of 90°; for angles of incidence greater than θc, total internal reflection occurs.

Point

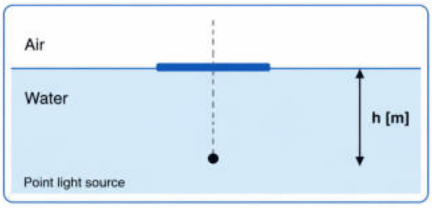
- Light travels at its greatest speed c in a vacuum; in a transparent medium it travels at a lower speed v. The absolute refractive index of that medium is $n = \frac{c}{v}$, and a medium with a larger refractive index is called optically denser — light travels more slowly in it, which does not necessarily mean that the material has greater mass density. Written with absolute indices, the law of refraction is $n_1 \sin\theta_1 = n_2 \sin\theta_2$.
- Passing from a denser medium to a rarer one, the angle of incidence at which the angle of refraction becomes 90° is called the critical angle. For angles of incidence greater than it, no refracted ray emerges and the light is entirely reflected back: this is total internal reflection. $\sin\theta c = n_2/n_1$.

Equation box

Absolute refractive index	$n = \dfrac{c}{v}$
Snell's law for light	$n_1 \sin\theta_1 = n_2 \sin\theta_2$
Critical angle (denser → rarer)	$\sin\theta_c = \dfrac{n_2}{n_1}$

Quick application: light is incident on a water–air surface from the water side. Take the refractive index of water as $\sqrt{2}$ (and $\sqrt{2} = 1.4$). **Determine** $\sin\theta_c$ for total internal reflection, using $\sin\theta_c = \frac{n_2}{n_1}$ with $air\ n_2 = 1$.

Worked example

Problem	**(a)** Light traveling in air (refractive index 1.0) enters a substance of refractive index 1.5. Take the speed of light in air $c = 3.0 \times 10^8$ m/s and the wavelength in air $\lambda_0 = 6.0 \times 10^{-7}$ m; **(i)** Find the speed v in the substance, in m/s. (ii) Find the wavelength λ in the substance, in m.  Figure 2-9-4 · A point source at depth h below the surface. **(b)** A point light source lies at depth $h\ m$ below the water surface. By floating a disk on the surface, find the minimum radius $r\ m$ of the disk that prevents the source's light from emerging into the air. Take the refractive index of air = 1 and of water = n.
Situation	**(a)** light passes from air ($n = 1.0$) into a denser substance ($n = 1.5$). **(b)** a point source sits at depth h in water (index n); a floating disk must block every ray that would otherwise refract out into the air.
Given and required	Given — **(a)** $n_{air} = 1.0,\ {}_{nsub} = 1.5,\ c = 3.0 \times 10^8\ \mathrm{m/s},\ \lambda_0 = 6.0 \times 10^{-7}\ \mathrm{m}$ · **(b)** depth $h,\ n_{air} = 1,\ n_{water} = n$ Required — **(a)(i)** v · **(a)(ii)** λ · **(b)** minimum disk radius r
Representation	Crossing into a denser medium, light slows and its wavelength shortens (frequency unchanged). Underwater, rays that meet the surface below the critical angle are refracted into the air; beyond it the surface produces total internal reflection. The escaping rays form a circle of radius r on the surface — cover it and no light emerges.
Strategy	Use the law of refraction with absolute indices: $n_{12} = \frac{n_2}{n_1} = \frac{c}{v} = \frac{\lambda_0}{\lambda}$. For total internal reflection, find the critical angle θ_c by setting the refraction angle to 90°: $n \sin\theta_c = \sin 90° \Longrightarrow \sin\theta_c = 1/n.\ Then\ r = h \tan\theta_c$.
Annotated solution	(a) Light entering the substance (n: $1.0 \to 1.5$) (i) speed: $\frac{3.0\times10^8}{v} = \frac{1.5}{1.0} \Longrightarrow v = 2.0\times10^8\mathrm{ms^{-1}}$ (ii) wavelength: $\frac{6.0\times10^{-7}}{\lambda} = \frac{1.5}{1.0} \Rightarrow \lambda = 4.0\times10^{-7}\mathrm{m}$ (b) Disk radius from the critical angle The disk must cover the area where light escapes, so its edge sits where total internal reflection just begins — at the critical angle θ_c. critical angle: $\frac{\sin\theta_c}{\sin 90^\circ} = \frac{1}{n} \Rightarrow \sin\theta_c = \frac{1}{n}$ From the right triangle (opposite 1, hypotenuse n, adjacent $\sqrt{n^2-1}$): $\tan\theta_c = \frac{1}{\sqrt{n^2-1}} \Longrightarrow r = h\tan\theta_c = \frac{h}{\sqrt{n^2-1}}\mathrm{m}$ $(a\text{-}i)\ v = 2.0\times10^8\ \mathrm{m/s}\ (a\text{-}ii)\ \lambda = 4.0\times10^{-7}\ \mathrm{m}\ (b)\ r = \frac{h}{\sqrt{n^2-1}}$

Data booklet

quote $n = \dfrac{c}{v}$ and $n_1\sin\theta_1 = n_2\sin\theta_2$ from the booklet. The critical-angle relation $\sin\theta_c = \dfrac{n_2}{n_1}$ is not printed — you build it by setting the angle of refraction equal to 90° in Snell's law. Read the geometry first, then substitute the numerical values.

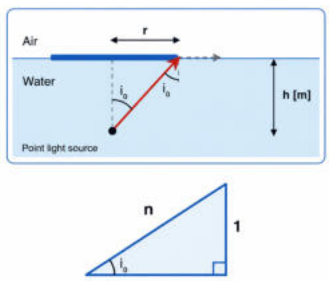


Figure 2-9-5 · The escaping cone has radius r, set by the critical angle.

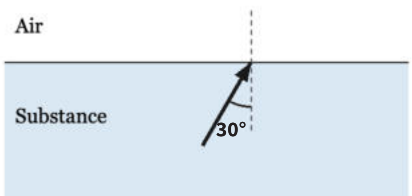
Reasonableness check	The radius (r) increases with depth (h) and decreases as (n) increases because a larger refractive index gives a smaller critical angle and a narrower escape cone: $r = \frac{h}{\sqrt{n^2 - 1}}$
Explain it yourself	For a source at the same depth h, why does a larger refractive index n make the blocking disk smaller? Idea: a larger n gives a smaller $\sin\theta_c = \frac{1}{n}$, hence a smaller critical angle θ_c. The cone of rays that can escape narrows, so the escape radius $r = h\tan\theta_c = \frac{h}{\sqrt{n^2-1}}$ shrinks.

Now you try: a lamp lies at depth **h = 0.30 m** under water of refractive index $\sqrt{2}$. **Determine** the smallest disk radius r that blocks every escaping ray, in m.

Put it to the test: in your cup, aim the narrow beam downward from the air into the water at a large angle of incidence and show that a transmitted ray is still produced - unlike the upward beam.

Try

(a) As shown in the figure, light traveling in a substance with a refractive index of $\sqrt{2}$ enters air with a refractive index of 1.0 at an angle of incidence of 30°. Let the speed of light in air c be 3.0×10^8 m/s, let the wavelength of light in air λ_0 be 5.8×10^{-7} m, and let $\sqrt{2} = 1.4$. **(i)** Determine the speed of light v in the substance, in m/s. **(ii)** Determine the wavelength of light λ; in the substance, in m. **(iii)** Let the angle of refraction be θ_2. Determine the value of $\sin\theta_2$.



(b) There is a point light source at a depth of 0.30 m below the water surface. To prevent light from the point source from emerging into the air by floating a disk on the water surface, determine the minimum radius r of the disk in m. Assume that the refractive index of air is 1 and that the refractive index of water is $\sqrt{2}$.

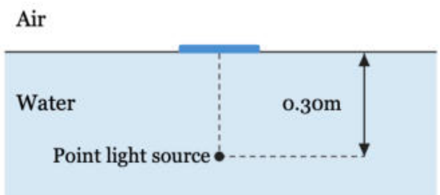


Figure 2-9-7 · A source 0.30 m below the water surface.

In a new context — the optical fibers that land at Alexandria

An optical fiber has a core with a refractive index greater than that of the surrounding cladding. Guided light traveling in the core can meet the core-cladding boundary at angles greater than the critical angle, so it undergoes total internal reflection and remains guided along the fiber.

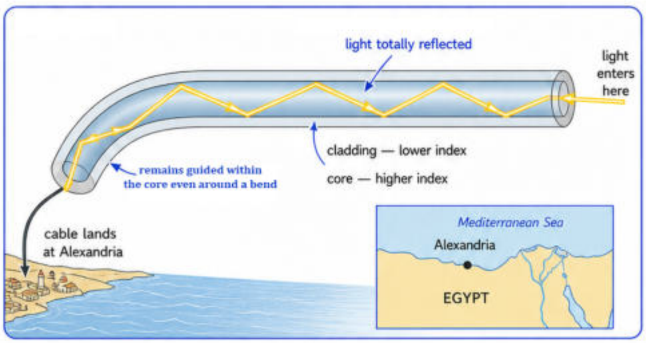


Figure 2-9-8 · Light stays in the core even around a bend in the cable.

Exam tip

show the relation you used — $n = c/v$, or $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, or $\sin\theta_c = \frac{n_2}{n_1}$ — and the substitution under it in every answer; examiners credit the working, not the final number alone.

Linking question

"How is total internal reflection used to guide light in optical fibers?"

the cladding's refractive index is lower than the core's, so light meeting their boundary can be totally reflected. Explain why the light stays trapped in the core, and state what would happen to the signal if the cladding were instead made of glass with a higher index than the core.

Nature of Science: Science as a shared endeavor — Charles Kao and the purest glass

Total internal reflection can guide light along a glass fiber, but early glass absorbed too much light for long-distance communication. In 1966, Charles Kao showed that the principal cause of the loss was impurities in the glass. He argued that sufficiently pure glass could transmit light over long distances. Later improvements in glass manufacturing made optical-fiber communication practical, and Kao shared the 2009 Nobel Prize in Physics for this work. Optical fibers became a major technology for global communication.

Your turn: suggest one reason why Kao's insight only changed the world after other scientists and engineers, in many countries, could test his calculation and build on it.

Data booklet

quote $n = \frac{c}{v}$ and $n_1 \sin\theta_1 = n_2 \sin\theta_2$ from the booklet; build $\sin\theta_c = \frac{n_2}{n_1}$ by derivation yourself by setting the refracted angle to $90°$. Your task is to choose the appropriate relation and use the diagram to identify the required geometrical quantities.

Lesson question, answered

Figure 2-9-9 · The tank revisited.

When light is incident from water into air, does it always cross the surface — or can the surface totally reflect the light? It does not always cross. From the denser side to the rarer one, the ray bends away from the normal. As the escaping ray bends further from the normal until, at the critical angle $\sin\theta_c = \frac{n_2}{n_1}$, it grazes the surface. Past that angle there is no refracted ray at all: all the light is reflected back, and the surface becomes a perfect mirror. Above the critical angle, a denser-to-rarer surface stops transmitting and reflects light totally — the principle that guides light signals along optical fibers.

Exercise

Determine the following.

Figure 2-9-10 · A ray meeting the boundary at 30°.

(a) As shown in Figure 2-9-10, light traveling in a substance with a refractive index of 1.5 enters air with a refractive index of 1.0 at an angle of incidence of 30°. Let the speed of light in air c be 3.0×10^8 m/s, and let the wavelength of light in the substance λ be 4.0×10^{-7} m.

(i) Determine the speed of light v in the substance, in m/s.

(ii) Determine the wavelength λ_0 of light in air, in m.

(iii) Let the angle of refraction be r. Determine the value of sin r.

(b) There is a point light source at a depth of 0.20 m below the glass–air surface. To prevent light from the point source from emerging into the air by placing a disk on the surface, determine the minimum radius r of the disk in m. Assume that the refractive index of air is 1.0 and that the refractive index of glass is $\sqrt{5}$.

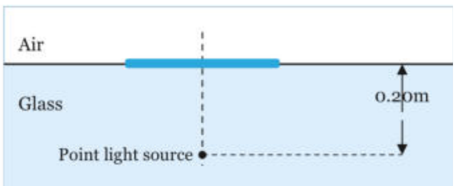
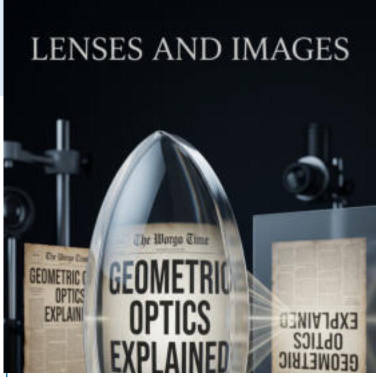


Figure 2-9-11 · A source 0.20 m below the glass surface.

Exam-style question

Light inside a glass block of refractive index 1.5 reaches the glass–air surface (air index 1.0).

Determine $\sin\theta_c$ for total internal reflection at this surface, and state what happens to a ray that meets the surface at an angle of incidence greater than θ_c.



Lesson 2-10 — Lenses and Images

❓ **Lesson question:** A magnifying lens produces an upright, enlarged image when it is close to a newspaper. Why can the image become inverted when the lens is moved farther from the page?

You will learn to

By the end of this lesson, you will be able to:

1. **Draw** the principal rays through a thin lens and locate the image from their intersection.
2. **Describe the image formed by a lens as real or virtual, inverted or upright**, magnified or diminished.
3. **Use** the lens equation and magnification relation to determine image position and size, using the sign convention stated in the lesson.

Safety: If a candle is used, it must be under teacher supervision and kept away from paper and other flammable materials.

🔍 The phenomenon — predict before you continue

Figure 2-10-1 · A magnifier makes the print look larger.

A grandfather holds a magnifier close over a newspaper, and the headline appears large and upright. He lifts the same lens higher above the page, and the letters become smaller and inverted. Only the distance between the page and the lens has changed. Why does changing this distance change the image from upright and magnified to inverted?

Predict before you continue

(a) Does a magnifier always make objects appear larger, whatever the distance?

(b) If you cover half the lens with your thumb, do you lose half of the image?

⭐ Key fact

Light is refracted when it passes from one medium into another with a different refractive index. A lens uses refraction at its curved surfaces to change the directions of light rays and form an image.

🔍 Explore — Form a real image with a convex lens

What you'll do: You will use a convex lens to form a real image on a screen and compare it with the virtual image observed when the object is inside the focal length.

Time: 12 min

You will need:

- A convex lens or magnifying glass.
- A small, bright object, such as a candle flame.
- A white screen or plain wall.
- A darkened room.

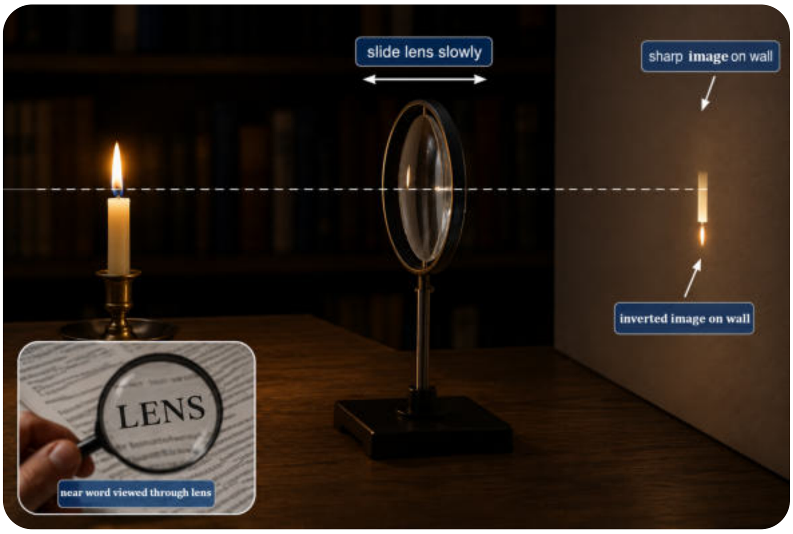


Figure 2-10-2 · Sliding the lens brings a sharp inverted image onto the wall.

Steps:

1. Place a small, bright object several meters from a white wall or screen.
2. Hold the convex lens between the object and the screen.
3. Move the lens slowly until a sharp image is formed on the screen.
4. Record whether the image is upright or inverted.
5. Place the same lens close to a printed word and look through the lens.
6. Move the lens slowly away from the word and observe how the image changes.

Object position relative to the lens	Image formed on screen?	Image orientation

Analyze what you collected:

(a) Was the image formed on the wall real or virtual? How do you know?

(b) When the printed word was very close to the lens, why could its image not be formed on a screen?

✏️ **Write your explanation in two lines before you read the next page.**

⭐ **A measurement note**

Slide the lens slowly near the sharp-focus position; the image on the wall blurs quickly if the distance is off by even a few millimeters.

From observation to model

In the Explore activity you formed a sharp image of the distant object on the wall, and it was inverted. The image of the nearby word seen through the lens could not be formed on any screen. You may well have predicted a lens simply magnifies; your two images already differ. Here is the model the rays indicate — drawn, no calculation yet.

Two lens shapes, one principal axis

A convex lens is thicker at its center than at its edges and acts as a converging lens for parallel rays. A concave lens is thinner at its center than at its edges and acts as a diverging lens for parallel rays.
The principal axis is the straight line passing through the optical center of the lens and perpendicular to the plane of the lens. The principal foci F and F' lie on this axis, one on each side of the lens, at a distance f, called the focal length, from the optical center. A convex lens converges parallel rays toward F'.

Locating the image with principal rays

To locate the image formed by a thin lens, draw two of the following principal rays from the top of the object:

1. A ray through the optical center of the lens continues undeviated.
2. A ray parallel to the principal axis is refracted through the far focus F' for a convex lens or appears to come from the near focus F for a concave lens.
3. A ray through the near focus F of a convex lens emerges parallel to the principal axis. For a concave lens, a ray directed toward the far focus F' emerges parallel to the principal axis.

If the refracted rays actually meet, the image is real and can be formed on a screen. If the refracted rays diverge and only their backward extensions meet, the image is virtual and cannot be formed on a screen.

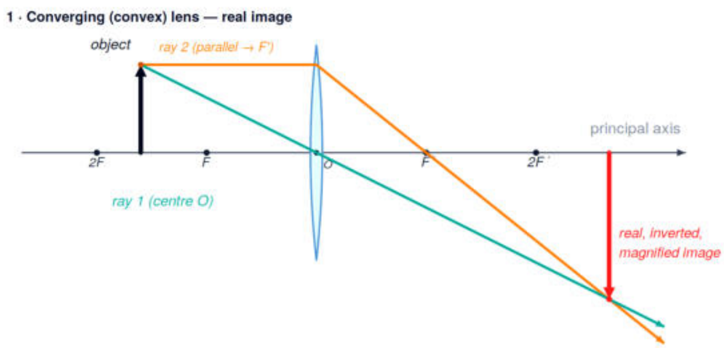
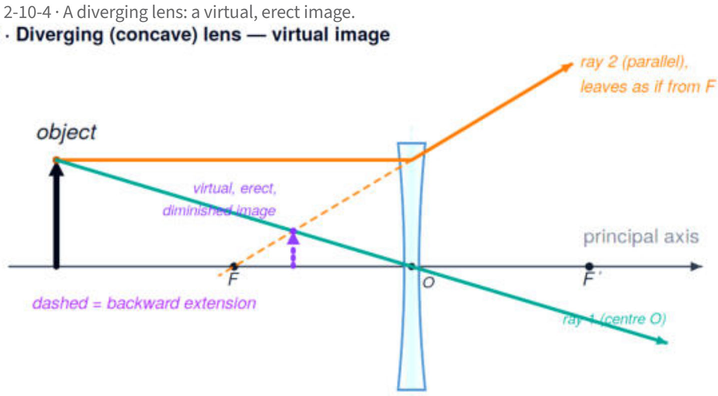


Figure 2-10-3 · A converging lens: a real, inverted image.

Figure 2-10-4 · A diverging lens: a virtual, erect image.



The six positions, shown on one grid

Move the object in from far away and the convex lens produces a fixed sequence of images. For a concave lens, the image remains virtual, upright and diminished, although its position and size change as the object moves. Read each row as a ray-diagram case — the algebra comes on the next page.

Principal focus (F)

For a convex lens, the principal focus is the point on the principal axis where rays initially parallel to the axis converge after refraction. For a concave lens, it is the point from which such rays appear to diverge. The focal length f is the distance from the optical center to the principal focus.

A grid note

On these diagrams one focal length f is a fixed number of squares, so $2F$ sits at twice that count from the lens — read object positions by counting squares, not by visual estimation.

Object position (convex unless stated)	Image type	Orientation and size	Image position
Beyond $2F$	Real	Inverted and diminished	Between F and $2F$ on the opposite side of the lens
At $2F$	Real	Inverted and same size	At $2F$ on the opposite side of the lens
Between F and $2F$	Real	Inverted and magnified	Beyond 2F on the opposite side of the lens
At F	Image at infinity; emergent rays are parallel	Inverted · highly magnified	At infinity on the opposite side of the lens
Inside F (closer than F)	Virtual	Upright and magnified	On the same side of the lens as the object
Concave lens — any position	Virtual	Upright and diminished	Between the lens and F, on the same side as the object

From the drawing to a number — the lens equation

The rays show where the image is; now we determine it quantitatively. Let a be the distance from object to lens, b the distance from image to lens, and f the focal length. In this lesson, we use the following sign convention: a is always positive, f is written as a positive magnitude, and the sign of b identifies the image.

For a convex (converging) lens: $\frac{1}{a}+\frac{1}{b}=\frac{1}{f}$ For a concave (diverging) lens: $\frac{1}{a}+\frac{1}{b}=-\frac{1}{f}$

The magnification M is the ratio of image size to object size: $M = \left|\frac{b}{a}\right|$

In this convention, M is taken as a positive size ratio. The image orientation is described separately as upright or inverted.

The decisive sign — the sign of *b*

The image distance b is the single signed quantity. Its sign tells you, with no extra drawing, which side the image is on — and therefore whether it is real or virtual.

Sign of b	Where the image sits	What it is
b positive (+)	Behind the lens — the far side, opposite the object	Real image
b negative (−)	In front of the lens — the same side as the object	Virtual image

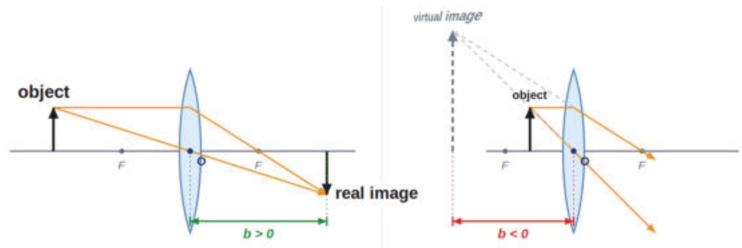


Figure 2-10-5 · The sign of the image distance separates real from virtual.

Exam tip

quote b with its sign, then read the image type directly from it: a plus means real and behind, a minus means virtual and in front. Never write a sign on M.

Point

- A lens that is thicker at the center than at the edges is called a convex lens and converges parallel rays; a lens that is thinner at the center than at the edges is called a concave lens and diverges them. The straight line through the optical center, perpendicular to the lens, is the principal axis.
- An image formed where the light rays themselves intersect is called a real image; an image formed on the extension lines of diverging rays is called a virtual image. For a convex lens with the object outside the focal point the image is real and inverted; with the object inside the focal point it is virtual and upright. For a concave lens the image is always virtual, upright and diminished, whatever the position of the object.
- Let a be the object distance, b the image distance and f the focal length. Convex lens: $\frac{1}{a}+\frac{1}{b}=\frac{1}{f}$. Concave lens: $\frac{1}{a}+\frac{1}{b}=-\frac{1}{f}$. The magnification is $M=|b/a|$.

b is positive for a real image and negative for a virtual image; *M* never carries a sign.

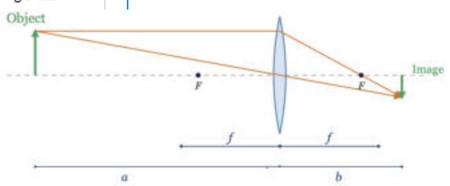
Equation box

Convex lens	$\frac{1}{a}+\frac{1}{b}=\frac{1}{f}$	Concave lens	$\frac{1}{a}+\frac{1}{b}=-\frac{1}{f}$	Magnification	$M=\left\lvert\frac{b}{a}\right\rvert$

Quick application: an object is placed 40 cm in front of a convex lens of focal length 10 cm. **Calculate** the image distance b from $\frac{1}{a}+\frac{1}{b}=\frac{1}{f}$, then the magnification M.

Worked example

Situation	An object is placed in front of a thin lens. Two separate cases are considered: a **convex** (converging) lens and a **concave** (diverging) lens.
Given and required	**Given: (a)** convex lens: object distance a = 60 cm · focal length f = 20 cm **(b)** concave lens: object distance $a = 20$ cm · focal length $f = 10$ cm **Required** (to 2 significant figures) **(i)** the type of image · **(ii)** the position of the image · **(iii)** the magnification
Representation	Apply the thin-lens equation, where a is the object distance and b is the image distance: $\frac{1}{a}+\frac{1}{b}=\frac{1}{f}$ • use $+\frac{1}{f}$ for a converging (convex) lens and $-\frac{1}{f}$ for a diverging (concave) lens, with f kept as a positive magnitude. • A positive b is a real image on the far side of the lens; a negative b is a virtual image on the same side as the object. • The linear magnification is $M = \left\lvert\frac{b}{a}\right\rvert$. Each image is then located with two principal rays: one parallel to the axis and one through the optical center of the lens.
Strategy	**1.** Assign the sign of the lens term: $+\frac{1}{f}$ for the convex lens, $-\frac{1}{f}$ for the concave lens. **2.** Solve the thin-lens equation for b; its sign gives the type and side of the image — positive is real (far side), negative is virtual (object side). **3.** Find $M = \left\lvert\frac{b}{a}\right\rvert$; $M < 1$ indicates a diminished image, $M > 1$ an enlarged one. **4.** Confirm with a ray diagram.
Annotated solution	**(a) Convex lens** **(i) Type of image** The lens is convex (converging) and the object (at 60 cm) lies beyond $2f = 40$ cm, so the image is real, inverted and diminished. **(ii) Position of the image** For a converging lens f = +20 cm: $\frac{1}{60}+\frac{1}{b}=\frac{1}{20}$ $\frac{1}{b}=\frac{1}{20}-\frac{1}{60}=\frac{1}{30}$ $\Rightarrow b = 30$ cm Since b is positive, the image is 30 cm behind the lens. A positive b means a real image, on the far side of the lens from the object. **(iii) Magnification** $M = \left\lvert\frac{b}{a}\right\rvert = \left\lvert\frac{30}{60}\right\rvert = \frac{1}{2} = 0.50$ $M = 0.50$ Convex lens — object beyond $2f$ gives a real, inverted, diminished image between f and $2f$. **(b) Concave lens** **(i) Type of image** The lens is concave (diverging), so for any object position the image is virtual, upright and diminished. **(ii) Position of the image** For a diverging lens f = 10 cm: $\frac{1}{20}+\frac{1}{b}=-\frac{1}{10}$: $\frac{1}{20}+\frac{1}{b}=\frac{1}{-10}=-\frac{1}{10}$ $\frac{1}{b}=-\frac{1}{10}-\frac{1}{20}=-\frac{3}{20}$ $\Rightarrow b=-\frac{20}{3}$ cm ≈ -6.7 cm Since b is negative, the image is 6.7 cm in front of the lens. A negative b means a virtual image, on the same side as the object. **(iii) Magnification** $M=\left\lvert\frac{b}{a}\right\rvert=\left\lvert\frac{-\frac{20}{3}}{20}\right\rvert=\frac{1}{3}=0.333\ldots\approx 0.33$ $M = 0.33$ Concave lens — a virtual, upright, diminished image forms between the object and the lens



Key fact

A virtual image (negative b) cannot be formed on a screen — the rays only seem to come from it. The upright word seen through a magnifier is exactly this kind of image.

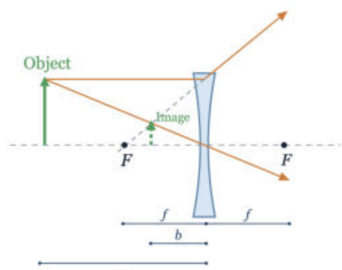


Figure 2-10-7 · Object and image distances for a diverging lens.

Reasonableness check	**Case (a):** an object beyond $2f$ gives a real image between f and $2f$. $b = 30$ cm lies between 20 and 40 cm, and $M = 0.50$ makes it diminished — as the diagram shows. **Case (b):** a diverging lens always gives a virtual, upright, diminished image. Here $\lvert b \rvert = 6.7$ cm $< \lvert f \rvert = 10$ cm and $M = 0.33$, again matching the diagram.
Explain it yourself	Why does a diverging (concave) lens always form a virtual, upright and diminished image, whatever the object distance?

Put it to the test: catch a sharp image on the wall. Then slide a card over half the lens. Note what changes — the shape of the image, or only its brightness.

Try

1.Given an object represented by an arrow and a lens with focal points *F* and *F′*, draw the image of the object formed by the lens.

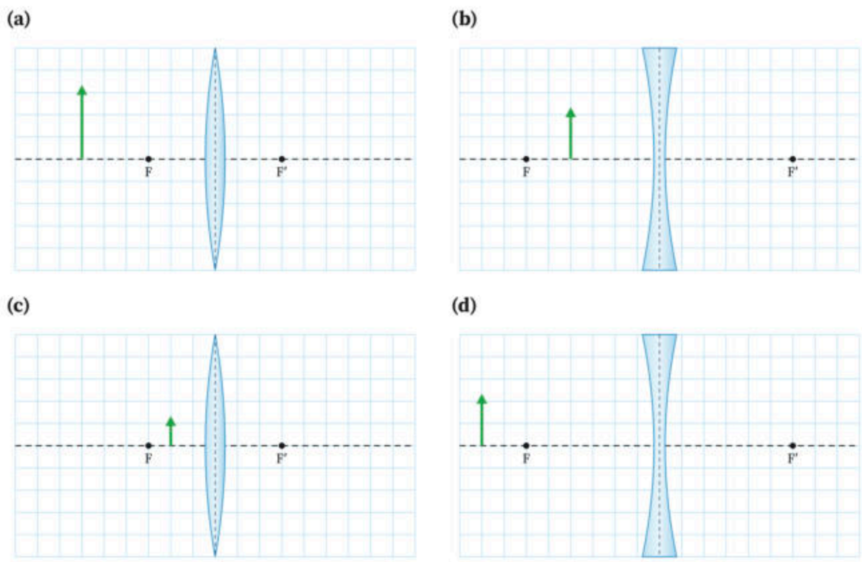


Figure 2-10-8 · Four lenses with the object at different distances.

2. Determine the following to 2 significant figures

(a) For each of the following cases, determine (A) the type of image, (B) the position of the image, and (C) the magnification.

(i) An object is placed 30 cm in front of a convex lens with a focal length of 20 cm.

(ii) An object is placed 4.0 cm in front of a convex lens with a focal length of 6.0 cm.

(iii) An object is placed 6.0 cm in front of a concave lens with a focal length of 12 cm.

(b) As shown in Figure 2-10-9, a screen is placed 90 cm from an object, and a convex lens with a focal length of 20 cm is moved between the object and the screen. Determine all values of the distance a between the lens and the object for which a clear image is formed on the screen.

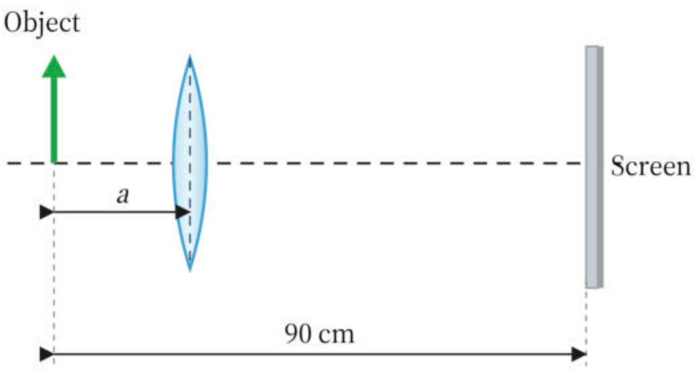


Figure 2-10-9 · An object 90 cm from the screen.

A working note

For the virtual-image cases (b), (c), and (d), the emerging rays diverge; extend them backward as dashed lines to find where the virtual image sits, on the object side of the lens.

In a new context — the lens inside your eye

The cornea and crystalline lens of the eye act together as a converging optical system. They form a real, inverted image on the retina. Accommodation changes the shape of the crystalline lens, altering the effective focal length so that images of near and distant objects can be focused on the retina.

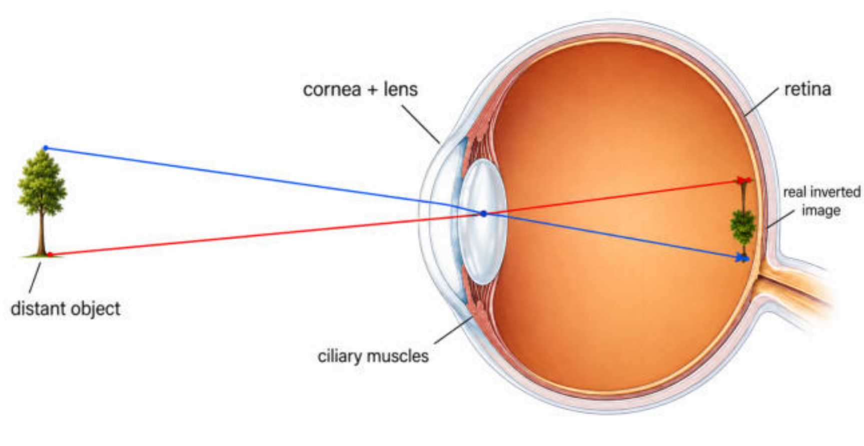


Figure 2-10-10 · The eye focuses a distant object onto the retina.

Apply your understanding: Explain why the image on your retina is real and inverted, and which principal rays you would draw to show it.

Nature of Science: Ibn al-Haytham and the Book of Optics — seeing by light that enters the eye

Ibn al-Haytham argued that vision occurs because light from objects enters the eye, not because rays leave the eye. His work in optics emphasized observation and experiment, which are central methods in modern science. (Source: Ibn al-Haytham, Kitāb al-Manāẓir, c. 1011–1021 CE.)

Your turn: suggest one everyday observation that shows we see by light entering the eye, not by rays leaving it.

> **Linking question**
>
> "How does refraction at a curved surface let a single lens form an image?" — links back to the refraction of Lesson 2-9. The bending you met there is what every principal ray obeys at the lens.

Lesson question, answered

Figure 2-10-11 · The magnifier revisited.

A magnifying glass enlarges the print when it is held close to a newspaper—but why do the letters become inverted as the lens is moved farther away?

When the newspaper is between the convex lens and its principal focus, the lens forms a virtual, upright, magnified image. As the lens is moved away, the object distance increases. Once the newspaper is beyond the principal focus, the lens forms a real, inverted image.

Thus, the properties of the image depend on the position of the object relative to the principal focus: an object placed between the lens and the principal focus produces a virtual, upright image, whereas an object placed beyond the principal focus produces a real, inverted image.

Exercise

1. Given an object represented by an arrow and a lens with focal points F and F', draw the image of the object formed by the lens.

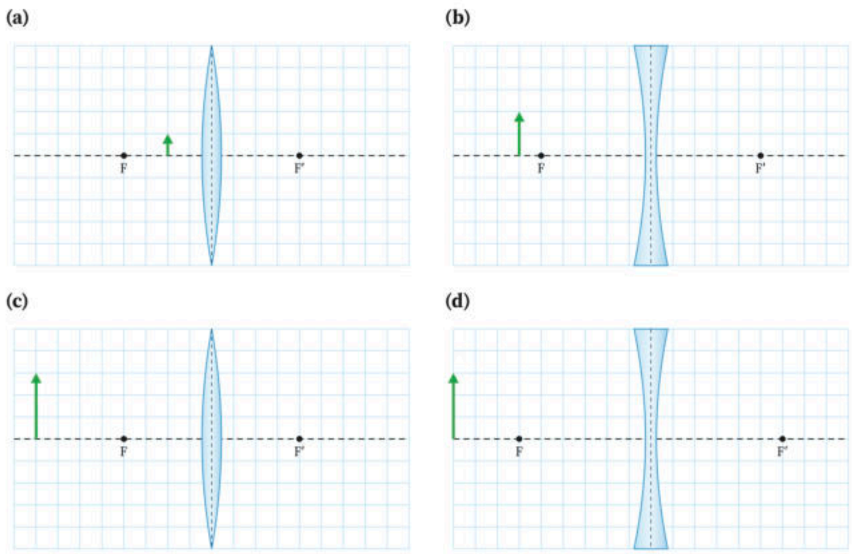


Figure 2-10-12 · Four more cases, for the exercise.

2. Determine the following to 2 significant figures.

(a) For each of the following cases, determine (A) the type of image, (B) the position of the image, and (C) the magnification.

(i) An object is placed 50 cm in front of a convex lens with a focal length of 30 cm.

(ii) An object is placed 8.0 cm in front of a convex lens with a focal length of 10 cm.

(iii) An object is placed 10 cm in front of a concave lens with a focal length of 15 cm.

(b) As shown in Figure 2-10-13, a screen is placed 50 cm from an object, and a convex lens with a focal length of 12 cm is moved between the object and the screen. Determine all values of the distance a between the lens and the object for which a clear image is formed on the screen.

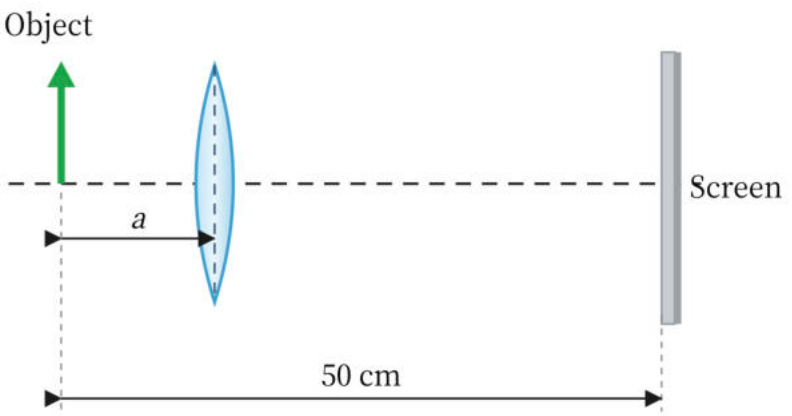
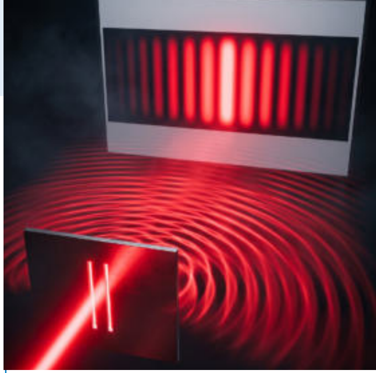


Figure 2-10-13 · An object 50 cm from the screen.

Exam-style question

An object is placed 30 cm in front of a convex lens of focal length 12 cm. Determine the image distance b, with its sign, and state whether the image is real or virtual.

Lesson 2-11 — Young's Double-Slit Experiment



❓ **Lesson question:** When coherent monochromatic light passes through two narrow slits, does the screen show two bright regions only, or an interference pattern of bright and dark fringes?

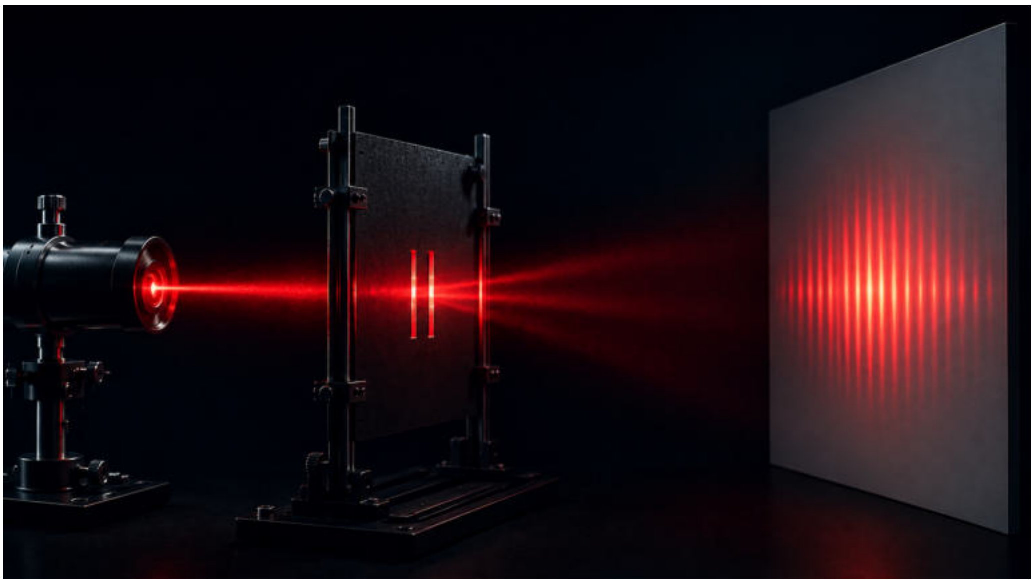
You will learn to

By the end of this lesson, you will be able to:

1. **Describe** how coherent light from two narrow slits produces bright and dark interference fringes on a screen.
2. **Relate** the path difference from the two slits to the position x on the screen using: $\Delta = \frac{dx}{D}$ for small angles.
3. **Use** the bright-fringe condition: $\Delta = n\lambda$ and the fringe-spacing relation:$\Delta y = \frac{\lambda D}{d}$ to solve problems and predict changes in the pattern.

🔍 The phenomenon — predict before you continue

Figure 2-11-1 · Laser light through a double slit gives bright and dark bands.



When monochromatic light from a laser passes through two closely spaced narrow slits, the light from the slits overlaps on the screen. Instead of two bright regions only, a pattern of bright and dark fringes is formed. The central fringe is bright because the light from the two slits travels equal distances to the center of the screen.

Predict before you continue

(a) With both slits open, will the wall show two bright spots (one per slit), or a row of many bright fringes — and where is the brightest?

(b) If a double slit with a smaller slit separation is used, do the fringes become closer together or farther apart?

Explore — measure fringe spacing at different screen distances

What you'll do: You will pass monochromatic laser light through a prepared double slit, observe the interference fringes on a screen, and measure the fringe spacing.

Time: 12 min

You will need:

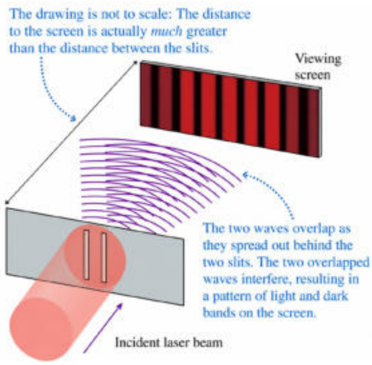
- A low-power red laser.
- A white screen or matte wall.
- A meter ruler.
- A prepared double slit.
- A tape measure.
- A darkened room.

Use only a low-power laser under teacher supervision. Never direct the laser beam toward any eye or reflective surface. Keep the beam below eye level and switch it off when not in use.

Steps:

1. Place the double slit in front of the laser and direct the beam onto a white screen several meters away.
2. Darken the room so that the fringe pattern can be seen clearly.
3. Identify the central bright fringe and the adjacent bright fringes on both sides.
4. Measure the distance across several adjacent bright fringes, then divide by the number of spacings to obtain the average fringe spacing s.
5. Measure the distance D from the double slit to the screen.
6. Move the screen farther away. Measure and record the new slit-to-screen distance D, then measure and record the new mean fringe spacing s.

Figure 2-11-2 · The two waves overlap beyond the slits and interfere on the screen.



Slit-to-screen distance D / m	Distance across several fringe spacings / mm	Number of spacings	Mean fringe spacing Δy / mm

Analyze what you collected:

(a) Did the screen show two bright regions only, or a pattern of several bright and dark fringes? Where was the central bright fringe located?

(b) What happened to the fringe spacing when the distance D from the slits to the screen was increased?

Write your explanation in two lines before you read the next page.

Measurement note

The fringes sit close together, so measure across several spacings at once and divide, rather than one tiny gap. For example, measuring ten spacings together and dividing by ten reduces the percentage uncertainty in the calculated mean fringe spacing compared with measuring a single spacing.

From observation to model

In Explore, you observed not two bright spots but a series of alternating bright and dark fringes, with a bright central fringe. You also found that moving the screen farther from the slits increased the fringe spacing. You may have expected the two slits to produce two separate bright spots. However, light from both slits reaches each point on the screen, and the resulting interference pattern depends on the path difference between the two waves.

Coherent light from two slits

In Young's double-slit experiment, one monochromatic beam illuminates two narrow slits. The two slits act as coherent sources because the light emerging from them has the same frequency and a constant phase difference. The two beams overlap on the screen and interfere.

Bright fringes occur where the two waves arrive in phase. Dark fringes occur where the two waves arrive in antiphase. When the coherent waves overlap, constructive interference produces bright fringes and destructive interference produces dark fringes.

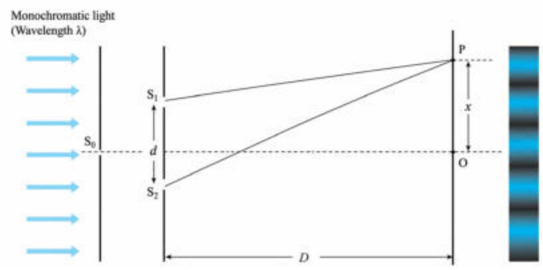


Figure 2-11-3 · The geometry that fixes the spacing of the bands.

The two paths differ by *dx*/*D*

For a point P at a distance x from the central line on a screen a distance D from two slits separated by distance d, the path difference is approximately:

$\Delta = |S_1P - S_2P| \approx d\sin\theta \approx \dfrac{dx}{D}$.

This approximation is valid when: $d \ll D$ and $x \ll D$, so that the angle is small

Bright fringes occur when: $\Delta = n\lambda$ where: $n = 0,1,2,3,\ldots\ldots$

Dark fringes occur when: $\Delta = \left(n + \frac{1}{2}\right)\lambda$, where: $n = 0,1,2,3,\ldots$

At the center of the screen: $x = 0$ so: $\Delta = 0$ and the central fringe is bright.

From the fringes to their spacing

For adjacent bright fringes, the path difference increases by one wavelength: (Δ increases by λ. Since: $\Delta = \dfrac{dx}{D}$ the separation between adjacent bright fringes is: $\Delta y = \dfrac{\lambda D}{d}$.

Key fact

When two coherent waves meet at a point, their displacements superpose. Constructive interference occurs when the path difference is a whole number of wavelengths: $\Delta = n\lambda$, where $n = 0, 1, 2, 3, \ldots$ Destructive interference occurs when the path difference is an odd number of half-wavelengths: $\Delta = \left(n + \frac{1}{2}\right)\lambda$, where $n = 0, 1, 2, 3, \ldots$

Path difference

The path difference is the extra distance traveled to point (P) by light from one slit compared with light from the other slit

Point

- When monochromatic light illuminates a slit S_0 and then the double slits S_1 and S_2 onto a screen, the light interferes and a pattern of bright and dark fringes — interference fringes — is observed. The two slits act as coherent sources because the light leaving them has the same frequency and a constant phase difference.
- Where S_1 and S_2 are in phase, at a point P on the screen:

  Bright fringes: $|S_1P - S_2P| = \dfrac{\lambda}{2} \times 2n$

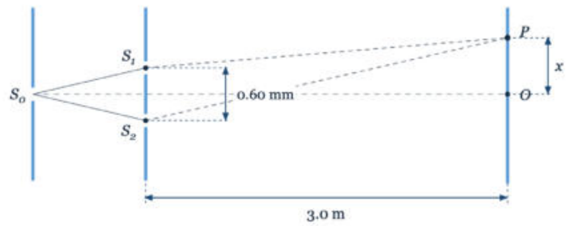
  Dark fringes: $|S1P - S2P| = \dfrac{\lambda}{2} \times (2n + 1)$ with $n = 0, 1, 2, \ldots$
- Let d be the slit separation and D the distance from the slits to the screen, with $d \ll D$ so that the angles are small. The spacing between adjacent bright fringes $= \dfrac{\lambda D}{d}$, so a longer wavelength, a greater D, or a smaller slit separation spreads the fringes farther apart.

Equation box

Fringe spacing in Young's double-slit experiment	$\Delta y = \dfrac{\lambda D}{d}$

A longer wavelength (λ), or a greater slit-to-screen distance (D), increases the fringe spacing (Δy).
Decreasing the slit separation (d) also increases the fringe spacing because d is in the denominator.
The key is: $\Delta y \propto \dfrac{1}{d}$
Therefore, moving the slits closer together makes the fringes farther apart.

Worked example

Situation	Monochromatic light passes through a single slit and then through a pair of slits, producing an interference pattern on a screen. The fringe spacing is measured, and the effect of placing water between the slits and the screen is then considered.
Given and required	**Given** fringe spacing $\Delta y = 2.9$ mm · slit separation $d = 0.60$ mm · slit–screen distance $D = 3.0$ m · refractive index of water $n_w = 1.3$ **Required** (a) the wavelength λ of the light · (b) the new fringe spacing $\Delta x'$ in water
Representation	Light from the single slit S_0 illuminates the two slits S_1 and S_2, separated by d. The waves from S_1 and S_2 overlap and interfere on the screen, a distance D away. A bright fringe forms at a point P, a distance x from the central point O, where the path difference from the two slits is a whole number of wavelengths. 
Strategy	1. For a point P a distance x from the center O, the path difference between the waves from the two slits is $\dfrac{xd}{D}$ (small-angle approximation). 2. Bright fringes occur where this path difference is a whole number of wavelengths (n whole wavelengths); dark fringes occur where it is an odd multiple of half a wavelength. 3. Consecutive bright fringes are then separated by the fringe spacing $\Delta y = \dfrac{\lambda D}{d}$; rearrange this to find λ. 4. Water reduces the wavelength to $\lambda' = \dfrac{\lambda}{n}$, which reduces the fringe spacing: recalculate to find $\Delta y'$.
Annotated solution	(a) Wavelength of the light $\lambda = \dfrac{d\Delta y}{D} = \dfrac{(0.60 \times 10^{-3})(2.9 \times 10^{-3})}{3.0} = 5.8 \times 10^{-7}$ m $\lambda = 5.8 \times 10^{-7}$ m (b) Fringe spacing in water In water the frequency is unchanged, so $\lambda' = \dfrac{\lambda}{n}$. The fringe spacing then becomes: $\Delta y' = \dfrac{D\lambda'}{d} = \dfrac{D\lambda}{nd}$ $\Delta y' = \dfrac{3.0 \times (5.8 \times 10^{-7})}{1.3 \times (0.60 \times 10^{-3})} = 2.23 \times 10^{-3} \approx 2.2$ mm $\Delta y' \approx 2.2$ mm

Exam tip

Always write the formula in symbolic form, rearranged for the required quantity, before substituting any numbers:

$\Delta y = \dfrac{\lambda D}{d}$

$\lambda = \dfrac{d\Delta y}{D}$

This makes your method explicit and earns the method marks. Then substitute and round carefully, since the final value is marked separately.

Reasonableness check	The wavelength $5.8 \times 10^{-7}\ m$ (580 nm) lies in the visible region of the spectrum, a physically reasonable value. In water the wavelength decreases ($\lambda' = \frac{\lambda}{n}$), so the fringe spacing decreases: $\Delta y' = 2.2\ \text{mm} < \Delta y = 2.9\ \text{mm}$, consistent with $\Delta y \propto \lambda$.
Explain it yourself	Explain why the spacing between the interference fringes decreases when the light passes from air into water.

Try

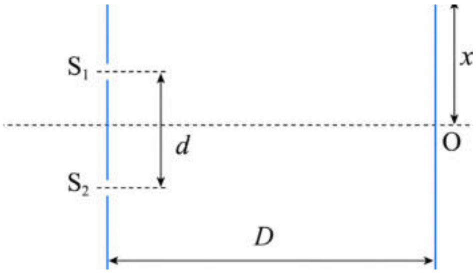


Figure 2-11-6 · Slit separation d and screen distance D.

As shown in Figure 2-11-6, monochromatic light of wavelength λ [m] passes through a single slit S_0, then two slits S_1 and S_2. Bright and dark fringes form on a screen. The slit separation is d [m] and the slit-to-screen distance is D [m].

(a) If D is **increased**, determine whether the spacing between adjacent bright fringes becomes smaller or larger.

(b) If d is **decreased**, determine whether the spacing between adjacent bright fringes becomes smaller or larger.

(c) The space between the slits and the screen is filled with a liquid of refractive index n. Determine by what factor the spacing between adjacent bright fringes changes.

In a new context — the spectral colors seen on a CD

The closely spaced tracks on a CD or DVD act like a diffraction grating. When white light reflects from these regularly spaced tracks, different wavelengths undergo constructive interference in different directions. Therefore, **different colors are seen at different viewing angles**. Changing the viewing angle selects different wavelengths, producing a changing sequence of colors.

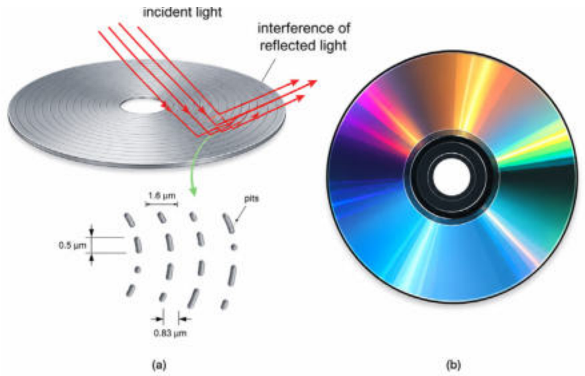


Figure 2-11-7 · Light reflected from the track of a disc interferes into colors.

Using the idea that different wavelengths undergo constructive interference at different angles, explain in one or two sentences why a CD produces a spread of colors rather than a single bright spot.

Nature of Science: Science as a shared endeavor — the gradual acceptance of Young's interference evidence

Young's double-slit experiment provided strong evidence for the wave nature of light because it produced interference fringes. At the time, the particle model of light was widely accepted, so the wave interpretation was not immediately accepted. Later work, especially by Augustin-Jean Fresnel, strengthened the wave model and helped establish it more widely.

Your turn: Suggest one reason why strong experimental evidence, like Young's fringes, can still take a generation to be accepted.

Data booklet

$\Delta y = \frac{\lambda D}{d}$ is printed; for part (a)(i) rearrange it to $\lambda = \frac{\Delta y d}{D}$, keep lengths in meters, and convert to the required unit only at the end.

Lesson question, answered

When coherent monochromatic light illuminates two narrow slits, the screen shows an interference pattern of bright and dark fringes rather than two isolated bright regions. The two slits act as coherent sources, and the emerging waves overlap and interfere. The central fringe is bright because the path difference there is zero. For small angles, the path difference at a point a distance x from the center is approximately $\Delta \approx \frac{dx}{D}$. Bright fringes occur when $\Delta = n\lambda$, and dark fringes occur when $\Delta = \left(n + \frac{1}{2}\right)\lambda$, where $n = 0,1,2,\ldots$. Therefore, adjacent bright fringes are separated by $\Delta y = \frac{\lambda D}{d}$.

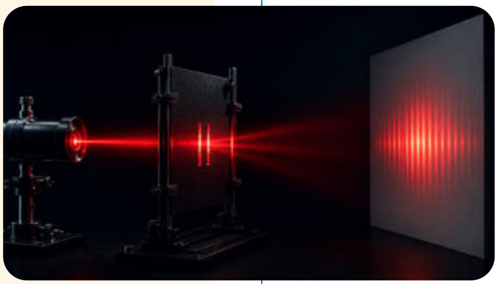


Figure 2-11-8 · The double-slit pattern revisited.

Exercise

Determine the following.

(a) Monochromatic light of wavelength λ [m] passes through a single slit S_0, then two slits S_1 and S_2, forming bright and dark fringes on a screen. The spacing between adjacent bright fringes is measured as 4.5 mm. The slit separation is 0.20 mm and the slit-to-screen distance is 1.5 m.

(i) Determine the wavelength λ [m] of the light.

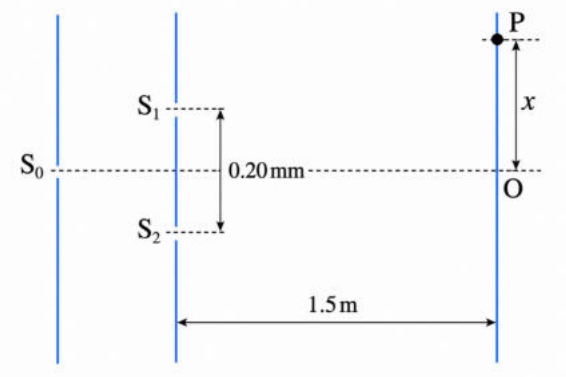


Figure 2-11-9 · Slits 0.20 mm apart, with the screen 1.5 m away.

(ii) The space between the slits and the screen is then filled with water (refractive index 1.3). Determine the new spacing between adjacent bright fringes, in mm.

(b) In a double-slit pattern, determine in each case below whether the spacing between adjacent bright fringes becomes **larger** or **smaller**:

(i) the slit separation is increased ______________

(ii) the screen is moved farther from the slits ______________

(iii) the light is changed to a longer wavelength ______________

Exam-style question

In a double-slit experiment the slit separation is $d = 0.40$ mm and the screen is $D = 2.0$ m away. The light has wavelength $\lambda = 6.0 \times 10^{-7}$ m.

Determine the spacing between adjacent bright fringes, in millimeters.

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