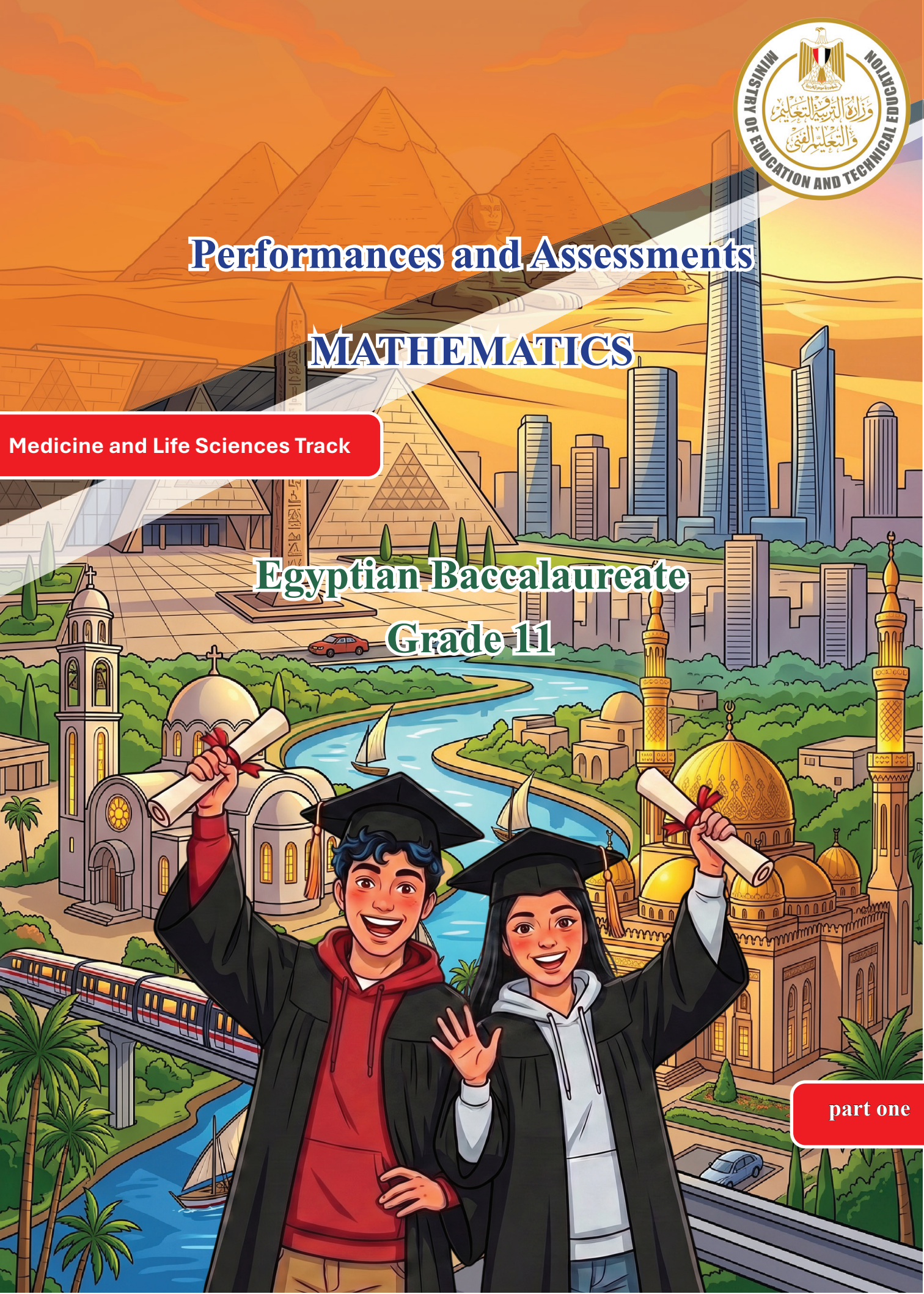


Performances and Assessments

MATHEMATICS

Medicine and Life Sciences Track

Egyptian Baccalaureate

Grade 11

part one

Introduction

Recognising the importance of equipping teachers with effective tools that improve classroom practice, enhance the quality of education and strengthen learning outcomes, the Ministry of Education and Technical Education has prepared this book in response to teachers' feedback and the needs identified in schools. It provides flexible, comprehensive resources to help teachers plan and carry out their instructional work effectively and assess learning continuously. Its content offers varied models of effective classroom activities, designed through an interactive approach that promotes active learning and places the student at the centre of the educational process.

The book includes formative and diagnostic questions and purposeful learning tasks for students to complete at home. These connect learning inside and outside the classroom and help students build lasting understanding. They also develop critical and creative thinking, deepen comprehension, strengthen motivation and give students a sense of achievement and distinction.

It also contains short weekly tests and periodic assessments that allow teachers to monitor progress towards learning outcomes, address learning gaps early and provide timely feedback to improve student performance. Each month concludes with graded practice that takes individual differences and different levels of thinking into account. The range, depth and integration of these tasks prepare students for the monthly assessments.

This book reflects the Ministry's commitment to easing the financial and practical burdens on families and to providing supporting educational content that enriches classroom learning. It gives teachers more time for instructional innovation and productive interaction with their students. It also supports the achievement of learning outcomes and a shift towards understanding, application and active participation.

In placing this work in the hands of teachers and students, we hope that it will help improve academic performance and educational quality, and contribute to preparing an informed and capable generation equipped with the skills and competencies needed in a world that calls for creativity and innovation.

MAY GOD GUIDE US TO WHAT IS RIGHT

Chapter One Algebraic Proofs

week: ..... **lesson: 1–1 Binomial Theorem** **Date :** ...../...../.....

Class performance

Group activity Find and correct the error

A student expanded $(x + 2)^4$ and wrote the following steps:

$(x + 2)^4 = {}_4C_0\, x^4 + {}_4C_1\, x^3(2) + {}_4C_2\, x^2(2)^2 + {}_4C_3\, x(2)^3 + {}_4C_4\, (2)^4$

$= x^4 + 4x^3(2) + 6x^2(4) + 4x(8) + 16$

$= x^4 + 8x^3 + 24x^2 + 16x + 16$

Work with your group:

- Analyse each step and check whether the binomial theorem has been applied correctly.
- Record your observations and correct any error you find.
- Discuss your answer with your group and record your final conclusion.

Homework

Choose the correct answer.

1. The sum of the coefficients in the expansion of $(x + y)^5$ is: (A) 16 (B) 32 (C) 64 (D) 5

Answer the following questions.

2. Find the coefficient of x^4 in the expansion of $(x^2 + \frac{1}{2})^8$.
3. In a population-growth experiment, the compound growth rate is calculated using a formula resembling the binomial expansion of $(1 + x)^n$. Use the binomial theorem to find the exact value of $(1.01)^4$.
4. Find the coefficient of x in the expansion of $(2x - \frac{1}{2})^9$.
5. Find the coefficient of x^3 in the expansion of $(x + 3)^5$.

week: ..... **lesson: 1–2 Multinomial Theorem** **Date :** ...../...../.....

Class performance

Group activity: What evidence supports your answer?

A student claims that the coefficient of x^2yz in the expansion of $(2x + y - z)^4$ is 12 and gives the following steps:

Step 1. Write the coefficient as ${}_4C_2 = 6$.

Step 2. Substitute into the combinations formula: the coefficient is $\frac{4!}{2!} = 12$.

Step 3. Conclude that the coefficient of x^2yz is 12.

Your tasks:

1. Examine $(2x + y - z)$. Identify the variables and numerical coefficients in $a = 2x$, $b = y$ and $c = -z$, and the appropriate exponents p, q and r.

2. Use the multinomial coefficient $\frac{n!}{P!q!r!}$ to calculate the actual coefficient of the term accurately.

3. Show, step by step, whether the student's claim is true or false and state the correct coefficient.

Discuss your answer with your group.

Homework

Choose the correct answer.

1. The coefficient of xy^2z in the expansion of $(x + y + z)^4$ is: (A) 4 (B) 6 (C) 12 (D) 24

Answer the following questions.

2. Find the coefficient of x^2y^3z in the expansion of $(x + 2y - z)^6$.

3. Expand $(x + y + z)^3$, then calculate the sum of the coefficients of all its terms.

4. Six distinct prizes are distributed among three students in various ways represented by the expansion of $(x + y + z)^6$. The first student receives two prizes, the second one prize and the third three prizes. Write the term representing this distribution and find its coefficient.

5. Find the coefficient of x^2yz in the expansion of $(x - 2y + 3z)^4$.

week: .....**lesson: 3 Applications of the Binomial Theorem** **Date :** .....\....\.....

Class performance

Group activity Verify it yourself

Consider the following claims about the properties of binomial coefficients:

$(1 + x)^n = {}_nC_0 + {}_nC_1 x + {}_nC_2 x^2 + {}_nC_3 x^3 + ... + {}_nC_n x^n$.

Substituting $x = 1$ gives ${}_nC_0 + {}_nC_1 + {}_nC_2 + {}_nC_3 + ... + {}_nC_n = 2^n$.

Substituting $x = -1$ gives ${}_nC_0 - {}_nC_1 + {}_nC_2 - {}_nC_3 + ... + (-1)^n {}_nC_n = 0$.

Your tasks:

1. Let $n = 4$. Expand $(1 + 1)^4$ and $(1 - 1)^4$ in full using the binomial theorem, and verify both identities by direct numerical substitution.

2. Find the sum ${}_4C_0 + {}_4C_2 + {}_4C_4$ and the sum ${}_4C_1 + {}_4C_3$.

Record what you notice about the relationship between these two sums.

3. State a general mathematical rule relating the two sums for any positive integer n.

Discuss your answer with your group.

Homework

Choose the correct answer.

1. The alternating sum ${}_6C_0 - {}_6C_1 + {}_6C_2 - {}_6C_3 + ... + {}_6C_6$ equals: (A) 64 (B) 36 (C) 0 (D) 1

Answer the following questions.

2. Use the binomial expansion to prove that ${}_nC_0 + 2{}_nC_1 + 2^2{}_nC_2 + 2^3{}_nC_3 + ... + 2^n{}_nC_n = 3^n$.

3. Use the expansion of $(1 + x)^5$ to prove that ${}_5C_0 + {}_5C_2 + {}_5C_4 = {}_5C_1 + {}_5C_3 + {}_5C_5 = 2^4$.

4. Use the binomial expansion to prove that ${}_nC_0 - 2{}_nC_1 + (-2)^2{}_nC_2 + (-2)^3{}_nC_3 + {}_nC_3 + (-2)^n{}_nC_n = (-1)^n$.

5. Use the binomial expansion to prove that ${}_nC_0 + (\frac{1}{3}){}_nC_1 + (\frac{1}{3})^2{}_nC_2 + (\frac{1}{3})^3{}_nC_3 + ... + (\frac{1}{3})^n{}_nC_n = (\frac{4}{3})^n$.

Weekly Assessment week: ..... Date : ...../...../.....

Group A

Choose the correct answer.

1. The coefficient of xyz^2 in the expansion of $(x + y + z)^4$ is: (A) 4 (B) 6 (C) 12 (D) 24

Answer the following questions.

2. Use direct substitution in the binomial theorem to prove that ${}_5C_0 + {}_5C_1 + {}_5C_2 + {}_5C_3 + {}_5C_4 + {}_5C_5 = 32$.

3. Find the coefficient of x^3 in the expansion of $(x + 2)^6$.

4. Find the coefficient of x^4y^2 in the expansion of $(x^2 + 2y + 1)^7$.

5. In the expansion of $(x^3 + y)^8$, the coefficient of $x^{12}y^4$ is k. Find k.

Group B

Choose the correct answer.

1. The coefficient of x^2yz in the expansion of $(x + y + z)^4$ is: (A) 4 (B) 6 (C) 12 (D) 24

Answer the following questions.

2. Use direct substitution in the binomial theorem to prove that ${}_4C_0 + {}_4C_1 + {}_4C_2 + {}_4C_3 + {}_4C_4 = 16$.

3. Find the coefficient of x^5 in the expansion of $(x + 3)^{10}$.

4. Find the coefficient of x^4y in the expansion of $(x^2 + 2y + 1)^5$.

5. Use the multinomial theorem to find the coefficient of $x^2y^3z^4$ in the expansion of $(x + y + z)^9$.

Group C

Choose the correct answer.

1. The coefficient of x^3yz in the expansion of $(x + y + z)^5$ is: (A) 5 (B) 10 (C) 20 (D) 60

Answer the following questions.

2. Use direct substitution in the binomial theorem to prove that

$${}_6C_0 + {}_6C_1 + {}_6C_2 + {}_6C_3 + {}_6C_4 + {}_6C_5 + {}_6C_6 = 64.$$

3. Find the coefficient of x^6 in the expansion of $(x + 1)^{10}$.

4. Find the coefficient of x^{12} in the expansion of $(x^2 + y + 1)^6$.

5. Use the multinomial theorem to find the coefficient of $x^3y^4z^5$ in the expansion of $(x + y + z)^{12}$.

week: ..... **lesson: 4 Division of Polynomials** **Date:**...../...../.....

Class performance

Group activity Correct the error

A student divided $P(x) = x^3 + 2x^2 - 5x - 6$ by $x - 2$ using long division and recorded the following working:

$x^2 + 4x + 13$

$x^3 + 2x^2 - 5x - 6 \mid x - 2$

$x^3 - 2x^2$

$4x^2 - 5x$

$4x^2 - 8x$

$-13x - 6$

$-13x - 26$

20

Student's result: quotient $x^2 + 4x + 13$; remainder 20.

Group tasks:

1. Examine the long-division steps carefully, checking the signs in multiplication and subtraction.

2. Carry out the correct long division to find the actual quotient.

3. Explain the error made by the student and how it can be avoided.

Discuss your answer with your group.

Homework

Choose the correct answer.

1. What is the remainder when $x^3 - 4x^2 + x + 6$ is divided by $x - 4$? (A) 4 (B) 6 (C) 8 (D) 10

Answer the following questions.

2. Use long division to divide $P(x) = 3x^5 + 10ax^3 + 5ax^2 + 8a^2x - 24a^3$ by $D(x) = x^2 - 4a$. State the quotient and remainder.

3. If $P(x) = 3x^3 + 5x^2 - x - 3$ and $D(x) = x + 1$, find the quotient and remainder.

4. The remainder when $P(x) = x^3 + x^2 - 4x + k$ is divided by $D(x) = x - 2$ is 7. Find k.

5. If $P(x) = -x^4 + 2x^3 + 5x^2 - 6x - 2$ and $D(x) = x^2 - 3$, find the quotient and remainder.

week: ..... **lesson: 5** **Quotient and Remainder of Polynomials** **Date :** ...../...../.....

Class performance

Group activity You are the teacher

A student was asked to explain how to check an answer consisting of the quotient $2x^2 - 3x$ and remainder $x + 8$ for the division of $P(x) = 2x^3 - 5x^2 + 4x + 8$ by $x - 1$. The student wrote:

1. Multiply the proposed quotient $2x^2 - 3x$ by $x - 1$ to obtain $2x^3 - 5x^2 + 3x$.

2. Subtract the remainder $x + 8$ and obtain $2x^3 - 5x^2 + 2x - 8$.

3. Conclude that the proposed quotient is incorrect.

Take the role of the teacher:

- Examine the student's steps carefully and identify any calculation error.
- Use long division to calculate the quotient and check the actual remainder.
- Record mathematically sound observations and state the correct quotient and remainder.

Discuss your answer with your group.

Homework

Choose the correct answer.

1. If $P(x) = x^3 - 4x^2 + x + 6$, the remainder when P(x) is divided by $x - 2$ is:

(A) $x + 4$ (B) $4x$ (C) 0 (D) 4

Answer the following questions.

2. The quotient when P(x) is divided by $x^2 + x + 1$ is $x - 1$, and the remainder is 8. Find P(x).

3. Find the quotient and the value of k for which the remainder when $x^4 - 3x^3 - 10x^2 + k$ is divided by $x + 1$ is 3.

4. When $P(x) = x^2 + x + 2$ is divided by D(x), the quotient is $x - 2$ and the remainder is 8. Find D(x).

5. When P(x) is divided by $3x^3 + 4x^2 - 5$, the quotient is $x - 2$ and the remainder is $x - 5$. Find P(x).

week: ..... **lesson: 6 Multiplication and Division of Rational Expressions** **Date:**...../...../.....

Class performance

Group activity: What is your evidence?

Consider the following discussion between two students about simplifying rational expressions. Work with your group to gather mathematical evidence for or against each view.

Question: Is $\frac{x^2 - x - 6}{x - 3}$ exactly equivalent to $x + 2$ for every real value of x?

Student A: "Yes. The numerator factors as $(x - 3)(x + 2)$. Cancelling the common factor $x - 3$ leaves $x + 2$, so the two expressions are always identical."

Student B: "No. They are not identical for every value, because the denominator of the original expression is zero for some values."

Group instructions:

- Analyse both the original and simplified expressions, paying attention to values of x that make a denominator zero.
- Provide mathematical evidence by evaluating the original expression at $x = 3$. What do you observe?
- Does cancelling a factor remove the restrictions on the original expression? Explain your conclusion.

Discuss your answer with your group.

Homework

Choose the correct answer.

1. Simplify $\frac{x - 1}{10} \times \frac{5x}{x^2 - x}$

(A) $\frac{1}{2}$ (B) $\frac{5}{2x}$ (C) $\frac{1}{2x}$ (D) 2x

Answer the following questions.

2. Simplify $\frac{x^2 - 25}{x^2 + 7x + 10} \times \frac{x + 2}{x - 5}$

3. Simplify $\frac{3x^2 - 12}{x^2 - 9} \div \frac{x^2 - 4}{x - 3}$

4. Simplify $\frac{x^3 - 8}{x^2 - 4} \times \frac{x + 2}{x^2 + 2x + 4} \div \frac{x - 2}{x}$

5. A rectangle has area $\frac{x^2 - 9}{x^2 - 4}$ and width $\frac{x - 3}{x - 2}$ Find its length in simplest form.

week: ..... **lesson: 7 Addition and Subtraction of Rational Expressions** **Date :** ...../...../.....

Class performance **Group activity: Check it yourself**

A student attempted to simplify $\frac{x + 3}{x^2 - 4} + \frac{2}{x - 2}$ as follows:

Step 1: $\frac{x + 3}{(x - 2)(x + 2)} + \frac{2}{x - 2}$

Step 2: $\frac{x + 3 + 2}{(x - 2)(x + 2)}$

Step 3: $\frac{x + 5}{x^2 - 4}$

Group tasks:

1. Identify the step containing the mathematical error.

2. Explain why it is incorrect.

3. Perform the calculation correctly and give the result in simplest form.

Discuss your answer with your group.

Homework

Choose the correct answer.

1. The least common denominator of and is:

(A) $12x^2(x - 1)$ (B) $6x^2(x - 1)$ (C) $6x(x - 1)$ (D) $12x(x - 1)$

Find each result in simplest form.

2. $\frac{5x}{x+2} - \frac{2}{x+3}$

3. $\frac{x-3}{x^2+5x-24} + \frac{x^2-4}{x^2+10x+16}$

4. $\frac{3}{x^2-9} + \frac{2}{x+3} - \frac{2}{x-3}$

5. Prove that: $\frac{-2}{x^2-1} = \frac{1}{x+1} - \frac{1}{x-1}$

week: ..... **lesson: 8 Compound Fractions** **Date :** ...../...../.....

Class performance

Group activity Think Pair Share

Simplify the complex fraction $\frac{\frac{2}{x}+1}{\frac{4}{x^2}-1}$

Group tasks:

1. **Think independently**: simplify the numerator and denominator by finding common denominators, then divide.
2. **Work with a partner**: multiply the numerator and denominator of the complex fraction by their least common denominator, then simplify.
3. **Share in your group**: compare results and discuss which method is faster and less prone to errors.

Discuss your result and preferred method with your group.

Homework

Choose the correct answer.

1. Simplify $\frac{\frac{2}{x}}{\frac{4}{x^2}}$, where $x \neq 0$. (A) $\frac{x}{2}$ (B) $\frac{2}{x}$ (C) $2x$ (D) $\frac{8}{x^3}$

Find each result in simplest form.

2. $\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{xy}}$

3. $\frac{\frac{x}{x-2}+1}{\frac{x}{x-2}-1}$

4. Prove that $\dfrac{\frac{x}{x-2}+1}{\frac{x}{x-2}-1} = \left(\dfrac{x}{x-2}\right)^2$

5. Simplify , showing every step $\dfrac{\frac{x}{x-2}-1}{1-\frac{x}{x+1}}$

Weekly Assessment **week:** ..... **Date :** ...../...../.....

Group A

Choose the correct answer.

1. The Simplest form of $\dfrac{\frac{x}{x+1}+\frac{1}{x}}{\frac{x}{x+1}-\frac{1}{x}}$ is...

(A) $\dfrac{x^2+x+1}{x^2-x-1}$ (B) $\dfrac{x^2+x+1}{x^2+x-1}$

(C) $\dfrac{x^2+x+1}{x^2-x+1}$ (D) $\dfrac{x^2-x-1}{x^2+x+1}$

Answer the following questions.

2. A construction mould has volume $V(x) = x^3 + 5x^2 + 2x - 8$. If its width is $x + 2$, find the product of its length and height.

3. If $P(x) = -x^4 + 3x^3 - 23x^2 - 75x - 25$ and $D(x) = x^2 - 25$, find the quotient and remainder.

4. Simplify $\dfrac{x^2+x+1}{x^2-x-1} \times \dfrac{x+2}{x^2+3x+9} \div \dfrac{x^3-27}{x^2-4}$

Group B

Choose the correct answer.

1. When $x^2 + 3x - 7$ is divided by $x - 5$, which quotient and remainder are obtained?

(A) Quotient $x - 8$; remainder 12 (B) Quotient $x - 8$; remainder 33

(C) Quotient $x + 8$; remainder 12 (D) Quotient $x + 8$; remainder 33

Answer the following questions.

2. A construction mould has volume $V(x) = x^3 + 5x^2 + 2x - 8$. If its length is $x + 4$, find the product of its width and height.

3. If $P(x) = -x^4 + 3x^3 - 23x^2 - 75x - 25$ and $D(x) = x^2 + 3x + 2$, find the quotient and remainder.

4. Simplify $\frac{x^3 - 8}{x^2 - 9} \times \frac{x + 3}{x^2 + 2x + 4} \div \frac{x - 2}{x}$

Group C

Choose the correct answer.

1. Dividing a polynomial f(x) by x − 2 gives quotient q(x) and remainder 5. What is the remainder when f(x) + 3x is divided by x − 2?

(A) 5 (B) 8 (C) 11 (D) 3x + 5

Answer the following questions.

2. A construction mould has volume $V(x) = x^3 + 5x^2 + 2x - 8$ cubic units. If its height is x − 1 units, find the product of its length and width.

3. If $P(x) = -x^4 + 3x^3 - 23x^2 - 75x - 25$ and $D(x) = x^2 - 3x - 10$, find the quotient and remainder.

4. Simplify $\frac{x^3 - 8}{x^2 - 9} \times \frac{x + 3}{x^2 + 2x + 4} \div \frac{x - 2}{x}$

week: ..... **lesson: 9 Identities** **Date :** ...../...../.....

Class performance **Group activity Find the constants**

Work with your group to find the values of the constants a, b and c that make each equation an identity in x:

1) $x^2 + 2x + 1 = a(x + 3)^2 + b(x + 3) + c$.

2) $x^2 - 2x + 1 = a(x - 1)^2 + b(x - 1) + c$.

Discuss your answers with your group.

Homework

Choose the correct answer.

1) If $x^2 - 9 = a(x - 1)^2 + b(x - 1) + c$, what is the product abc?

(A) 12 (B) −12 (C) 16 (D) −16

Find the values of the constants that make each equation an identity in x.

2) $3x^2 - 7x + 2 = (x - 2)(ax + b) + c$.

3) $(ax + b)(x + 5) = 5x^2 + cx + 15$.

4) $\frac{9x + 11}{9x^2 + 2x - 3} = \frac{A}{x + 1} + \frac{B}{x + 3}$

week: ..... **lesson: 10 Proving Equalities 1** Date : ...../...../.....

Class performance **Group activity Prove the identity**

Work with your group to prove that $(2x - 3)^2 - (x + 1)(x - 1) = 3(x^2 - 4x) + 10$.

Discuss your answer with your group.

Homework

Prove the following identities.

1. $(a - b)^3 + 3ab(a - b) = a^3 - b^3$.
2. $(x - 2y)^3 + 6x^2y + 8y^3 = x(x^2 + 12y^2)$.
3. $(x^2 - x - 1)(x^2 + x + 1) = (x + 1)(x^3 - x^2 - 2) + 1$.
4. $(3x + y)^2 - (3y + x)^2 = 8(x^2 - y^2)$.
5. $(x^2 - 1)(y^2 - 1) = (xy + 1)^2 - (x + y)^2$.

week: ..... **lesson: 11 Proving Equalities 2** Date : ...../...../.....

Class performance **Group activity A constant result**

If $x + y + z + w = 0$, prove that

$$\frac{x}{w + z + y} + \frac{y}{w + z + x} + \frac{z}{w + y + x} + \frac{w}{z + y + x} = -4.$$

Discuss your answer with your group.

Homework

Prove the following statements.

1. If $x + y + z = 0$, then $x^2 + y^2 - z^2 = 2y(y + z)$.
2. If $x + y = 1$, then $x^3 + y^3 - 1 = -3y(1 - y)$.
3. If $x + y + z = 0$, then $x(x + z) = y(y + z)$.
4. If $x - y = 0$, then $(3x + y)^2 - (3y - x)^2 = 8(x^2 - y^2)$.
5. If $x + y = 0$, then $(x^2 - 1)(y^2 - 1) = (xy + 1)^2 - (x + y)^2$.

week: ..... **lesson: 12 Proving Equalities 3** **Date :** ...../...../.....

Class performance **Group activity Check the proof and find the error**

Let $\frac{a}{b} = \frac{c}{d} = k$, where $k \neq 0$. A student tries to prove that $\frac{a^2 + c^2}{b^2 + d^2} = \frac{ac}{bd}$:

Step 1. Substitute $a = kb$ and $c = kd$: $\frac{a^2 + c^2}{b^2 + d^2} = \frac{(kb)^2 + (kd)^2}{b^2 + d^2}$

Step 2. Factor k from the numerator and conclude that the left-hand side equals k.

Step 3. Substitute on the right: $\frac{ac}{bd} = \frac{(kb)(kd)}{bd} = k^2$

Step 4. Conclude that the equality is false because one side is k and the other k^2.

Group tasks: identify the error in applying the exponent rules in Step 2; correct the proof to show that both sides equal k^2; record your observations and calculations. Discuss your answer with your group.

Homework

Answer the following questions.

1. If $\frac{y}{5} = \frac{x}{3} = k$, find the value of $\frac{x + y}{3 + 5}$

2. If $\frac{x}{y} = \frac{z}{w} = \frac{2}{3}$, find the numerical value of $\frac{2z + 3x}{2w + 3y}$

3. If $\frac{x}{y} = \frac{z}{w}$, prove that $\frac{x - y}{x + y} = \frac{z - w}{z + w}$

4. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, prove that $\frac{x - 2y + z}{a - 2b + c} = \frac{y - z}{b - c}$

5. If $x : y : z = 3 : 5 : 2$, find $\frac{y + z - x}{y - z + x}$.

week: ..... **lesson: 13 Proving Inequalities 1** **Date:**...../...../.....

Class performance **Group activity Check it yourself**

Work with your group to prove the following inequalities:

1. If $x > 5$ and $y > -3$, prove that $xy + 3x > 5y + 15$.

2. If $a > b$ and $x > y$, prove that $ax + by > bx + ay$.

Discuss your answers with your group.

Homework

Choose the correct answer.

1. If $x > y$, which of the following must hold? (A) $3x > 7y$ (B) $x + y < -2$ (C) $-x > y$ (D) $x - y > 0$

Prove the following inequalities.

2. If $x > y > z$, prove that $x^2 + yz > xy + xz$.

3. If $x > y$, prove that $x^3 + 3xy^2 > y^3 + 3yx^2$.

4. If $y > x$, prove that $x^2 > 2xy - y^2$.

5. If $x > y > z$, prove that $x^2 + 2y^2 + z^2 > 2xy + 2yz$.

Weekly Assessment week: ..... Date : ...../...../.....

Group A

Choose the correct answer.

1. Simplify $\frac{x+3}{x^2-4} + \frac{2}{x-2} + \frac{5}{x+2}$

(A) $17 - 2x$ (B) $x^2 - 4$ (C) $\frac{2x-3}{4-x^2}$ (D) $\frac{17-2x}{x^2-4}$

Answer the following questions.

2. Prove that $\frac{1 - \frac{2}{(x+1)}}{1 + \frac{2}{(x-1)}} = \left(\frac{x-1}{x+1}\right)^2$

3. Find the constants that make $2x^2 - 5x + 2 = (2x - 1)(ax + b) + c$ an identity.

4. If $x > y$, prove that $2xy - y^2 < x^2 + y^2$

5. If $\frac{x}{y} = \frac{z}{y}$, prove that $\frac{2z+3x}{2w+3y} = \frac{3z-x}{3w-y}$

Group B

Choose the correct answer.

1. Simplify $\frac{x+3}{x^2-4} + \frac{2}{x-2} + \frac{5}{x+2}$

(A) $3 - 6x$ (B) $x^2 - 4$ (C) $\frac{6x-3}{4-x^2}$ (D) $\frac{6x-3}{x^2-4}$

Answer the following questions.

2. Prove that $\dfrac{1 + \dfrac{2}{(x+1)}}{1 - \dfrac{2}{(x-1)}} = \dfrac{x+3}{x-1}$

3. Find the constants that make $2x^2 - 5x + 2 = (2x - 1)(ax + b) + c$ an identity.

4. If $x > y$, prove that $2xy < x^2 + y^2$.

5. If $\dfrac{x}{y} = \dfrac{z}{w}$, prove that $\dfrac{3z+2x}{3w+2y} = \dfrac{z-x}{w-y}$

Group C

Choose the correct answer.

1. Simplify $\dfrac{x-3}{x^2-4} + \dfrac{2}{x-2} + \dfrac{5}{x+2}$

(A) $8x - 9$ (B) $x^2 - 4$ (C) $\dfrac{8x-3}{4-x^2}$ (D) $\dfrac{8x-9}{x^2-4}$

Answer the following questions.

2. Prove that $\dfrac{1 - \dfrac{2}{(x+1)}}{1 + \dfrac{2}{(x+1)}} = \dfrac{x-1}{x+3}$

3. Find the constants that make $x^2 - 5x + 4 = (x - 1)(ax + b) + c$ an identity.

4. If $x > y$, prove that $(x + y)^2 > 4xy$.

5. If = , prove that = If $\dfrac{x}{y} = \dfrac{z}{w}$, prove that $\dfrac{3z+2x}{3w+2y} = \dfrac{z-3x}{w-3y}$

week: ..... **lesson: 14 Proving Inequalities 2** **Date:**...../...../.....

Class performance **Group activity Observe and learn**

Work with your group to prove the following inequalities:

1. $x^2 + 25y^2 \geq 10xy$.

2. $4x^2 + 4y^2 \geq 8(x + y - 1)$.

Discuss your answer with your group.

Homework

Prove the following inequalities:

1. $9x^2 + 4y^2 \geq 12xy$.

2. $x^2 + 4x \geq -y^2 + 6y - 13$.

3. $x^2 \geq y(2x - y)$.

4. $y^2 \geq 2xy - x^2$.

5. $(a^2 + b^2)(x^2 + y^2) \geq (ax - by)^2$.

week: ..... **lesson: 15 Proving Inequalities 3** **Date:**...../...../.....

Class performance

Activity Check the validity of the steps

A student attempted to prove the following inequality for all positive real numbers x and y:

$\sqrt{2(x + y)} \geq \sqrt{x} + \sqrt{y}$.

The student's solution proceeds as follows:

1. Assume $(\sqrt{x} - \sqrt{y})^2 \geq 0$, since the square of every real number is nonnegative.

2. Expand the perfect square: $x - 2\sqrt{xy} + y \geq 0$.

3. Move the middle term to the other side: $x + y \geq 2\sqrt{xy}$

4. Add $(x + y)$ to both sides: $2(x + y) \geq x + y + 2\sqrt{xy}$

5. Factor the right-hand side as a perfect square: $2(x + y) \geq (\quad)^2$

6. Take the nonnegative square root of both sides: $\sqrt{2(x + y)} \geq \sqrt{x} + \sqrt{y}$

Group tasks: check each step and assess the validity of the mathematical reasoning; write a mathematical justification for Steps 1–6; discuss your answer with your group.

Homework

1. Which of the following inequalities is always true for positive real numbers x and y?

(A) $\sqrt{x} + \sqrt{y} = \sqrt{x+y}$ (B) $\sqrt{x} + \sqrt{y} < \sqrt{x+y}$

(C) $\sqrt{x} + \sqrt{y} > \sqrt{x+y}$ (D) $\sqrt{x} + \sqrt{y} \geq \sqrt{x+y}$

Prove the following inequalities:

2. If $x > 0$, then $x + 3 > \sqrt{4x + 8}$.

3. If $x > 0$ and $y > 0$, then $2(x + y) < \sqrt{8(x^2 + x^2)}$

4. If $x > 0$ and $y > 0$, then $3x + 3y > \sqrt{18xy}$

5. If $x > 0$ and $y > 0$, then $\sqrt{x} + 4\sqrt{y} > \sqrt{16y + x}$

week: ..... **lesson: 16 Proving Inequalities 4** Date:...../...../.....
(Arithmetic–Geometric Mean Inequality)

Class performance

Inequality activity Solve it yourself

Solve the following inequalities individually, then work with your group to prove them:

1. If $x > 0$ and $y > 0$, then $\frac{x}{y} + \frac{4y}{x} \geq 4$.

2. If $x > 0$ and $y > 0$, then $(x + y)(\frac{2}{x} + \frac{2}{y}) \geq 8$.

Discuss your answers with your group.

Homework

Prove the following inequalities:

1. If $x > 0$, then $\frac{x}{2} + \frac{2}{x} \geq 2$.

2. If $x > 0$ and $y > 0$, then $x + y + \frac{4}{x + y} \geq 4$.

3. If $x > 0$ and $y > 0$, then $(x + \frac{1}{x})(y + \frac{1}{y}) \geq 4$.

4. If $x > 0$ and $y > 0$, then $(3x + 4y)(\frac{4}{x} + \frac{3}{y}) \geq 48$.

5. If $x > 0$ and $y > 0$, then $(x^3 + y^3)(\frac{2}{x^2} + \frac{2}{y^2}) \geq 8$.

Weekly Assessment

week: ..... Date : ...../...../.....

Group A

Choose the correct answer.

1. Simplify $\frac{x - 3}{x^2 - 4} + \frac{-2}{x - 2} - \frac{5}{x + 2}$

(A) $8x - 9$ (B) $x^2 - 4$ (C) $\frac{6x - 3}{4 - x^2}$ (D) $\frac{8x - 3}{x^2 - 4}$

Prove the following inequalities.

2. If $x > y > z$, then $yz - y^2 > xz - xy$.

3. $x^2 - 4x \geq -y^2 - 6y - 13$.

4. If $x > 0$ and $y > 0$, then $(x + y)(\frac{4}{x} + \frac{1}{y}) \geq 9$.

5. If $0 < x < 1$, then $x + \frac{1}{x} > 2$.

Group B

Choose the correct answer.

1. Simplify $\frac{x-3}{x^2-4} + \frac{-2}{x-2} - \frac{5}{x+2}$

(A) $8x - 9$ (B) $x^2 - 4$ (C) $\frac{6x-3}{4-x^2}$ (D) $\frac{8x-3}{x^2-4}$

Prove the following inequalities.

2. If $x > y > z$, then $yz + xz < xy + y^2$.

3. $x^2 - 4x \geq -y^2 - 6y - 13$.

4. If $x > 0$ and $y > 0$, then $(x + y)(\frac{6}{x} + \frac{1}{y}) \geq 7 + 2\sqrt{6}$.

5. If $x > 0$ and $x \neq 1$, then $9x + \frac{1}{x} \geq 6$.

Group C

Choose the correct answer.

1. Simplify $\frac{x-3}{x^2-4} + \frac{-2}{x-2} + \frac{5}{x+2}$

(A) $4x - 17$ (B) $x^2 - 4$ (C) $\frac{8x-3}{4-x^2}$ (D) $\frac{4x-17}{x^2-4}$

Prove the following inequalities.

2. If $x > y > z > 0$, then $xy + yz - y^2 > xz$.

3. $x^2 - 4x \geq -y^2 - 6y - 13$.

4. If $x > 0$ and $y > 0$, then $(x + y)(\frac{4}{x} + \frac{2}{y}) \geq 6 + 4\sqrt{2}$.

5. If $x > 0$ and $x \neq 1$, then $x + \frac{4}{x} \geq 4$.

Chapter 2 Sequences

week: .....lesson: 1 Sequences and General Terms Date : ...../...../.....

Class performance

Activity: Check it yourself

For the sequence defined by $a_n = 3n - 1$, a student claims that the difference between any two consecutive terms is always constant and equals 3.

Group tasks:

1. Calculate the first four terms (a_1, a_2, a_3, a_4).

2. Test the claim: calculate the difference between each term and the preceding term. What do you notice? Calculate the ratio of each term to its predecessor. Is this ratio constant?

3. Decide whether the claim is true. Identify the constant change in this sequence: is it a constant difference or a constant ratio, and what is this constant called? Discuss your answer and record your observations and final proof.

Homework

Choose the correct answer.

1. If $a_n = 2n - 1$, then a_3 equals: (A) 3 (B) 5 (C) 7 (D) 9.

Answer the following questions.

2. Write the first four terms of the sequence whose general term is $a_n = 3^n + 2$.

3. Find the position of the term whose value 43 in the sequence 3, 7, 11, 15, ...

4. Khaled saves EGP 50 in the first week and increases the amount saved by EGP 15 each subsequent week. Derive the general term a_n for his savings in week n, and calculate the amount he saves in week 12.

5. A sequence has general term $a_n = xn + y$. If $a_2 = 7$ and $a_5 = 16$, find x and y, then determine a_{10}

week: ..... lesson: 2 Arithmetic Sequences 1 Date : ...../...../.....

Class performance

Activity: Provide evidence

A student claims that any sequence is arithmetic if the difference between its first two terms is positive. The student tests this claim using 2, 5, 9, 14, 20, ...

The student's reasoning: $a_1 = 2$ and $a_2 = 5$; $a_2 - a_1 = 5 - 2 = 3 > 0$; therefore, the sequence is arithmetic with common difference $d = 3$.

Present evidence to your classmate showing whether the claim is correct.

Discuss your answer with your group and record your observations and final proof.

Homework

Choose the correct answer.

1. For the arithmetic sequence 5, 9, 13, 17, ... , the common difference d and tenth term a_{10}, respectively, are:

(A) d = 4, a_{10} = 41 (B) d = 4, a_{10} = 45 (C) d = 3, a_{10} = 32 (D) d = 5, a_{10} = 50.

Answer the following questions.

2. Find the general term a_n of an arithmetic sequence with a_1 = 7 and d = −3; then find a_8.

3. An arithmetic sequence has a_3 = 11 and a_7 = 23. Find a_1 and d, then write its first four terms.

4. An engineer begins work on a monthly salary of EGP 6,000 and receives a regular increase of EGP 500 at the end of each year. Write the general term a_n for the monthly salary in year n and calculate the monthly salary in year 8.

5. Prove that a_n = 5n − 4 defines an arithmetic sequence; then find its twelfth term.

week: ..... **lesson: 3 Arithmetic Sequences 2** **Date :** ...../...../.....

Class performance

Activity: Try it yourself

If x, 3, x^2 form an arithmetic sequence in this order, what is its common difference d?

Use Think–Pair–Share, discuss the answer with your group and record your observations.

Homework

Choose the correct answer.

1. If a_8 = 37 and d = 5 in an arithmetic sequence, its first term is:

(A) 2 (B) 5 (C) 9 (D) 13.

Answer the following questions.

2. Find the general term a_n of an arithmetic sequence with a_3 = 15 and a_8 = 55.

3. An arithmetic sequence has d = 3 and a_8 = 25. Find the position of the term equal to 112; then find the position of the first term greater than 300.

4. An engineer saves an amount each month according to an arithmetic sequence. The amount saved in month 3 is EGP 1,000 and that saved in month 8 is EGP 5,000. Find the general term for the amount saved in month n and the first month in which the monthly amount saved exceeds EGP 10,000.

5. If x, 8, $(x^2 - x)$ form an arithmetic sequence in this order, find all possible values of x and write the terms of the sequence in each case.

week: ..... lesson: 4 Sum of an Arithmetic Sequences 1 Date : ...../...../.....

Class performance

Activity: What is your evidence?

A group wants to establish the relationship between the two formulas for the sum of the first n terms of an arithmetic sequence:

1. In terms of its last term L: $S_n = \frac{n}{2}(a_n + L)$.

2. In terms of its common difference d: $S_n = \frac{n}{2}[2a_1 + (n - 1)d]$.

Apply each formula independently to the first 10 terms of the arithmetic sequence with $a_1 = 4$, $d = 3$ and $L = 31$. Record whether the two answers agree. Discuss your answer with your group.

Homework

Choose the correct answer.

1. An arithmetic sequence has seven terms, with first term 9 and last term 23. Its sum is:

(A) 70 (B) 112 (C) 140 (D) 224.

Answer the following questions.

2. Find the sum of the arithmetic sequence with first term 4, common difference 3 and last term 49.
3. In the arithmetic sequence 89, 83, 77, ... , find its tenth and twentieth terms. Then find the sum from the ninth term through the twentieth term.
4. A child saves EGP 20 in the first month, then increases the monthly saving by EGP 5 each month for 13 months. Calculate the total amount saved in this period.
5. An arithmetic sequence has first term 2 and common difference 4. If the sum of its first n terms is 288, find n.

week: .....lesson: 5 Sum of an Arithmetic Sequences 2 Date : ...../...../.....

Class performance

Group activity Calculate it yourself

Work with your group to divide the integers from 1 to 100 into ten groups. Find the sum of each group. Then find the sums from 1 to 50 and from 51 to 100.

Compare the results and record your observations.

Discuss your answer with your group.

Homework

Choose the correct answer.

1. The sum of the first 50 natural numbers (1 through 50) equals:

(A) 1225 (B) 1275 (C) 2550 (D) 1300.

Answer the following questions.

2. Find the sum of the natural numbers from 3 through 39.

3. Find the sum of the even natural numbers between 1 and 100.

4. Find the sum of the odd natural numbers between 101 and 200.

5. Find the sum of the two-digit multiples of 3, from 12 through 99.

week: .....lesson: 6 Sum of an Arithmetic Sequences 3 Date : .....\.....\.....

Class performance

Activity: You be the judge

Two students address this question: "An arithmetic sequence has first term 25 and common difference −4.

Find the number of terms starting from the first that gives the maximum possible sum; then find this sum."

Student 1: To find the greatest sum, set $S_n = 0$ and solve for n.

Student 2: To find the greatest sum, add only the positive terms; find the position of the last positive term from $a_n > 0$.

Think and discuss with a partner: decide whose answer is correct and explain why.

Calculate the number of terms giving the greatest sum and the value of that sum.

Discuss with your group and record your observations.

Homework

Choose the correct answer.

1. For an arithmetic sequence with $a_1 = 7$ and $d = 3$, if the sum starting with its first term is 65, the number n of terms is: (A) 4 (B) 5 (C) 6 (D) 10.

Answer the following questions.

2. An arithmetic sequence has $a_1 = 11$ and $d = -3$. Find the sum of its first six terms.

3. An arithmetic sequence has $a_1 = -68$ and the sum of its first 14 terms is −224.

Find its general term.

4. An arithmetic sequence has $a_1 = 69$ and $d = -5$. Find the number of terms, starting with the first, needed to obtain the greatest possible sum; then calculate this sum.

5. An arithmetic sequence has $a_1 = 2$ and $d = 3$. How many terms, starting with the first, must be added to obtain a sum of 155?

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: choose the correct answer.

1. If the general term of a sequence is $a_n = 2n + 3$, what is its fifth term?

(A) 10 (B) 11 (C) 13 (D) 15.

Second: answer the following questions.

1. Find the sum of the natural numbers from 103 through 200.
2. Find the sum of the multiples of 3 between 1 and 100.
3. Find the general term of an arithmetic sequence if the sum of its first 10 terms is 100 and the sum of its first 20 terms is 400.
4. Deduce the general term a_n of the sequence 24, 20, 16, 12, ...

Group B

First: choose the correct answer.

1. For the sequence with general term $a_n = n^2 + n$, its fourth term is:

(A) 12 (B) 16 (C) 20 (D) 24.

Second: answer the following questions.

1. Find the sum of the natural numbers from 101 through 300.
2. Find the sum of the multiples of 5 between 1 and 200.
3. Find the general term of an arithmetic sequence if the sum of its first 15 terms is 225 and the sum of its first 25 terms is 625.
4. Deduce the general term a_n of the sequence 28, 24, 20, 16, 12, ...

Group C

First: choose the correct answer.

1. If $a_n = 3^n$, its first three terms are:

(A) {1, 3, 9} (B) {3, 9, 27} (C) {3, 9, 12} (D) {1, 3, 12}.

Second: answer the following questions.

1. Find the sum of the natural numbers from 51 through 160.
2. Find the sum of the multiples of 7 between 1 and 200.
3. Find the general term of an arithmetic sequence if the sum of its first 12 terms is 144 and the sum of its first 18 terms is 324.
4. Deduce the general term an of the sequence 20, 16, 12, 8, ...

week: .....lesson: 7 Geometric Sequences 1 Date : .....\.....\.....

Class performance

Activity: Folding paper

Take a blank A_4 sheet. Fold it in half, open it, and count the parts. Fold it in half and then in half once more; count the parts. Repeat, recording the number of parts after each fold.

Number of folds	none	first	second	third	fourth.
Number of parts:	1	2	...	...	.....

Complete the table, then answer:

(1) Describe the pattern in the number of parts.

(2) Derive a general rule for the pattern.

(3) How many parts would there be after the sixth fold?

Homework

Find the general term of the geometric sequence in each case:

1. First term 4; common ratio 3.

2. First term 2; common ratio −2.

3. First term 4; common ratio $\frac{1}{2}$.

4. First term 10; common ratio 3.

5. −3, 12, −48, 192, ...

week: .....lesson: 8 Geometric Sequences 2 Date : .....\.....\.....

Class performance

Activity: Bacterial cell division

One bacterial cell becomes three cells each minute. Work with a classmate and use suitable objects to model the division. Complete the table and answer the questions.

Time:	original cell	minute 1	minute 2	minute 3	minute 4
Number of cells:	1	3	....	....	....

(1) Describe the pattern in the number of cells.

(2) Derive a general rule for the pattern.

(3) How many cells would there be at minute 8?

Homework

1. Find the general term of the geometric sequence in each case:

   (a) Third term 9; fifth term 81.

   (b) Second term 20; fourth term 320.

   (c) Fourth term 192; eighth term 12.

2. If 1, x, 81 are consecutive terms of a geometric sequence, find x.

3. If 3, x, 75 are consecutive terms of a geometric sequence, find x.

week: .....lesson: 9 Sum of a Geometric Sequence 1 Date : ...../...../.....

Class performance

Activity: Good news

Imagine hearing unexpectedly good news at a school with 510 students. You tell two people in the first minute. In the next minute, each of them tells two other people, and so on. Complete the table.

Time:	when you learn the news	minute 1	minute 2	minute 3	minute 4
Number of people:	1	2	....	.....	.....

(1) Describe the pattern.

(2) Derive a general rule.

(3) Calculate the total number of people reached by the end of minute 6.

(4) If the news spreads at this rate, how many minutes are needed for it to reach all the students?

Homework

Find the sum of the geometric sequence in each case:

1. $a_1 = 3, r = 2, n = 8.$

2. $a_1 = 4, r = -5, n = 5.$

3. $a_1 = 2, r = -3, n = 10.$

4. $a_1 = 8, r = \frac{1}{2}, n = 6.$

5. $1, -6, 36, -216, \ldots .$

week: .....lesson: 10 Sum of a Geometric Sequence 2 Date : ...../...../.....

Class performance

Activity: Planting trees in the neighborhood

A school joins a neighborhood tree planting project. The number of trees planted grows by a fixed ratio each month. A damaged report gives only these totals:

- Months 1, 2 and 3: 35 trees in total.
- Months 3, 4 and 5: 140 trees in total.

With a classmate, find the number planted in month 1 and the monthly multiplication factor.

Homework

1. The sum of the first and second terms of a geometric sequence is 45, and the sum of its first and third terms is 205. Find its common ratio.
2. A geometric sequence has common ratio 3 and the sum of its first three terms is 117. Find its first term.
3. The second term of a geometric sequence is 2 and its common ratio is −4. Find the sum of its first five terms.
4. The sum of the second and fourth terms of a geometric sequence is 156; the sum of its third and fifth terms is 234. Find its common ratio and first term.
5. A geometric sequence has positive terms, second term 6 and a sum of 26 for its first three terms. Find its first term and common ratio, then write the sequence.

week: .....lesson: 11 Summation Symbol ∑ (1) Date : ...../...../.....

Class performance

Activity: Data analyst

You are a data analyst for a marketing firm reporting the total new followers during the first week of a social media campaign. The firm gained 100 new followers on day 1, and the number of new followers doubled each day for a week.

(a) Write the total number of new followers over the week using summation notation.

(b) Calculate the total after one week.

Homework

1. Express each situation using summation notation, then calculate its total:
   (a) Loujain saved EGP 5 on day 1 and tripled her daily saving for six days.
   (b) Amir ran 500 meters on day 1 and doubled the daily distance for four days.
   (c) A furniture factory made 10 pieces in week 1 and quadrupled its weekly production for six weeks.
2. Write each sum as an expanded series:

   (a) $\sum_{k=1}^{3}(3k - 3)$.

   (b) $\sum_{k=1}^{5}(4k - 23)$.

week: .....lesson: 12 Summation Symbol ∑ (2) Date : ...../...../.....

Class performance

Activity: Physical fitness

Mohamed starts a six day exercise plan with 10 pushups on day 1 and increases his daily count by four. Work with a classmate:

(a) How many pushups does he do on day 6?

(b) Express the total over the training period in summation notation.

(c) Calculate the total.

Homework

Find each sum:

1. $\sum_{k=1}^{10}(4k + 5)$.
2. $\sum_{k=1}^{8}(2k - 1)$.
3. $\sum_{k=1}^{n}(4k - 3)$.
4. $\sum_{k=1}^{5}(k - 1)(2k - 2)$.
5. $\sum_{k=1}^{20}(k - 2)(3k - 1)$.

week: .....**lesson: 13 Summation Symbol $\sum$ (3) Date :** .....\.....\.....

Class performance

Activity: Jump rope

During physical education, Amir begins a six minute jump rope challenge with five jumps in the first minute and doubles the number in every subsequent minute. Work with a classmate:

1. Identify the pattern in the number of jumps each minute.

2. Use summation notation to express the total number of jumps over the training period.

3. To win, Amir needs 72 jumps in four minutes. Does he succeed?

Homework

Find each sum:

1. $\sum_{k=1}^{n}(-2)^k$
2. $\sum_{k=1}^{10}(-3)^{k-1}$
3. $\sum_{k=1}^{5}(-2)^{k-1}$
4. $\sum_{k=1}^{n} 8 \times (-3)^{k-1}$
5. $\sum_{k=1}^{4} 4 \times 2^{k-1}$

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: choose the correct answer.

1. The sum of the first two terms of a geometric sequence with $a_1 = 3$ and $r = 2$ is:

(A) 5 (B) 6 (C) 9 (D) 12.

Second: answer the following questions.

1. Find the sum of the first eight terms of a geometric sequence with $a_1 = 3$ and $r = 2$.

2. Find the common ratio if the sum of the first and third terms is 21 and the sum of the third and fifth terms is 84.

3. Express the series $2 + 5 + 8 + \ldots + (3n - 1)$ in summation notation.

4. If 5, c, 20 are consecutive terms of a geometric sequence, find c.

Group B

First: choose the correct answer.

1. The sum of the first two terms of a geometric sequence with $a_1 = 4$ and $r = 2$ is:

(A) −6 (B) 8 (C) 10 (D) 12.

Second: answer the following questions.

1. Find the sum of the first six terms of a geometric sequence with $a_1 = 4$ and $r = 3$.

2. Find the common ratio if the sum of the first and second terms is 12 and the sum of the third and fourth terms is 48.

3. Express the series $4 + 9 + 14 + \ldots + (5n - 1)$ in summation notation.

4. If 8, c, 72 are consecutive terms of a geometric sequence, find c.

Group C

First: choose the correct answer.

1. The sum of the first two terms of a geometric sequence with $a_1 = 5$ and $r = 3$ is:

(A) 8 (B) 15 (C) 20 (D) 25.

Second: answer the following questions.

1. Find the sum of the first seven terms of a geometric sequence with $a_1 = 5$ and $r = 2$.

2. Find the common ratio if the sum of the second and fourth terms is 30 and the sum of the fourth and sixth terms is 120.

3. Express the series $1 + 5 + 9 + \ldots + (4n - 3)$ in summation notation.

4. If −3, c, −75 are consecutive terms of a geometric sequence, find c.

First Month Exercises

First: choose the correct answer.

1. The sum of the coefficients in the expansion of $(1 + x)^5$ is:

   (A) 16 (B) 32 (C) 64 (D) 5.

2. The sum of the coefficients in the expansion of $(1 + x)^6$ is:

   (A) 32 (B) 64 (C) 128 (D) 6.

3. The remainder when $x^2 - 5x + 6$ is divided by $x - 2$ is:

   (A) 0 (B) 2 (C) −2 (D) 4.

4. Find the least common denominator of $\frac{1}{4X}$ and $\frac{1}{X^2}$.

   (A) $4x$ (B) $4x^2$ (C) x^2 (D) $4x^3$.

5. In the arithmetic sequence 5, 9, 13, 17, ... , the common difference and tenth term, respectively, are:

   (A) $d = 4, a_{10} = 41$ (B) $d = 4, a_{10} = 45$ (C) $d = 3, a_{10} = 32$ (D) $d = 5, a_{10} = 50$.

6. An arithmetic sequence has $a_8 = 43$ and $d = 5$. Its first term is:

   (A) 2 (B) 5 (C) 8 (D) 13.

7. The sum of the seven terms of an arithmetic sequence with first term 11 and last term 53 is:

   (A) 70 (B) 112 (C) 140 (D) 224.

8. The sum of the positive odd integers less than 100 is:

   (A) 1225 (B) 1275 (C) 2500 (D) 1300.

9. A geometric sequence has $a_1 = 12$ and $r = \frac{1}{3}$. Its fourth term is:

   (A) $\frac{1}{3}$ (B) $\frac{4}{9}$ (C) $\frac{4}{27}$ (D) $\frac{4}{81}$.

10. A geometric sequence has $a_1 = 3$ and $r = -3$. The sum of its first four terms is:

    (A) −60 (B) 60 (C) −30 (D) 30.

Answer the following questions.

1. Simplify $\frac{9}{x - 3} - \frac{x^2}{x - 3}$

2. Simplify $\frac{3}{x^2 - 4} + \frac{1}{x - 2}$

3. Divide $x^3 + 3x^2 - x - 3$ by $x + 3$ using long division.

4. If $x > y > z$, prove that $xy + yz - y^2 > xz$.

5. Prove that $x^2 - 4x \geq -y^2 - 6y - 13$.

6. If $x > 0$ and $y > 0$, prove that $(x + y)(\frac{2}{x} + \frac{2}{y}) \geq 8$.

7. Write the first four terms when $a_n = 3n + 2$.
8. Find the general term of an arithmetic sequence with $a_3 = 15$ and $a_8 = 55$.
9. An engineer saves an amount monthly in an arithmetic pattern. In month 3 the saving is EGP 2,000; in month 8 it is EGP 6,000. Find the general term for month n and the first month in which the monthly saving exceeds EGP 11,000.
10. An arithmetic sequence has $a_1 = 1$ and $d = 4$. If its first n terms have sum 276, find n.
11. Find the sum of the two digit numbers that are multiples of 4 numbers (stated range: 12 to 100).
12. A geometric sequence has first term 16 and common ratio 2. Find its sixth term.
13. A geometric sequence has $a_1 = 2$ and $r = -3$. Find the sum of its first ten terms.

week: .....lesson: 14 Sums of Fractional Sequences Date : ...../...../.....

Class performance

Activity: Data compression

Students in the school's computer club are analyzing an algorithm used for data compression and automatic error correction. The algorithm consists of a fractional sequence of 10 terms in the following form:

$$\frac{1}{2}+\frac{1}{2\,x3}+\frac{1}{3\,x4}+\frac{1}{4\,x5}+\ldots+\frac{1}{10\,x11}$$

With a classmate:

(1) Write the decomposed forms of the first and last terms using the appropriate rule.

(2) Identify the intermediate terms that cancel.

(3) Find the total sum of the series.

Homework

1. Three friends share a large pizza according to this fractional pattern:

$$\frac{1}{2}+\frac{1}{2\,x3}+\frac{1}{3\,x4}+\frac{1}{4\,x5}+\ldots+\frac{1}{15\,x16}$$

(a) As the pattern continues, find the term immediately after [the reference term is missing in the source].

(b) Identify the intermediate terms that cancel. (c) Find the sum of the series.

2. Find the following sums:

(a) $\frac{1}{3\,x5}+\frac{1}{4\,x6}+\frac{1}{5\,x7}+\ldots+\frac{1}{(n+2)(n+4)}$

(b) $\sum_{k=1}^{n}\frac{1}{\sqrt{k}+\sqrt{k+2}}$

week: ..... lesson: 15 Sum of (Arithmetic Sequences) ×(Geometric Sequences) Date : ...../...../.....

Class performance

Activity: Video games

In a video game, a player receives daily bonus points that multiply and increase based on their level.

On the first day (Level 1), the player receives 1 point, and on the following days, the number of points increases according to the following pattern:

(11) + (22) + (34) + (48) +....

With a classmate:

(1) Identify the first term and common difference of the arithmetic factor, and the first term and common ratio of the geometric factor.

(2) Calculate the points over the first three days.

(3) Calculate the points over the first five days.

(4) Find the points scored on day 10.

Homework

Find the sum of the terms in each series:

1. $(12) + (34) + (58) + (716) + \ldots + [(2n - 1)2^n]$.

2. $(11) + (23) + (39) + (427) + \ldots + [n3^{n-1}]$.

3. $(21) + (44) + (616) + (864) + \ldots + [2n4^{n-1}]$.

4. $(15) + (210) + (320) + (440) + \ldots + [n\,(52^{n-1})]$.

week: .....lesson: 16 Sequences Where General Term Is a Sum Date : ...../...../.....

Class performance

Activity: Educational platforms

On an educational platform, a student earns bonus points upon completing each grade level n, where the number of points earned at each level can be expressed by the sum: 1 + 2 + 3 + ... + n. If the number of bonus points can be expressed by the following table:

levels	the bonuses
First	1
Second	3
third	6

With a classmate:

(1) Find the bonuses at levels 4 and 5.

(2) Write a formula for the general term in n.

(3) Find the bonus at level 10.

(4) At which level is the bonus 36 points?

Homework

1. A sequence has general term $a_n = 2 + 4 + 6 + ... + 2n$. Find its fifth term.
2. A sequence has general term $a_n = 5 + 10 + 15 + ... + 5n$. Find the sum of its first 2n terms.
3. The terms of a sequence are $1; 1 + 2; 1 + 2 + 3; 1 + 2 + 3 + 4; ...$.

Find:

(a) the fifth term,
(b) the kth term and
(c) the sum of its first n terms.

week: .....**lesson: 17 Sum of Sequence and General Term** **Date :** ...../...../.....

Class performance

Activity: Phone credit

If we consider S_n to be the total balance topped up by a student on their phone line from the beginning of the year up to month number n, where S_1 is the top-up amount for month 1, S_2 is the top-up amount for month 1 + month 2, and so on.

Let us assume that the telecommunications company offers a promotion, and the student's cumulative total top-up amount is calculated by the relation: $S_n = 10 + 5n$

With your group:

(1) Calculate the cumulative amount after two months.

(2) Using $a_n = S_n - S_{n-1}$, find the amount added in second month alone.

(3) Find the general term a_n.

Homework

1. If the sum of the first n terms is $S_n = n^2$, find the third term.
2. If the sum of the first n terms is $S_n = 7^n - 2$, find the fourth term.
3. If a sequence has $S_n = 2(3^n - 1)$, find its first and third terms.
4. If a sequence has $S_n = 7^{n+5}$, derive its general term starting with the second term.
5. If a sequence has $S_n = 3 \times 2^n - 3$, write its first three terms.

week: .....lesson: 18 Difference Sequences Date : ...../...../.....

Class performance **Activity: Number of likes**

Agroup of students tracked the reach of a short video clip they posted on social media platforms. The number of likes during the first 4 hours was as follows:

Hour	liked
first	16
second	10
third	8
fourth	4

The student noticed that the number of reactions per hour doesn't change at a constant rate, but the differences between hours of reaction form a regular sequence.

Work with your classmates and answer the following.

With your classmates:

(1) Write the sequence of differences between adjacent values.

(2) Find its general term in k.

(3) Predict the total number of interactions in hour 5.

Homework

Find the general term of each sequence:

1) 1, 5, 11, 19, 29, ...

2) 2, 5, 10, 17, 26, ...

3) 5, 7, 11, 19, 35, ...

4) 4, 1, −2, −5, ...

5) 2, 3, 1, 5, −3, ...

week: .....lesson: 19 Group Sequences Date : ...../...../.....

Class performance **Activity: Personal savings**

A student decided to save an amount from his daily allowance to buy his favorite electronic device at the end of the month. He started by putting 10 pounds on the first day, then increased the daily amount by 5 pounds each day compared to the previous day, forming an arithmetic sequence of daily contributions. The student wanted to calculate how much he had saved by the end of a certain period without needing to sum the amounts manually each day.

With your group:

(1) Write the general term of the daily saving on day k.

(2) Express the total over ten days using summation notation.

(3) Find the total through day 10.

(4) Find the total after n = 15 days.

Homework

Consider the pattern in groups: {1}, {3, 5}, {7, 9, 11}, {13, 15, 17, 19}, ...

1. How many terms are in the fifth group?
2. Find the first term of the fifth group.
3. Find the last term of the fifth group.
4. Find the sum of the terms of the fifth group.
5. Identify the group containing 31.

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: choose the correct answer.

1. The sum of the first 20 terms of 1, 3, 5, 7, ... is:

(A) 200 (B) 250 (C) 300 (D) 400.

Second: answer the following questions.

1. Find the sum $\frac{1}{1 \times 4} + \frac{1}{3 \times 6} + \frac{1}{5 \times 8} + \ldots + \frac{1}{(2n-1)(2n+2)}$

2. A sequence its first trem 10 and its difference sequence is −1, −2, −3, ... Find its first four terms.

3. In the pattern {2}, {4, 6}, {8, 10, 12}, {14, 16, 18, 20}, ... , find the number of terms in the fifth group.

4. If $S_n = 3 \times 4^{n-3}$, find the third term of the sequence.

Group B

First: choose the correct answer.

1. For the geometric sequence 8, 4, 2, 1, ... , the general term is:

(A) $a_n = 16^{-n}$ (B) $a_n = 16^n$ (C) $a_n = 2^{4-n}$ (D) $a_n = 2^{4+n}$

Second: answer the following questions.

1. Find the sum $\frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \frac{1}{7 \times 9} + \ldots + \frac{1}{(2n+1)(2n+3)}$

2. A sequence begins with 5 and its difference sequence is 2, 3, 4, ... Find its first four terms.

3. In the pattern {2}, {4, 6}, {8, 10, 12}, {14, 16, 18, 20}, ... , find the last term of the tenth group.

4. If $S_n = \ + 5$, find the fourth term of the sequence.

Group C

First: choose the correct answer.

1. If $a_1 = 1$ and $a_{n+1} = a_n + n$, the fifth term is:

(A) 11 (B) 13 (C) 15 (D) 17.

Second: answer the following questions.

1. Find the sum $\frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \frac{1}{7 \times 9} + \ldots + \frac{1}{(2n+1)(2n+3)}$

2. A sequence begins with 1 and its difference sequence is 2, 4, 6, ... Find its first four terms.

3. In the pattern {2}, {4, 6}, {8, 10, 12}, {14, 16, 18, 20}, ... , find the last term of the tenth group.

4. If $S_n = 7^n - 1$, find the second term of the sequence.

week: ..... **lesson: 20 Recurrence Relations (1)** **Date :** ...\....\.....

Class performance

Activity: Charging a battery

Amir charges his mobile phone before going to school. The battery is at 20% when charging begins, and its charge increases by 15 percentage points every 10 minutes. With your classmates:

1. Find the first term, which represents the initial battery charge.

2. Write the recurrence relation a_{n+1} , expressing the increase every 10 minutes.

3. Identify whether the resulting sequence is arithmetic or geometric.

4. Find a_2 and a_3, the charge after 10 and 20 minutes.

Homework

1. For $a_1 = 3$ and $a_{n+1} = a_{n+4}$, identify the first term and common difference in the Recurrence relation.

2. Identify the type of sequence generated by $a_{n+1} = 5a_n$.

3. For $a_1 = 1$ and $a_{n+1} = a_n + 2$, find a_2 and a_3.

4. For $a_1 = 2$ and $a_{n+1} = 3a_n$, find a_2 and a_3

5. Find the general term of the sequence with $a_1 = 4$ and $a_{n+1} = a_n - 4$

week: ..... lesson: 21 Recurrence Relations (2) Date : ...\....\.....

Class performance

Activity: Tree growth

An agricultural engineer monitors a tree. Its height at planting, in the first month, was 100 cm. It grows 5 cm each month. With your classmates:

1. Write the heights in the first and second months.
2. Express its height in the fourth month in terms of k.
3. Find its actual height in the sixth month.
4. After how many months will its height reach 2 m?

Homework

Find the general term of each sequence under the stated conditions:

1. $a_{n+1} = a_n + 4n$; $a_1 = 4$.
2. $a_{n+1} = a_n + 2^n$; $a_1 = 1$.
3. $a_{n+1} = a_n + 6n - 1$; $a_1 = 5$.
4. $a_{n+1} = a_n - 2n$; $a_1 = 2$.
5. $a_{n+1} = a_n + 3n^2 + n$; $a_1 = 4$.

week: ..... lesson: 22 Recurrence Relations (3) Date : ...\....\.....

Class performance

Activity: Physical fitness

A student undertakes a month-long fitness challenge. Each day, the student performs half as many repetitions as on the previous day, plus 15 additional repetitions. The student begins with 40 repetitions on the first day. With your classmates:

1. Write the recurrence relation for the number of repetitions on day n + 1.
2. Find the general term of the repetitions.
3. Find the number performed on the third day.

Homework

Find the general term in each case:

1. $a_1 = -2$; $a_{n+1} = -2a_n + 4n$.

2. $a_1 = 1$; $a_{n+1} = a_n + 2$.

3. $a_1 = -3$; $a_{n+1} = 4\,a_n + 2$.

4. $a_1 = 2$; $a_{n+1} = a_n - 3$.

5. $a_1 = 4$; $a_{n+1} = 3\,a_n - 2$.

week: .....lesson: 23 Mathematical Induction for Proving Equalities Date : ...../...../.....

Class performance

Activity: Organizing study hours

A secondary-school student starts with one hour of study on the first day and increases the time by two hours each subsequent day. The total after n days is $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.
With a classmate, prove this identity by mathematical induction.

Homework

Prove each statement by mathematical induction, for positive integers n:

1. $2 + 4 + 6 + \ldots + 2n = n(n + 1)$.

2. $1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}$

3. $1 + 3 + 5 + \ldots + (2n - 1) = n^2$, where n is positive.

4. $5^n - 1$ is divisible by 4.

5. $3 + 3^2 + 3^3 + \ldots + 3^n = \dfrac{3(3^n - 1)}{2}$

week: .....lesson: 24 Mathematical Induction for Proving Inequalities Date : ...../...../.....

Class performance

Activity: Power charging

Mohamed develops a video game offering two ways to increase character power after stage n. Under the first choice power doubles exponentially to 2^n; under the second it triples linearly to 3n. Starting at stage 4, the first becomes greater. Prove by induction that $2^n > 3n$ for $n \geq 4$.

Homework

Prove by mathematical induction:

1. $2^n > n$, for natural numbers n.

2. $2^n > 4n$, for natural numbers $n \geq 5$.

3. $1 + 2 + 3 + .. + n < n^2$, for natural numbers $n \geq 2$.

4. $(\frac{3}{2})^n < n$, for natural numbers $n \geq 3$.

5. $1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} > \frac{n}{n+1}$, for $n \geq 2$.

week: .....lesson: 25 Proof of Being a Multiple by Mathematical Induction Date : ...../...../.....

Class performance

Activity: Internet bundles

A telecommunications company calculates bonus points collected in month n using

$A(n) = n^3 + (n + 1)^3 + (n + 2)^3$.

The company wants to distribute these points equally among nine main branches. With a classmate, use mathematical induction to prove that the total is divisible by 9.

Homework

Use mathematical induction to prove, for natural numbers n:

1. $4^n + 15n - 1$ is divisible by 9.

2. $n^3 + 2n$ is divisible by 3.

3. $n^3 - n$ is divisible by 6.

4. $7^n - 2^n$ is divisible by 5.

5. $5^{2n} - 1$ is divisible by 24.

week: .....lesson: 26 Recurrence Relations and Mathematical Induction Date : ...../...../.....

Class performance

Activity: The physiotherapist

A physiotherapist designs a rehabilitation program that gradually increases the number of strengthening exercises each week. A patient begins with four exercises in week 1. In each following week, three exercises are added: $a_{n+1} = a_n + 3$. With your classmates:

1. Prove by induction that $a_n = 3n + 1$.

2. Use the recurrence relation to calculate the number of exercises in each of the first four weeks.

Homework

1. If $a_1 = 2$ and $a_{n+1} = a_n + 5$, prove by induction that $a_n = 5n - 3$ for $n \geq 1$.

2. If $a_1 = 3$ and $a_{n+1} = 2\,a_n$, prove by induction that $a_n = 3 \cdot 2^{n-1}$ for $n \geq 1$.

3. If $a_1 = 1$ and $a_{n+1} = a_n + (2n + 1)$, prove by induction that $a_n = n^2$.

4. A rehabilitation program starts with 50 minutes in week 1 and decreases by four minutes each week. The recurrence relation is $a_{n+1} = a_n - 4$, for $n \geq 1$.

(a) Prove by induction that $a_n = 54 - 4n$.

(b) In which week will the treatment program end completely?

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: Choose the correct answer.

1. A sequence has first term 3 and recurrence relation $a_{n+1} = a_n + 5$. Which formula represents its general term?

(A) $a_n = 5n - 2$ (B) $a_n = 3n + 5$ (C) $a_n = 5n + 2$ (D) $a_n = 3n + 2$.

Second: Answer the following.

1) Prove by induction:

(a) $1 + 3 + 5 + \ldots + (2n - 1) = n^2$ for $n \geq 1$.

(b) $5^n - 1$ is divisible by 4 for $n \geq 1$.

2) By using the induction, prove that:

(a $2^n > n + 1$ for $n \geq 2$.

(b) $5^n > 2^n$ for $n \geq 1$.

Group B

First: Choose the correct answer.

1. A sequence has first term 4 and recurrence relation $a_{n+1} = 2a_n$. Which formula represents its general term?

(A) $a_n = 4 \times 2^n$ (B) $a_n = 2 \times 4^{n-1}$ (C) $a_n = 4 \times 2^{n-1}$ (D) $a_n = 2^{n-1}$.

Second: Answer the following.

1) Prove by induction

(a) $1 + 2 + 3 + \ldots + n = \quad n = \dfrac{n(n+1)}{2}$ "where n is a natural number "

(b) $7^n - 2^n$ is divisible by 5 for $n \geq 1$.

2) By using the induction, prove that:

(a) $2^n > n^2$ for $n \geq 5$.

(b) $1 + 2 + 3 + .. + n < (n + 1)^2$ for $n \geq 1$.

Group C

First: Choose the correct answer.

1. A sequence has first term 20 and recurrence relation $a_{n+1} = a_n - 3$. Which formula represents its general term?

(A) $a_n = 23 - 3n$ (B) $a_n = 3n - 23$ (C) $a_n = 3n - 20$ (D) $a_n = 20 - 3n$.

Second: Answer the following.

1) Prove by induction

(a) $2 + 2^2 + 2^3 + ... + 2^n = 2^{n-1} - 2$ for $n \geq 1$.

(b) $n^2 + n$ is even for $n \geq 1$.

22) By using the induction, prove that:

(a) $2^n > n + 5$ for $n \geq 4$.

(b) $3^n > 2^n$ for $n \geq 1$.

Chapter 3 Complex Numbers

week: ..... **lesson: 1 Complex Numbers** **Date :** ...../...../.....

Class performance

Activity: No network coverage

When mobile coverage weakens or disappears, your phone repeatedly searches for the nearest tower. A digital search signal is represented by a complex number a + bi, where the real part a describes signal strength and the imaginary part b describes signal direction. Work with your classmates:

1. For the signal 6 − 8i, identify its real and imaginary parts.

2. Two successive signals are 1 + 4i and 3 + 2i. Calculate their combined signal.

Homework

Find the real numbers x and y satisfying the following:

1) x + yi = 2 − 3i.
2) 5x + 4i = 20 − yi.
3) x − yi = −2.
4) 1 − i = 2x − yi.
5) 3 + i = (2 − i) + (x + yi).

week: ..... **lesson: 2 Operations with Complex Numbers (1)** **Date :** ...../...../.....

Class performance

Activity: A favorite drink

The temperature of a drink interacts with the surrounding cold air. In a complex-number model, the real part represents its actual temperature and the imaginary part estimates change in temperature over time. Work with your classmates:

1. Find the total temperature represented by $70 + 5i^2$.

2. Milk represented by $-20 + 2i^2$ is added to a drink represented by $80 - 3i^2$. What is their combined total?

Homework

Simplify each expression:

1) (1 + i) − (−2 + i).

2) i(i − 5).

3) (3 − 2i)(1 − 2i).

4) (1 − i)(i − 10).

5) $i + 2i^2 + 3i^3 + 4i^4$.

week: ..... **lesson: 3 Operations on Complex Numbers (2)** Date:...../...../.....

Class performance

Activity: Why use the conjugate?

A student attempts to simplify $\frac{3+2i}{3-2i}$, writing that the result is $\frac{3+2i}{3-2i} = 1$

Discuss whether this is correct and why; identify the factor by which to multiply numerator and denominator to make the denominator real; find the correct result; and decide whether it is real or complex.

Homework

1. Find the complex conjugate of:

(a) $3 + 4i$ (b) $7 - 2i$ (c) 5 (d) $-6i$.

2. Compute $(4 - i)(2 + 3i)$ and express the answer as $a + bi$.

3. Simplify $\frac{5+2i}{1-i}$ and write the result in standard form.

4. If $z = a + 3i$ and the sum of z and its conjugate is 10, find a and then their product.

week: ..... **lesson: 4 Square Roots of Negative Numbers** Date : ...../.../.....

Class performance

Activity: Find and correct the error

One student states $\sqrt{-36} = -6$; another states $\sqrt{-36} = 6i$.

Discuss: Which answer is correct?

Why is $\sqrt{-36}$ not -6?

Find both square roots of -36 and verify each by squaring.

Can a negative number have a real square root? Explain.

Homework

1. Express $\sqrt{-9}$, $\sqrt{-25}$ =, and $\sqrt{-18}$ = in terms of i.

2. Simplify in the simplest form $\sqrt{-48}$.

3. Find $\sqrt{-24} \times \sqrt{-6}$

4. Simplify in the simplest form $\sqrt{-32} + \sqrt{-18}$

5. If $x^2 = -20$, find both complex values of x in a + bi form.

week: ..... lesson: 5 Solution of Quadratic Equations Date : ...../...../.....

Class performance

Activity: Choose a solution method

Consider $x^2 - 9 = 0$, $(x - 3)^2 + 4 = 0$, and $3x^2 - 5x + 3 = 0$.

Discuss

- which can be solved directly by factoring?
- which calls for square roots?
- which can be solved by the quadratic formula?
- which has complex solutions?

Then solve all three equations?

Homework

Solve these quadratic equations:

1. $(x - 3)^2 + 4 = 0$.
2. $2x^2 + 5x - 3 = 0$.
3. $3x^2 + 2\sqrt{3} = x + 5 = 0$.
4. $4x^2 - 6x + 13 = 0$.
5. $3(5x - 2)^2 = 7$.

week: ..... lesson: 6 Discriminant of Quadratic Equations Date : ...../...../.....

Class performance

Activity: Change the equation

Given: $x^2 - 4x + 5 = 0$

(1) Solve in the complex numbers and state whether there are real solutions.
(2) Change the constant term to produce two distinct real roots.
(3) Change it to produce a repeated real root.
Write all three equations.

Homework

1. Find the value or values of a for which $x^2 - (a + 3)x + 9 = 0$ has a repeated root.
2. Find the values of m for which $x^2 + (m + 4)x + m - 3 = 0$ has two distinct non - real roots.
3. Find the values of m for which $x^2 - 4mx + 3m^2 - 4 = 0$ has real roots.
4. For real a, determine the nature of the roots of $x^2 + ax - 2 = 0$.
5. For real m, consider $x^2 - (m + 2)x + (m - 2) = 0$. Find the values of m that produce

(a) two distinct real roots;
(b) a repeated real root;
(c) two non-real complex roots.

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: Choose the correct answer.

1. If $x^2 - 2(k + 1)x + (k^2 + 5) = 0$ has real roots, what is the greatest listed value of k that gives a repeated root?

(A) 2 (B) 1 (C) −2 (D) 4.

Second: Answer the following questions.

2. If $x \in R$, $y \in R$ find x and y satisfying $(3 + 2i)x + (1 - i)y = 7 + 3i$.

3. Simplify $(2 - i)^2 + i^{27} + \frac{\sqrt{-48}}{\sqrt{-3}}$ into a + bi form.

4. If $z = \frac{2 + i}{1 - i}$, express z in standard form.

5. Find all real k for which $x^2 - (k + 2)x + k^2 = 0$ has two distinct real roots.

Group B

First: Choose the correct answer.

1. If $x^2 - 2(k - 1)x + (k^2 + 3) = 0$ has real roots, which listed value of k gives equal roots?

(A) −1 (B) 1 (C) 2 (D) No real value of k.

Second: Answer the following questions.

2. Find real x and y satisfying $(1 + 2i)(x + i) = x(y + i)$.

3. Simplify $(3 - i)^2 + i^{35} + \frac{\sqrt{-75}}{\sqrt{-3}}$ into a + bi form.

4. If $z = \frac{4 + 8i}{1 - i}$, express z in standard form.

5. Find all real m for which $x^2 - 4mx + 3m^2 - 4 = 0$ has distinct real roots.

Group C

First: Choose the correct answer.

1. If $x^2 - 4mx + 4m^2 + k = 0$ has nonreal complex roots, which condition on k is necessary?

(A) $k > 0$ (B) $k < 0$ (C) $k = 0$ (D) k is any real number.

Second: Answer the following questions.

2. Find real x and y satisfying $(4 - i)x + (2 + 3i)y = 10 - i$.

3. Simplify $(1 - 2i)^2 + i^{19} + \frac{\sqrt{-32}}{\sqrt{-2}}$ into a + bi form.

4. If $z = \frac{5 + i}{3 - i}$, express z in standard form.

5. Prove that $x^2 + ax - 2 = 0$ has two distinct real roots for every nonzero real a.

week: ..... **lesson: 7 Relationships Between Roots and Coefficients (1)** **Date :** ...../...../.....

Class performance

Activity: Construct a new expression

If α + β = 5 and αβ = 6, without finding α or β calculate:

(1) $\alpha^2 + \beta^2$; (2) $(\alpha - \beta)^2$; (3) $\alpha^3 + \beta^3$.

Write the algebraic identity used in each calculation.

Homework

Answer the following:

1. If α and β are roots of $x^2 - 7x + 10 = 0$, find α + β.
2. If α and β are roots of $2x^2 - 6x + 5 = 0$, find αβ.
3. If α + β = 6 and αβ = 5, find $\alpha^2 + \beta^2$.
4. If α and β are roots of $2x^2 - 8x + 3 = 0$, find $\alpha^2 + \beta^2$.
5. If α and β are roots of $x^2 - 6x - 2 = 0$, find $\alpha^3 + \beta^3$ without finding either root.

week: ..... **lesson: 8 Relationships Between Roots and Coefficients (2)** **Date :** ...../...../.....

Class performance

Activity: A mathematical model for population growth

A researcher uses a simplified model for a microorganism population. The associated equation is $x^2 - 6x + 5 = 0$. Its roots α and β represent two possible growth rates. Without solving,

find α + β, αβ, and $\alpha^2 + \beta^2$.

Explain how the relationships between roots and coefficients make the calculation more efficient, and whether the actual roots are always needed.

Activity 2: Analyze before calculating. A team models the difference between a drug concentration and its reference level as $D(t) = t^2 - 8t + 15$, where t is measured in hours. At D(t) = 0 the concentration reaches the reference level. Let α and β be the roots of $t^2 - 8t + 15 = 0$. Without initially solving, find α + β and αβ. What can you infer about the two times? Can you find their actual values? Explain.

Homework

1. One root of $x^2 + 7x + m = 0$ is −2. Find m and the other root.
2. If α and β are roots of $2x^2 - 9x + 4 = 0$, find α + β and αβ.
3. If α and β are roots of $3x^2 + 5x - 2 = 0$, find $\alpha^2 + \beta^2$.
4. If α and β are roots of $x^2 - 6x + 5 = 0$, find $\alpha^3 + \beta^3$.
5. One root of $x^2 - (k + 2)x + 6 = 0$ is 2. Find k and the other root.

week: ..... lesson: 9 Factorisation of Quadratic Expressions Date : ...../...../.....

Class performance **Activity: Decode the electrocardiogram signal**

Cardiologists analyze signals and oscillations in models of the heart's electrical changes. The team is asked to factor the "heart-signal stability equation" $2x^2 - 4x + 10 = 0$. Find its complex roots α and β using the quadratic formula, then express $2x^2 - 4x + 10$ as a product of complex factors.

Homework

Factor each expression over the complex numbers:

1. $x^2 - 7x + 12$.
2. $x^2 + 5x + 6$.
3. $x^2 + 9$.
4. Factor $x^2 - 4x + 13$, then use the factorization to solve $x^2 - 4x + 13 = 0$.
5. One factor of $x^2 - 4x + c$ is $[x - (2 - 3i)]$. Find c and express the quadratic as a product of complex factors.

week: ..... lesson: 10 a Quadratic Equations with Two Given Solutions Date : ...../...../.....

Class performance **Activity: Symmetric growth**

In a mathematical model of a scientific phenomenon, one solution is 2 + 3i.

Write the two factors corresponding to its conjugate roots,

Form the quadratic equation, put it in standard form, and check that its coefficients are real.
Why do nonreal complex roots occur in conjugate pairs for polynomials with real coefficients?

Homework

1. Form a quadratic equation with each pair of roots: (a) 2 + 3i and 2 − 3i; (b) 1 + 5i and 1 − 5i.
2. Find two numbers given: (a) sum 6, product 13; (b) sum −4, product 9.
3. α and β are roots of $x^2 - 4x + 5 = 0$. Find a quadratic equation with roots α + 1 and β + 1.
4. α and β are roots of $x^2 - 2x + 6 = 0$. Find a quadratic equation with roots α^2 and β^2.
5. α and β are roots of $3x^2 - 5x + 2 = 0$. Find a quadratic equation with integer coefficients and roots and .

week: .....**lesson: 11 Signs of Two Solutions** **Date :** .....\.....\.....

Class performance **Activity: Monitoring pulse and oxygen levels in intensive care**

Intensive-care monitors set pulse and breathing alarms using quadratic inequalities involving a real control variable m. For the pulse readings to have two distinct positive boundary values, consider $x^2 - 2mx + m + 12 = 0$. Write the three simultaneous conditions $\Delta > 0$, $\alpha + \beta > 0$, and $\alpha\beta > 0$; solve each inequality independently and find their common interval for m.

Homework

1. Find all real numbers m for which $x^2 - 2(m - 2)x + m + 4 = 0$ has two distinct positive roots.
2. Find all real numbers k for which $x^2 + 2kx - k^2 - 6 = 0$ has roots of opposite signs.
3. Find all real numbers k for which $x^2 + (k + 2)x + 3k + 1 = 0$ has two distinct negative roots.
4. For $x^2 + (m - 3)x + m = 0$, find all real numbers m giving distinct roots that are

(a) both positive; (b) both negative; (c) of opposite signs.

5. Find all real numbers m for which $mx^2 - 4x + 2 = 0$ has two distinct positive real roots.

Weekly Assessment week: ..... Date : .....\.....\.....

Group A

First: Choose the correct answer.

1. If α and β are the roots of $x^2 - 7x + 10 = 0$, then $\alpha\beta$ equals:

(A) −10 (B) 7 (C) 10 (D) −7.

Second: Answer the following questions.

2. Given $\alpha + \beta = 8$ and $\alpha\beta = 12$, find $\alpha^2 + \beta^2$.
3. Factor $x^2 - 4x + 13$ over the complex numbers.
4. Form a quadratic equation with roots $2 + 3i$ and $2 - 3i$.
5. Find the roots of: $x^2 - 6x + 10 = 0$.

Group B

First: Choose the correct answer.

1. If α and β are the roots of $x^2 - 9x + 14 = 0$, then $\alpha\beta$ equals:

(A) 9 (B) 14 (C) −14 (D) −9.

Second: Answer the following questions.

2. Given $\alpha + \beta = 7$ and $\alpha\beta = 10$, find $\alpha^2 + \beta^2$.
3. Factor $x^2 - 6x + 13$ over the complex numbers.
4. Form a quadratic equation with roots $3 + 2i$ and $3 - 2i$.
5. Find the roots of: $x^2 - 2x + 17 = 0$.

Group C

First: Choose the correct answer.

1. If α and β are the roots of $x^2 - 8x + 15 = 0$, then αβ equals:

(A) 8 (B) 15 (C) −15 (D) −8.

Second: Answer the following questions.

2. Given $\alpha + \beta = 9$ and $\alpha\beta = 14$, find $\alpha^2 + \beta^2$.

3. Factor $x^2 - 8x + 17$ over the complex numbers.

4. Form a quadratic equation with roots $1 + 4i$ and $1 - 4i$.

5. Find the roots of: $x^2 - 8x + 17 = 0$.

week: .....**lesson: 12 The Remainder Theorem (1)** **Date :** ...../...../.....

Class performance

Activity: Prediction

Researchers model the variation of a scientific quantity over time with $F(x) = x^3 - 5x^2 + 6x + 2$.

They want its value at $x = 2$. Instead of lengthy operations, use the remainder theorem.

(1) Find F(2).

(2) Relate F(2) to the remainder on division by $x - 2$.

(3) What could this mean if x represents time?

Discuss why evaluating the function may be quicker than long division.

Homework

1. Find the remainder when $F(x) = x^2 - 2x - 3$ is divided by $x - 1$.

2. Find the remainder when $F(x) = 3x^3 + 2x - 5$ is divided by $x - 2$.

3. Find the remainder when $F(x) = 6x^3 - x^2 + 3x + 4$ is divided by $2x - 1$.

4. If the remainder when $F(x) = x^3 + ax + 2$ is divided by $x - 2$ is 8, find a.

5. If $x^3 + ax^2 + bx + 6$ is divisible by $x - 2$ and its remainder on division by $x + 1$ is 9, find a and b.

week: .....**lesson: 13 The Remainder Theorem (2)** **Date :** ...../...../.....

Class performance

Activity: Residual drug concentration

A hospital pharmacy predicts the residual concentration F(x) of a compound. Division by $x - 1$ leaves 5 units, and division by $x + 4$ leaves −10 units. Find the remainder in the form $ax + b$ when F(x) is divided by

$x^2 + 3x - 4$.

Homework

1. If $F(1) = 5$ and $F(-2) = -1$, find the remainder when $F(x)$ is divided by $(x - 1)(x + 2)$.

2. If $F(2) = 4$ and $F(-3) = 7$, find the remainder on division by $(x - 2)(x + 3)$.

3. The remainders on division by $x - 2$ and $x + 1$ are 3 and -6, respectively. Find the remainder on division by $(x + 1)(x - 2)$.

4. The remainders on division by $x - 1$ and $x + 3$ are 4 and -2, respectively. Find the remainder on division by $x^2 + 2x - 3$.

5. If the polynomial $f(x)$ satisfies: $F(2) = 3$ and $F(-1) = -6$, find the remainder on division by $x^2 - x - 2$.

week: .....**lesson: 14 The Remainder Theorem (3)** **Date :** ...../...../.....

Class performance

Activity: Analyzing a biological model

In a laboratory experiment, researchers model a biological indicator by $F(x) = x^3 - 5x^2 + 2x + 8$. They want only the remainder on division by $x^2 - 4x + 3$, without performing long division.

Homework

1. $F(x)$ has remainder 3 on division by $x^2 - 3x - 4$, and remainder $x + 5$ on division by $x^2 - 9$. Find its remainder on division by $x^2 - x - 6$.

2. $F(x)$ has remainder $2x + 1$ on division by $x^2 - 4x + 3$, and remainder $3x - 2$ on division by $x^2 - 5x + 6$. Find its remainder on division by $x^2 - 7x + 12$.

3. $F(x)$ has remainder $x - 4$ on division by $(x + 2)^2$, and remainder 5 on division by $x - 3$. Find its remainder on division by $(x + 2)(x - 3)$.

4. $F(x)$ has remainder 4 on division by $x^2 - 5x + 6$, and remainder $2x - 1$ on division by $x^2 - 7x + 12$. Find its remainder on division by $x^2 - 6x + 8$.

5. $F(x)$ has remainder $2x + 3$ on division by $(x - 1)^2$, and remainder 7 on division by $x + 2$. Find its remainder on division by $(x + 2)(x - 1)$.

week: .....**lesson: 15 Synthetic Division** **Date :** ...../...../.....

Class performance

Activity: Find and correct the error

A researcher wants to analyze $F(x) = x^3 - 6x^2 + 11x - 6$ and divide it by $x - 1$.

(1) Find the quotient by synthetic division.

(2) Find the remainder.

(3) Verify with the remainder theorem.

Discuss why synthetic division can be more efficient than long division.

Homework

Use synthetic division to find quotient and remainder when polynomial A is divided by B:

	A		B
1.	$x^3 - 6x^2 + 11x - 6$	;	$x - 1$.
2.	$2x^3 - 5x^2 - 4x + 3$	;	$x - 3$.
3.	$x^4 - 5x^3 + 2x^2 + 8x - 4$	;	$x - 2$.
4.	$x^3 - 4x^2 - x + 4$	;	$x - 1$.

week: ..... **lesson: 16 Factor Theorem** **Date :** ...../...../.....

Class performance

Activity: Testing a scientific hypothesis

A student has $F(x) = x^3 - 4x^2 + x + 6$ and believes $x - 2$ is a factor.

(1) Test the hypothesis with the factor theorem.

(2) If correct, factor the polynomial and find every solution of $F(x) = 0$.

(3) Verify the solutions.

Why is the factor theorem a quick way to test candidate roots before dividing?

Homework

1. Check whether $x - 2$ is a factor of $x^3 - 5x^2 + 6x$.
2. Check whether $x + 1$ is a factor of $x^3 - 2x^2 - x + 2$.
3. Factor $x^3 - 6x^2 + 11x - 6$.
4. Factor $x^3 - 4x^2 + x + 6$.
5. If $x^3 + ax^2 - 4x + 4$ is divisible by $x - 2$, find a and then factor $F(x)$ completely.

week: .....**lesson: 17 Higher Degree Equations** **Date :** ...../...../.....

Class performance

Activity: A scientific model

The variable x represents a possible value in a scientific model satisfying $x^3 - 2x^2 - 5x + 6 = 0$.
A researcher believes $x = 1$ is a solution.
(1) Check by the factor theorem.
(2) Use synthetic division.
(3) List all solutions.
(4) Verify by substitution.
How did the known root (x - 1) make finding all solutions easier?

Homework

Solve and verify the following equations:

1. $x^4 - 5x^2 + 4 = 0$.

2. $x^4 - 13x^2 + 36 = 0$.

3. $x^3 - 2x^2 - 5x + 6 = 0$.

4. $x^4 - 6x^3 + 8x^2 = 0$.

5. Solve $x^3 - 3x^2 + 4x - 12 = 0$ over the complex numbers.

week: .....lesson: 18 Determining a Cubic Equation Date : ...../...../....

Class performance

Activity: Modeling the movement of a drug in the body

After a drug is injected, its concentration in blood varies with time x in hours. The cubic model is

$x^3 + ax^2 + bx + 16 = 0$

Doctors know that x = 1 and x = 2 are roots. (1) Find the real constants a and b. (2) Find the third root, at which the effect of the drug completely ends.

Homework

1. If $x^3 - ax^2 - 4x + b = 0$ has roots -2 and 3, find real a and b and its remaining root.

2. If $x^3 + ax^2 + bx - 12 = 0$ has roots 1 and 3, find real a and b and its remaining root.

3. If $x^3 - kx^2 + (2k + 1)x - 18 = 0$ has root 3, find real k and the other roots.

4. If $x^3 + ax^2 + bx - 26 = 0$ has root $1 - 5i$, find real a and b and the remaining two roots.

5. If $x^3 + ax^2 + bx + 30 = 0$ has root $2 + i$, find real a and b and the remaining two roots.

week: ..... lesson: 19 Cube Roots of Unity Date:...../...../.....

Class performance

Activity: A simple periodic model

Some mathematical models of periodic phenomena use complex quantities repeating every three steps. Let ω be a nonreal cube root of unity, so $\omega^3 = 1$.

Find

(1) ω^4 (2) ω^5 (3) ω^6 (4) ω^7 (5) ω^{2010}.

Discuss how division of the exponent by 3 replaces repeated multiplication.

Homework

Let ω be non-real number, cube root of unity. Evaluate:

1) ω^{14}.

2) $\frac{1}{\omega} + \frac{1}{\omega^2}$

3) $(1 - \omega^2)(1 - \omega)$.

4) $1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5$.

5) $\frac{3\omega + 3}{\omega^2}$

week: ..... **lesson: 20 Cubic Equations with Repeated Roots** **Date :** ...../...../.....

Class performance

Activity: Repeated roots

A simplified model gives $x^3 - 7x^2 + 15x - 9 = 0$; $x = 1$ is suspected to be a root.

(1) Verify it.

(2) Use the factor theorem.

(3) Factor into irreducible factors.

(4) Identify the repeated root.

(5) State all roots and their multiplicities.

(6) Challenge yourself: construct a cubic equation with repeated root $x = 2$ and state its roots.

Homework

1. Find a so that $x = 1$ is a root of $x^3 - 5x^2 + 7x + a$.

2. Find a so that $x = -2$ is a root of $x^3 + 3x^2 - 4x + a$.

3. Find a so that $x = 2$ is a root of $x^3 - 6x^2 + 11x + a$.

4. Use the factor theorem to find the roots and the repeated root of $x^3 - 7x^2 + 15x - 9 = 0$.

5. Find the roots and the repeated root of $F(x) = x^3 - 8x^2 + 19x - 12 = 0$.

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: Choose the correct answer.

1. If F(x) is divisible by x − 3, its remainder on division by x − 3 is:

(A) 3 (B) −3 (C) 0 (D) F(0).

Second: Answer the following questions.

2. If the remainders on division of F(x) by x − 2 and x + 3 are 5 and −10, find its remainder on division by $x^2 + x - 6$.

3. Use synthetic division to find the quotient and remainder when $x^3 - 5x^2 + 2x + 8$ is divided by x − 4.

4. Check whether x − 1 is a factor of $F(x) = x^3 - 6x^2 + 11x - 6$, then find all roots of f(x) = 0.

5. If $x^3 + ax^2 + bx - 20 = 0$ has roots 2 and 5, find real a and b and the third root.

Group B

First: Choose the correct answer.

1. If a cubic equation with real coefficients, has root 2 + i, then one of its other roots is:

(A) −2 − i (B) 2 − i (C) −2 + i (D) 2 + i.

Second: Answer the following questions.

2. If F(x) has remainder 2x + 3 on division by (− 1) and remainder 5 on division by x − 2, find the remainder on division by (x − 1)(x − 2).

3. Use synthetic division to find the quotient and remainder when $2x^3 - 7x^2 + 4x + 4$ is divided by x − 2.

4. If $F(x) = x^3 + ax^2 - 7x + 6$ is divisible by x − 1, find a and factor F(x) completely.

5. If $x^3 + ax^2 + bx - 15 = 0$ has root 1 − 2i, find real a and b and the remaining roots.

Group C

First: Choose the correct answer.

1. Divide the polynomial $F(x) = x^3 - 2x^2 + kx + 4$ by x − 1 using synthetic division yields remainder 5. Find k:

(A) 1 (B) 2 (C) 3 (D) 4.

Second: Answer the following questions.

2. If F(3) = 7 and F(−1) = −1, find the remainder on division by $x^2 - 2x - 3$.

3. Use synthetic division to find all real and complex roots of $x^4 - 3x^3 - x^2 + 3x = 0$.

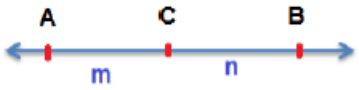
4. Test whether x + 2 is a factor of $x^3 - 3x^2 - 4x + 12$ using the factor theorem.

5. If $x^3 - kx^2 + (3k - 1)x - 24 = 0$ has root 4, find real k and the other complex roots.

Chapter 4

week: .....lesson: 1 Internal and External Division on a Number Line Date : ...../...../.....

Class performance **Activity: Precise division of a path**



In a modern physics laboratory, a straight laser path lies along a number line. The emission point is A(2) and the target is B(12). Researchers wish to locate C between them so that AC:CB = 2:1.

(1) Draw the number line and find the full path length.

(2) Is C between A and B?

(3) Which segment is longer, and why?

(4) Find the coordinates of C

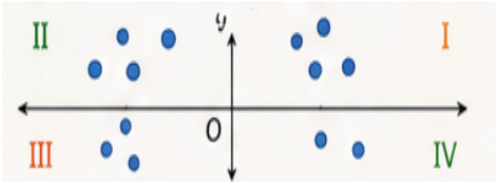
(5) Verify the 2:1 ratio.

Homework

1. Find the coordinate of C dividing the line segment between the two points A(2) and B(8) internally in the ratio 1:2.
2. Find D dividing the line segment between the two points A(3) and B(15) internally in the ratio AD:DB = 4:3.
3. Find C dividing the line segment between the two points A(4) and B(10) externally in the ratio AC:CB = 2:1.
4. Find the midpoint of the segment with endpoints A(−6) and B(8).
5. Find the point dividing the line segment between the two points A(−5) and B(7) externally in the ratio 3:2, measured from A.

week: ..... lesson: 2 Points in the Coordinate Plane Date : ...../...../.....

Class performance **Activity: Precisely locating cells**



Imagine a medical researcher using a coordinate system to locate cells in a tissue sample under a microscope. Each point marks a cell; the origin marks the sample centre.

The positions are A(2, 3), B(−2, 3), C(−2, −3), D(−3, 2), E(4, 0), N(−4, 0), and M(0, 0).

(1) Plot the points.

(2) Identify the quadrant of A, B, C and D.

(3) Which cells lie on the x axis?

(4) Which lie on the y axis?

(5) If a new cell appears at (4, −5), where is it in the sample? Explain.

Homework

1. Identify the quadrant of A(3, 4), B(−2, 5), C(−3, −4), and D(6, −1).
2. Reflect H(3, −5) in the x axis and write the new coordinates.
3. Reflect A(−4, 2) in the y axis.
4. Reflect R(−3, −6) in the origin.
5. If the point C (−a, b) lies in quadrant II, with a > 0 and b > 0. Write its coordinates of reflections in the x axis, y axis and origin.

week: ..... **lesson: 3 Distance Between Two Points** **Date :** ...../...../.....

Class performance

Activity: Distance between two devices

Two devices in a laboratory plan are at A(2, 3) and B(8, 11). An engineer needs their shortest straight-line separation.

(1) Find the difference between x coordinates.

(2) Find the difference between y coordinates.

(3) Form a right triangle.

(4) Apply the Pythagorean theorem.

(5) Find AB.

Homework

1. Find the distance between A(2, 3) and B(6, 6).
2. Find the distance between A(−1, 4) and B(5, −4).
3. Find x if the distance between A(x, 7) and B(3, −5) is 13 units.
4. Determine whether the triangle whose vertices A(0, 0), B(3, 4), C(6, 0) is an isosceles or not.
5. Find x if the distance between A(x, 7) and B(3x − 1, −5) is 13 units.

week: ..... **lesson: 4 Points Equidistant from Two Points** **Date :** ...../...../.....

Class performance

Activity: Locating a monitoring device

Two devices in a laboratory are at A(2, 4) and B(−1, 1).

A researcher wants a monitoring device on the x axis equidistant from both.

Write P = (x, 0); express PA and PB, equate the distances, and find P.

Explain why P has y coordinate zero.

Homework

1. Find P on the x axis equidistant from A(2, 4) and B(−1, 1).
2. Find P on the y axis equidistant from A(2, −5) and B(5, 3).
3. Find P on y = 3x + 1 equidistant from A(−5, 4) and B(−3, 2).
4. Find Q on y = x − 2 equidistant from A(1, 5) and B(7, −1).
5. Find Q on y = −x + 1 equidistant from A(1, 2) and B(−3, 6).

week: .....lesson: 5 Internal and External Division Points in the Coordinate Plane Date : ....\.....\.....

Class performance

Activity 1 : Travel map

A(1, −2) and B(5, 6) lie on a map.

Find M dividing AB internally in the ratio AM:MB = 2:1, and plot A, B and M.

Then find W dividing AB externally in the ratio AW:WB = 1:2.

Activity 2: The Garden

A triangular garden has vertices A(0, 0), B(4, 2), C(2, 6).

Find the midpoint of AB; find its centroid M; find D dividing BC internally in the ratio BD:DC = 1:1; and plot A, B, C, D, M and W in the coordinate plane.

Homework

1. Choose the correct answer:

A(2, 1) and B(8, 7). What are the coordinates of M if AM:MB = 2:1 internally?

(A) (4, 3) (B) (6, 5) (C) (5, 4) (D) (7, 6).

2. Find M dividing the line segment AB, A(6, 7) and B(2, 7) internally in the ratio 2 : 3 .
3. Find N dividing the line segment AB, A(−2, 4) and B(4, 4) externally in the ratio 1 : 2.
4. Find the midpoint M of the line segment AB, A(5, −3) and B(−1, 9).
5. Find the centroid of the triangle whose vertices A(2, 0), B(4, 6), C(8, 3).

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: Choose the correct answer.

Which point lies on the x axis and is equidistant from A(4, 4) and B(6, 4)?

(A) (4, 0) (B) (2, 0) (C) (6, 0) (D) (5, 0).

Second: Answer the following questions.

1. One root of $x^3 - 5x^2 + 7x + a = 0$ is $x = 1$.

(a) Find real a.
(b) Find its remaining roots.

2. Find C dividing the directed segment from A(2) to B(4) internally in the ratio AC:CB = 3:1.

3. P(−3, 4) lies in the coordinate plane. (a) Identify its quadrant. (b) Find Q, its reflection in the x axis, and R, its reflection in the origin.

4. Find D on the x axis equidistant from A(2, 4) and B(−1, 1).

Group B

First: Choose the correct answer.

A(−3, 4) and B(0, −5) are divided internally by C with AC = 2CB. What is C?

(A) (−1, −2) (B) (−1, 2) (C) (1, −2) (D) (1, 2).

Second: Answer the following questions.

1. One root of $x^3 - 6x^2 + 11x + a = 0$ is $x = 2$.

(a) Find real a.
(b) Find the remaining roots and identify the repeated root.

2. Find C dividing the directed segment A(3) to B(15) internally in the ratio AC:CB = 1:3.

3. P(4, −5) lies in the coordinate plane. (a) Identify its quadrant. (b) Find its reflection in the x axis and in the origin.

4. Find D on the x axis equidistant from A(2, −5) and B(5, 3).

Group C

First: Choose the correct answer.

Which point on the x axis is equidistant from A(−2, 5) and B(−4, −1)?

(A) (1, 0) (B) (3, 0) (C) (0, 1) (D) (0, 3).

Second: Answer the following questions.

1. One root of $x^3 - 7x^2 + 15x + a = 0$ is $x = 1$.

(a) Find real a.

(b) Find the remaining roots and identify the repeated root.

2. Find C dividing the directed segment from A(−5) to B(7) externally in the ratio AC:CB = 3:2.

3. P(−2, −6) lies in the coordinate plane. (a) Identify its quadrant. (b) Find its reflection in the x axis and in the origin.

4. Find D on y = 3x + 1 equidistant from A(−5, 4) and B(−3, 2).

Second Month Exercises

Choose the correct answer:

1. If 3, x, 27 are consecutive terms of a geometric sequence, then x = ...

(A) ±3 (B) ±9 (C) 9 (D) 3.

2. In a geometric sequence, the sum of the first two terms is 6 and the sum of the second and third is 12. Find the common ratio.

(A) −2 (B) 2 C) 3 (D)

3. Evaluate (1 × 2) + (2 × 4) + (3 × 8).

(A) 10 (B) 20 (C) 34 (D) 40

4. Write $\frac{(2 - 3i)}{(1 + 2i)}$ in standard form.

(A) $\frac{4}{5} - (\frac{7}{5})i$ (B) $-\frac{4}{5} + (\frac{7}{5})i$ (C) $-\frac{4}{5} - (\frac{7}{5})i$ (D) $\frac{4}{5} + (\frac{7}{5})i$.

5. Solve $2x^2 - 4x + 10 = 0$ in the set of the complex numbers.

(A) x = 1 ± 2i (B) x = −1 ± 2i (C) x = ±2i (D) x = 1 ± 3i.

6. If α and β are roots of $3x^2 - 7x + 2 = 0$, find $\alpha^2 + \beta^2$.

(A) $\frac{9}{37}$ (B) $\frac{37}{9}$ (C) $\frac{9}{49}$ (D) $\frac{2}{3}$

7. A quadratic equation has real coefficients and one root is 2 + 3i. Which equation is it?

(A) $x^2 - 4x + 13 = 0$ (B) $x^2 + 4x + 13 = 0$ (C) $x^2 - 3x + 13 = 0$ (D) $x^2 - 4x - 13 = 0$.

8. The remainder when $F(x) = x^3 - 2x^2 + ax + 5$ is divided by x + 2 is 7. Find a.

(A) −9 (B) −7 (C) 7 (D) 9.

9. If $\omega^3 = 1$ and $\omega \neq 1$, evaluate $(\omega - 1)(\omega^2 - 1)$.

(A) 1 (B) 2 (C) 3 (D) −3.

Answer the following

10. Express 10 + 12 + 14 + ... + 30 using sigma notation.

11. Find the sum $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + ... + \frac{1}{n(n + 1)}$

12. The nth term of a sequence is the sum of the natural numbers from 1 to n. Find the fourth and fifth terms.

13. Find the general term of a geometric sequence whose second term is −12 and fourth term is −48.

14. In a geometric sequence, the fourth and fifth terms sum to 24, and the fifth and sixth sum to 48. Find the first three terms.

15. Find the sum of the first n terms of (2 × 5) + (4 × 10) + (6 × 20) + (8 × 40) + ..., whose nth term is $2n \times 5 \times 2^{n-1}$.

16. For the sequence $a_n = 4n^2$, find the sum of its first two terms.

17. A sequence starts at 2, and its successive differences are 3, 5, 7, ... Find the sequence.

18. Simplify $\left(\frac{(3 + 4i)}{(2 - 1)}\right)$ into standard form, then find the complex conjugate of the result.

19. Find all real m for which $x^2 - (m - 2)x + 9 = 0$ has two distinct nonreal roots.

20. If α and β are roots of $3x^2 - 7x + 2 = 0$, find α + β and αβ; use these to find $\alpha^2 + \beta^2$ without solving for the roots.

21. If 2 + 3i is one root of $x^2 + ax + b = 0$, where a and b are real numbers, find $a^2 + b$.

22. Use the remainder theorem to find the remainder when $F(x) = 2x^4 - 3x^3 + 5x^2 - x + 7$ is divided by x − 2. Then write $F(x) = (x - 2)Q(x) + R$.

23. By converting the divisor into linear factors, find the remainder when $F(x) = x^3 - 2x^2 + 4x + 1$ is divided by $x^2 - 3x + 2$, without long division.

24. Use the factor theorem to prove that x − 4 is a factor of $F(x) = x^3 - 5x^2 + 2x + 8$. Then factor F(x) completely.

25. Solve $x^6 - 5x^4 + 4x^2 = 0$ over the real numbers. Explain the substitution that reduces the equation to a quadratic.

26. If $\omega^3 = 1$, evaluate $(2 + \omega^5 + \omega^2)^{(2^6)}$.

27. Points A(−4) and B(8) lie on a number line. Find the point that divides AB internally in the ratio 1:2, then the point that divides it externally in the ratio 3:2. Explain the different locations.

28. The distance between A(x, 7) and B(3x − 1, −5) is 13 units. Find every possible value of x and verify each.

29. Find the point P on y = 3x + 1 that is equidistant from A(−4, 5) and B(3, −2).

week: ..... **lesson: 6 Symmetric Points** **Date :** ...../...../.....

Class performance

Activity: the Coordinate Grid

Draw the coordinate plane. A(1, −1) and B(−x, 3) are symmetric about M(2, 1).

Find x and the coordinates of B.

Homework

1. Choose the correct answer: Which point B is symmetric to A(3, −1) about (5, 2)?

(A) (5, 7) (B) (7, 4) (C) (3, 2) (D) (7, 5).

2. Find B, symmetric of A(−3, −2) with respect to E(1, 6).

3. Find Q, symmetric of E(2, −4) with respect to A(1, −1).

4. A(−3, −2) and B(4, x) are symmetric about M(2, 1). Find x and the coordinates of point B.

5. B(5, m) and C(n, 7) are symmetric about A(1, −3). Find m + n.

week: ..... lesson: 7 Shapes of Triangles Date : ...../...../.....

Class performance

Activity 1: The dream garden.

An architect is planning a triangular garden.

Classify triangle ABC with vertices A(0, 4), B(4, 4), and C ($2\sqrt{3}$, 2).

Activity 2: A kite flight.

Salma wants to make a triangular kite.

Classify triangle ABC with vertices A(0, 0), B(6, 0), and C(3, 2).

Homework

Choose the correct answer:

1) Determine the shape of the triangle A(1, 2), B(5, 2), C(1, 5).

(A) Equilateral (B) Isosceles (C) Right angled (D) Right angled and isosceles.

2. Determine the shape of each triangle using its vertices:

(a) A(0, 0), B(4, 0), C(2, 2);
(b) A(0, 0), B(4, 0), C(4, 4);
(c) A(0, 0), B(0, 6), C(3, 3);
(d) A(0, 0), B(0, 6), C(2, 1).

week: ..... lesson: 8 Equation of a Straight Line Date : ...../...../.....

Class performance

Activity 1: Discover the equation of a line.

Find the equation of the line

(1) through (11, 2) and (3, 2);
(2) through (4, −3) with slope 2;
(3) through (−1, −2) and (5, 4);
(4) parallel to the x axis passes through (−5, 7).

Activity 2: A line in real life.

A delivery company charges a fixed fee of 20 Egyptian pounds plus 5 pounds per kilometer.

(1) Write the cost y in terms of distance x.

(2) Interpret the slope.

(3) Interpret the y intercept.

(4) Calculate the cost of an 8 km trip.

(5) Graph the relationship.

(6) If the trip costs 70 pounds, find the distance.

Homework

Choose the correct answer:

1. the equation of the line passes through (4, −3), parallel to the x axis:

(A) y = 4 (B) x = −3 (C) y = −3 (D) x = 3.

2. A line passes through A(−1, 2) and B(7, 6). Find its slope and equation, and express it as ax + by + c = 0.

3. A line L passes through (4, −3) with slope 2. Find its equation; check whether (12, 1) lies on it.

4. The lines 2x − 3y + 1 = 0 and x + 2y − 3 = 0 intersect. Find their intersection and the equation of the line through it and (3, −1).

5. Equation of one of two lines is y = 3x + 2; another passes through (1, 4) and (5, 6).

(a) Are they parallel, intersecting or perpendicular? Explain.

(b) Find their intersection if it exists.

week: ..... lesson: 9 Applications of the Equations of a Straight Line Date: ...../...../.....

Class performance

Activity 1: Design, write and draw.

In a coordinate plane, draw two streets. Street 1 passes through (2, −2) and (2, 8); street 2 passes through (1, 1) and (3, 5).

(1) Find the equation of each street in the form y = mx + b where possible.

(2) Determine whether they are parallel, intersecting or perpendicular, and explain.

(3) If street 1 is extended to meet the y axis, find its y intercept, if any.

Activity 2: A real life application.

A company installs lamp posts along a straight road. The line through the installations on one side passes through A(−4, 2) and B(4, 6).

(1) Find its equation.

(2) Do equally spaced posts along this line give a linear model? Why?

(3) Find the y intercept and interpret it in context.

Homework

Choose the correct answer:

1) Equation of the line passes through (2, 3) and (6, 11):

(A) y = 2x + 1 (B) y = x + 3 (C) y = 2x − 1 (D) y = 3x − 2.

2. Find the equation of line passes through (1, 2) and (5, 6), and find its y intercept.

3. Write the equation of the line passes through (3, 2) parallel to y = 2x − 5 in two different forms.

4. Find the equation of the line passes through (1, 1) with x intercept (4, 0). Find its y intercept.

5. Trip cost y in pounds increases at a constant rate with distance x in kilometres, and the graph passes through A(2, 30) and B(6, 50). Find (a) the relation between x and y; (b) the cost of a 10 km. trip.

week: .....**lesson: 10 Parallel and Perpendicular Lines** **Date :** ...../...../.....

Class performance

Activity 1: Think and discuss.

Line L_1 passes through (1, 2) and (6, 5), while L_2 passes through (−1, 5) and (3, 4).

(1) Find the slope of each of the two straight lines L_1 and L_2.

(2) Write the equation of L_1.

(3) Are L_1 and L_2 parallel? Explain.

Activity 2: A grid of columns in a modern building.

Vertical columns and horizontal beams are represented by lines. A column is x = 3 and a beam is y = −2.

(1) Are they perpendicular? Explain.

(2) Give an equation of a different vertical column parallel to x = 3.

(3) Give an equation of a different horizontal beam parallel to y = −2.

Homework

Choose the correct answer:

1. Which of the following equations represents a straight line which is parallel to the line whose equation: to y = 4x − 3?

(A) y = −4x + 5 (B) y = x + 2 (C) y = 4x + 7 (D) y = −x − 3.

2. Find the equation of the line passes through (2, 2) and (7, 6); then find a parallel line through (1, −2).

3. Find the equation of the line passes through (1, −4) perpendicular to y = −2x + 7.

4. If y = 4x − 3 and y = −hx + 5 are perpendicular, find h.

5. Find the equation of the line passes through (5, 0) parallel to $y = \frac{1}{2}x - 4$

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: Choose the correct answer.

1. If A(3, 4) and B(9, 10), what is the coordinates of the midpoint of AB?

(A) (6, 7) (B) (7, 6) (C) (5, 7) (D) (6, 8).

Second: Answer the following questions.

1. For A(1, 3) and B(7, 9), find the coordinates of the point dividing AB internally in the ratio 2:1.

2. For A(−4, 2) and B(4, 2), find the coordinates of the point dividing AB externally in the ratio 3:1.

3. Find the line through (3, −2) parallel to $y = 2x + 4$.

4. Find the line through A(2, 1) and B(5, 7). Determine whether the triangle with C(5, 1) is right angled.

Group B

First: Choose the correct answer.

1. If A(4, 2) and B(10, 8), what is the coordinates of the midpoint of AB?

(A) (7, 5) (B) (5, 7) (C) (6, 5) (D) (7, 6).

Second: Answer the following questions.

1. For A(2, 5) and B(8, 11), find the coordinates of the point dividing AB internally in the ratio 2:1.

2. For A(−2, 3) and B(6, 3), find the coordinates of the point dividing AB externally in the ratio 3:1.

3. Find the line through (4, −3) parallel to $y = 3x + 2$.

4. Find the line through A(1, 3) and B(4, 9). Determine whether the triangle with C(4, 3) is right angled.

Group C

First: Choose the correct answer.

1. If A(2, 5) and B(8, 11), what is the coordinates of the midpoint of AB?

(A) (5, 8) (B) (8, 5) (C) (6, 8) (D) (5, 7).

Second: Answer the following questions.

1. For A(3, 2) and B(9, 8), find the coordinates of the point dividing AB internally in the ratio 2:1.

2. For A(−6, 4) and B(2, 4), find the coordinates of the point dividing AB externally in the ratio 3:1.

3. Find the line through (2, −4) parallel to $y = 4x + 1$.

4. Find the line through A(2, 2) and B(5, 8). Determine whether the triangle with C(5, 2) is right angled.

week: ..... lesson: 11 Perpendicular Bisectors and Symmetric Points with Respect to a line Date: ...../...../.....

Class performance

Activity 1: Symmetric laser task.

Two points A and B represent light sources. We want a line whose points are equally distant from both.

(1) Draw AB.

(2) Find its midpoint.

(3) Draw the perpendicular line through the midpoint.

(4) Explain why every point on this line is equidistant from A and B.

(5) If A(2, 5) and B(8, 5), find the midpoint and the equation of the perpendicular bisector.

Activity 2: Mirror code.

A(2, 3) is reflected in the line $x = 5$.

(1) Find the distance from A to the axis of symmetry.

(2) Move the same distance to the other side.

(3) Find A'.

(4) Show that $x = 5$ is the perpendicular bisector of AA'.

Homework

1. Choose the equation of the perpendicular bisector of A(1, 5) and B(7, 1):

(A) $3x - 2y + 6 = 0$ (B) $3x - 2y - 6 = 0$ (C) $2x + 3y - 6 = 0$ (D) $2x - 3y + 6 = 0$.

2. For A(1, 4) and B(7, 4), find the coordinates of the midpoint and the equation of the perpendicular bisector, showing your steps.

3. A(2, 3) and A' are symmetric in $x = 6$. Find A' and explain the relation between $x = 6$ and AA'.

4. Given M(4, 7) and N(10, 1), find the midpoint of MN and the slopes of MN and its perpendicular bisector.

5. A(3, 5) and B(9, 5) lie on opposite sides of $x = 6$. Prove that they are symmetric in this line, and explain why it bisects AB perpendicularly.

week: ..... lesson: 12 Distance between a Point and a Line Date : ...../...../.....

Class performance

Activity 1: The shortest distance to a road.

An ambulance station is at (2, 3) and the main equation of road is $3x + 4y - 12 = 0$.
The engineer wants to know the shortest distance between the ambulance station and road.
Required:
Find the shortest distance between the point and the road using the distance formula between a point and a line.

Activity 2: A building near a street.

A public-service building is at $(-1, 2)$ and the equation of the straight street is $4x - 3y + 6 = 0$. Find the shortest distance from the building to the street.

Homework

Choose correct answer:

1) The distance between the point $(3, -2)$ and the line $6x + 8y - 10 = 0$ is:

(A) 0.8 (B) 2 (C) 3 (D) 4.

Answer the following:

(a) Find the shortest distance between the point $(2, -3)$ and the line $3x + 4y + 6 = 0$.

(b) Find the perpendicular distance between the origin and the line $4x + 3y - 10 = 0$.

(c) Find the shortest distance between the point $(-2, 1)$ and the line $5x - 12y + 13 = 0$.

(d) Find the perpendicular distance between the point $(3, 4)$ and the line $3x + 4y - 25 = 0$.

week: ..... lesson: 13 Area of a Triangle Date : ..../...../.....

Class performance

Activity 1: A coordinate map.

Given the points A (2, 1), B(4, 7) and C(3, −2):

(1) find the equation of AB;

(2) find the altitude from C to AB;

(3) find the area of triangle ABC.

Activity 2: Smart surveying.

A triangle has vertices A(3, −2), B(4, 1) and C(7, 6).

(1) Find the equation of AB.

(2) Find the distance between C and AB.

(3) Find the area of triangle ABC.

Homework

Choose the correct answer:

The area of triangle whose vertices A(1, 2), B(5, 2), C(3, 6): "all in square units."

(A) 6 (B) 8 (C) 10 (D) 12

2. Find the area of triangle whose vertices A(1, 2), B(0, 6), C(−2, 4).

3. Find the line equation of passes through A(3, 8) and B(7, 5), then the shortest distance from C(2, −1) to it.

4. Use coordinates to find the area of triangle A(−3, 2), B(2, 0), C(8, 1).

5. Find the area of triangle A(−4, −1), B(2, 0), C(6, −3), showing your steps.

week: ..... **lesson: 14 Line Passing Through a Fixed Point** **Date :** ....\....\.....

Class performance

Activity: The fixed-point key.

The family of lines $(k - 4)x + (k + 3)y + 6 = 0$ passes through a fixed point independent of k.

(1) Collect the terms containing k.

(2) Set its coefficient to zero and locate the fixed point.

(3) Verify by substitution.

Homework

1. The line whose equation $(k + 2)x + (k + 3)y - 6 = 0$ passes through a fixed point for every k. Find that point.

2. The line $(k + 2)x + (k - 2)y - 3 = 0$ passes through a fixed point independent on the value of k , Find the coordinates of this point.

3. The line $(k - 2)x + (k + 3)y + 10 = 0$ passes through a fixed point independent on the value of k , Find the coordinates of this point.

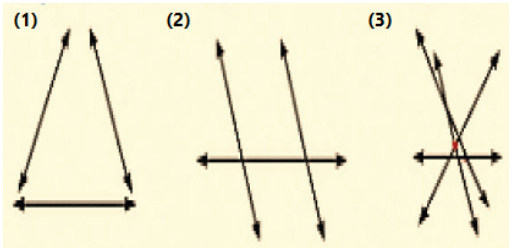
4. The line $(k + 4)x + (k - 4)y + 2 = 0$ passes through a fixed point independent on the value of k , Find the coordinates of this point.

5. The line $(k - 1)x + (k + 2)y + 7 = 0$ passes through a fixed point independent on the value of k , Find the coordinates of this point.

week: ..... L: 15 Conditions for Not Forming a Triangle Date : ....\.....\.....

Class performance

Activity: Check the situation.



Study the three diagrams: one has three lines meeting pairwise at distinct vertices, one has two parallel lines, and one has three concurrent lines.

Decide whether each set forms a triangle and explain why.

Homework

1. Three lines x, y, z satisfy x + y + z = 18, x + y = 8 and y + z = 7. Can they form the sides of a triangle? Explain.
2. Three lines x, y, z satisfy x + y = 10, y + z = 10 and x + z = 5. Can they form the sides of a triangle? Explain.
3. The lines $y = -\frac{3}{2}x - 3$, x = 2y − 1, and kx = 3y − 2 do not form a triangle. Find the possible values of k.

week: ..... lesson: 16 Equation of a Circle 1 Date : .....\.....\.....

Class performance

Activity 1: Radar screen.

A fountain in a public garden has centre (2, −3) and radius 5 meters.

(1) Write its equation in standard form.

(2) Identify its centre and radius from this equation.

(3) If its centre moves to (−2, 4) while the radius remains 5 meters, find its new equation.

(4) Compare the equations.

Activity 2: City map.

A circular district is modelled by $x^2 + y^2 - 6x + 4y = 12$.

(1) Using perfect square , convert the equation into the standard form of a circle's equation.

(2) Find the centre and radius.

(3) Write the equation for another district with centre (1, −1) and radius 4.

(4) Which district is larger? Explain.

Homework

1. For $(x - 3)^2 + (y + 2)^2 = 25$, choose the centre and radius, respectively:

(A) (3, −2), 5 (B) (−3, 2), 5 (C) (3, 2), 25 (D) (−3, −2), 25.

2. Write the equation of a circle with centre (−5, 2) and radius 6; give one point on it.

3. Convert $x^2 + y^2 + 8x - 4y - 5 = 0$ into standard form and find its centre and radius.

4. A circle centred at (−3, 1) has twice the radius of a circle centred at (−1, 2) with radius 3. Write both equations.

5. For $x^2 + y^2 - 10x + 6y + 9 = 0$: (a) prove that it describes a circle; (b) find its centre and radius.

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: Choose the correct answer.

Which line is parallel to $3x - 2y + 5 = 0$?

(A) $6x - 4y + 1 = 0$ (B) $3x + 2y - 1 = 0$ (C) $2x + 3y + 4 = 0$ (D) $4x + 6y - 2 = 0$.

Second: Answer the following questions.

1. Find the equation of line passes through $(2, -3)$ and parallel to $y = 2x + 1$.
2. Find the perpendicular bisector of the segment with endpoints A(1, 3) and B(5, 7).
3. Find the distance from point $(3, -2)$ to $4x + 3y - 6 = 0$.
4. Find the area of triangle whose vertices A(1, 2), B(5, 2), C(3, 6).

Group B

First: Choose the correct answer.

Which line is parallel to $2x - 3y + 7 = 0$?

(A) $4x - 6y + 1 = 0$ (B) $2x + 3y - 5 = 0$ (C) $3x + 2y + 1 = 0$ (D) $6x + 4y - 3 = 0$.

Second: Answer the following questions.

1. Find the equation of line passes through $(-1, 4)$ and parallel to $y = -3x + 2$.
2. Find the perpendicular bisector of the segment with endpoints A(−2, 1) and B(4, 5).
3. Find the distance from point (2, 3) to $3x + 4y - 8 = 0$.
4. Find the area of triangle whose vertices A(−1, 2), B(3, 2), C(1, 7).

Group C

First: Choose the correct answer.

Which line is parallel to $5x - 2y + 4 = 0$?

(A) $10x - 4y + 3 = 0$ (B) $5x + 2y - 6 = 0$

(C) $2x + 5y + 1 = 0$ (D) $4x + 10y - 2 = 0$.

Second: Answer the following questions.

1. Find the equation of line passes through $(3, -1)$ and parallel to $y = 4x - 5$.
2. Find the perpendicular bisector of the segment with endpoints A(0, 2) and B(6, 8).
3. Find the distance from point $(4, -1)$ to $3x - 4y + 5 = 0$.
4. Find the area of triangle whose vertices A(2, 1), B(6, 1), C(4, 6).

week: ..... lesson: 17 Equation of a Circle 2 Date : ...../...../.....

Class performance

Activity 1: Radar screen.

A radar station monitors a circular region defined by $x^2 + y^2 - 8x + 6y - 11 = 0$.

(1) Convert the equation into standard form.

(2) Find the centre and radius.

(3) If the engineer wants to make the radius of the observation field 10 units, while keeping the center the same, what is the new equation ?

(4) Is a signal at (4, 2) inside, on or outside the original boundary? Explain.

Activity 2: A circular playground.

A public park contains a circular playground centred at (−2, 5) with radius 6 metres.

(1) Write the equation of its boundary in standard form.

(2) Increase the radius to 8 metres without moving the centre; find the new equation.

(3) Change the centre to (−4, 1) and radius to 5 metres; find the equation.

(4) Decide whether (−2, 9) and (3, 1) lie inside, on or outside the original playground; justify your answers.

Homework

1. Choose the equation of the circle with centre (3, −2) and radius 5:

(A) $(x - 3)^2 + (y + 2)^2 = 25$ (B) $(x + 3)^2 + (y - 2)^2 = 25$

(C) $(x - 3)^2 + (y + 2)^2 = 5$ (D) $(x + 3)^2 + (y - 2)^2 = 5$.

2. Find the equation of the circle centred at (−3, 3) with radius 4, and give a point on the circle in quadrant I.

3. Convert $x^2 + y^2 + 6x - 4y - 12 = 0$ into standard form; find the centre and radius.

4. A circle centred at (3, 2) passes through (7, 1). Find its radius and equation.

5. Find the equation, centre and radius of the circle passes through (1, 1), (1, 5) and (5, 3).

week: ..... lesson: 18 Equation of a Circle 3 Date : ...../...../.....

Class performance

Activity 1: A circular garden.

A city authority is designing a fountain. Write the circle equation in each case.

(1) Its centre is (3, −2) and it touches the x axis.

(2) It passes through (−3, 5) and touches both coordinate axes.

(3) Its centre lies on $y = x - 1$ and it passes through (6, 2) and (4, 2).

(4) Its center lies on $y = 2x + 1$ and it passes through (5, 2) and (2, −1).

Activity 2: A medical service area.

A mobile clinic serves a circular region centred at (−1, 4) whose boundary is tangent to the x axis. Find its equation.

Homework

Choose the correct answer:

1) The center and radius of the circle whose equation $(x - 3)^2 + (y + 2)^2 = 25$:

(A) (3, −2), 5 (B) (−3, 2), 5 (C) (3, −2), 25 (D) (−3, 2), 25.

2. Find the equation of the circle whose centred at (3, 2) touches the x axis.

3. Find every circle passing through (2, 4) and touches both the x and y axes.

4. Find the equation of the circle whose centre lies on $y = x - 1$ and which passes through (5, 2) and (2, −1).

5. Find the equation of the circle whose centre lies on $y = 2x + 3$ and which passes through (1, 0) and (5, 4).

week: ..... lesson: 19 Conditions for Forming a Circle Date: ..../.../.....

Class performance

Activity 1: Designing a circular fountain.

The proposed equation is $x^2 + y^2 - 8x + 4y + k = 0$, where k is adjustable. Find all values of k for which it represents a real circle.

Activity 2: A cleaning robot.

Its planned circular path is $x^2 + y^2 + 6x - 2y + 3k = 0$. Find all values of k for which it represents a real circle.

Homework

1. Choose the condition on k for $x^2 + y^2 + 6x - 8y + k = 0$ to be a circle:

(A) $k > 25$ (B) $k < 25$ (C) $k \geq 25$ (D) $k \leq 25$.

2. Find all k for which $x^2 + y^2 + 4x - 6y + 2k = 0$ represents a real circle.

3. Find all k for which $x^2 + y^2 - 10x + 2y + 2k = 0$ represents a real circle.

4. Find all k for which $x^2 + y^2 + 2x + 8y + 5k = 0$ represents a real circle.

5. Find all k for which $x^2 + y^2 - 6x - 4y + k = 0$ represents a real circle.

week: ..... **lesson: 20 Points of Intersection of a Circle and a Line 1** **Date:** ...../...../.....

Class performance

Activity 1: An exploration of circles and boundaries.

For a small park, find the intersections of $x^2 + y^2 = 36$ and $y = 3x - 6$.

Activity 2: From space to Earth.

A telecommunications engineer studies circular satellite paths.

Find the intersections of $x^2 + y^2 = 32$ and $y = x - 4$.

Homework

Choose the correct answer:

1. How many points of intersection do $x^2 + y^2 = 25$ and $y = 3$ have?

(A) One (B) Two (C) Three (D) None.

2. Solve the following problems. Find the coordinates of the intersection points of the following circles and lines.

(a) $x^2 + y^2 = 25$ and $y = -3x + 15$;

(b) $x^2 + y^2 = 17$ and $x = y + 3$;

(c) $x^2 + y^2 = 50$ and $y = -x + 6$;

(d) $x^2 + y^2 = 10$ and $y = 2x - 1$.

week: ..... lesson: 21 Points of Intersection of a Circle and a Line 2 Date : ...../...../.....

Class performance

Activity: Observe and examine.

A watchtower at an Egyptian port is at the origin, and its radar covers $x^2 + y^2 = 25$. A ship travels on $y = x + 1$. (1) Plot the circle and line and identify their intersections.

(2) Substitute the line equation into the circle equation to obtain a quadratic in x.

(3) Solve it, then calculate the corresponding y values.

(4) Compare the algebraic and graphical results and describe the position of the line relative to the circle.

Homework

Choose the correct answer

If the circle is $x^2 + y^2 = 25$ and the line is $3x + 4y = k$, choose k for which it is tangent:

(A) ±25 (B) ±24 (C) ±20 (D) ±30.

2. The line $y = x + m$ is tangent to $x^2 + y^2 = 18$. Find m.

3. The line $y = 2x + n$ has no intersection with $x^2 + y^2 = 13$. Find all possible n.

4. The line $y = 3x + m$ has exactly one intersection with $x^2 + y^2 = 10$. Find m.

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: Choose the correct answer.

1. Find the condition on k for the following equation $x^2 + y^2 - 6x + 4y + k = 0$ to represent a real circle.

(A) $k < 13$ (B) $k > 13$ (C) $k = 13$ (D) $k \leq 13$.

Second: Answer the following questions.

2. Find the equation of the circle with centre (3, −2) and radius 5.

3. Find the equation of the circle whose diameter has endpoints A(1, 3) and B(5, −1).

4. Find the equation of the circle centred at (−2, 4) touches to the x axis.

5. Find the points of intersections of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ and the straight line $y = 1$.

Group B

First: Choose the correct answer.

1. Find the condition on k for the following equation $x^2 + y^2 + 8x - 2y + k = 0$ to represent a real circle.

(A) $k < 17$ (B) $k > 17$ (C) $k = 17$ (D) $k \leq 17$.

Second: Answer the following questions.

2. Find the equation of the circle with centre $(-4, 3)$ and radius 6.

3. Find the equation of the circle whose diameter has endpoints A$(-2, 4)$ and B$(4, -2)$.

4. Find the equation of the circle centred at $(-5, 2)$ touches y axis.

5. Find the points of intersections points of $x^2 + y^2 - 2x - 2y - 23 = 0$ and the straight line $y = 1$.

Group C

First: Choose the correct answer.

1. Find the condition on k for the following equation $x^2 + y^2 - 10x + 4y + k = 0$ to represent a real circle.

(A) $k < 29$ (B) $k > 29$ (C) $k = 29$ (D) $k \leq 29$.

Second: Answer the following questions.

2. Find the the equation of circle with centre $(-2, -3)$ and radius 4.

3. Find the the equation of circle whose diameter has endpoints A$(0, 2)$ and B$(6, 8)$.

4. Find the the equation of circle centred at $(-3, 5)$ touches the x axis.

5. Find the points of intersections points of $x^2 + y^2 - 4x + 4y - 17 = 0$ and the straight line $y = -2$.

week: ..... **lesson: 22 Length of a Chord** **Date :** ...../...../.....

Class performance

Activity 1: A bridge across a pond.

A straight bridge joins two points on the rim of a circular pond. The pond radius is 5 metres, and the perpendicular distance from its centre to the bridge is 3 metres. How long is the bridge?

Activity 2: A lamp support.

A circular lamp shade has radius 5 cm. A technician fits a straight support between two points on its edge, 4 cm from the centre measured perpendicularly. Find the support length.

Homework

Choose the correct answer:

the length of the chord cut from $x^2 + y^2 = 25$ by $x + y - 1 = 0$:

(A) 7 (B) 4 (C) 6 (D) 2.

2. Find the chord length cut from $x^2 + y^2 - 2y = 25$ by $y = x + 1$.

3. Find k if the segment joining the intersections of $x^2 + y^2 - 4y + 4 = 20$ and $y = x + k$ has length $6\sqrt{2}$.

4. Find the chord length cut from $(x - 3)^2 + (y + 2)^2 = 49$ by $y - x = 1$.

5. Find k if the segment joining the intersections of $x^2 + y^2 + 4y = 16$ and $y = 2x + k$ has length 6.

week: ..... **lesson: 23 Tangents to a Circle** **Date :** ...../...../.....

Class performance

Activity 1: Smart metro tickets.

On the first day, you entered the metro at (2, 2) and exited at (7, 3). On another day, you entered at (5, 5) and exited at (9, −2). Find the circle equations for

(a) the first day and

(b) the second day. Then find the circle centred at (1, 4) tangent to the x axis.

Activity 2: Al Azhar Park.

A circular fountain was initially centred at (−3, 2). In a new design, its centre moves to (3, 4), and the fountain becomes tangent to $y = -2$. Find the new circle equation. Also write the equation of another circle centered at (1, −1) with radius 6.

Homework

Choose the correct answer:

1. The equation of the tangent to the circle $x^2 + y^2 = 49$ at (7, 0):

(A) $x = 7$ (B) $y = 7$ (C) $x + y = 7$ (D) $x - y = 7$.

For each pair, describe the relative positions of the circles:

(a) $x^2 + y^2 = 5$ and $(x - 2)^2 + (y + 2)^2 = 5$;

(b) $x^2 + y^2 = 9$ and $(x + 3)^2 + (y - 2)^2 = 4$;

(c) $x^2 + y^2 = 16$ and $(x - 4)^2 + (y + 3)^2 = 9$;

(d) $x^2 + y^2 = 1$ and $(x + 3)^2 + (y + 3)^2 = 3$.

2. Find the equation of a circle centred at (2, 2) that intersects $x^2 + y^2 = 16$ at two points.

3. Find the equation of a circle centred at (4, 4) internally tangent to $x^2 + y^2 = 25$.

week: ..... **lesson: 24 Positional Relationship Between Two Circles** Date:...../...../.....

Class performance

Activity 1: A visitors' amphitheatre at the pyramids

The circular amphitheatre is represented by $x^2 + y^2 = 25$. Find its tangent at (5, 0).

Activity 2: Ain Sokhna.

In Ain Sokhna city, there's a huge Ferris wheel that spins on a horizontal level. Its circular rim is represented by $x^2 + y^2 = 81$. Find the tangent at (0, −9).

Homework

Choose the correct answer:

1. Two circles have radii 5 cm and 3 cm, and their centres are 8 cm apart. Choose their relation:

(A) intersect in two points (B) externally tangent

(C) internally tangent (D) one lies inside the other without touching.

2. Find equations of the following tangents:

(a) Touches the circle to $x^2 + y^2 = 100$ at (6, 8);

(b) Touches the circle to $x^2 + y^2 = 25$ at (4, 3);

(c) all tangents from (1, 5) to $x^2 + y^2 = 9$;

(d) all tangents from (2, 6) to $x^2 + y^2 = 16$.

week: ..... **lesson: 25 Points of Intersection of Two Circles** Date:...../...../.....

Class performance

Activity: A broadcasting station.

Two transmitters cover $x^2 + y^2 = 25$ and $(x - 3)^2 + y^2 = 16$. Find the points covered by both. Expand the second equation, subtract the equations to obtain a linear equation,

substitute back to find the intersection points, and check them on a coordinate grid.

Homework

Choose the correct answer:

1. The two intersections of $x^2 + y^2 = 25$ and $(x - 6)^2 + y^2 = 25$:

(A) (3, 4), (3, −4) (B) (4, 3), (−4, 3) (C) (5, 0), (−5, 0) (D) (0, 5), (0, −5).

2. Find the circle passing through the intersections of $x^2 + 3x + y^2 + 2y + x = 0$ and $x^2 + y^2 + 2y + 3x - y = 5$, and also through the origin.

3. Find the circle passing through the intersections of $x^2 + y^2 + 2x - 4y + 8 = 0$ and

$x^2 + y^2 - 2x + y = 14$, and through (1, 2).

4. Find the circle passing through the intersections of $x^2 + 2x + y^2 + 4y + x + 1 = 0$ and

$x^2 + y^2 + 2x - 5y = 20$, and through $(1, -3)$.

5. Find the circle passing through the intersections of $x^2 + y^2 + 3x - 4y - 7 = 0$ and

$x^2 + y^2 - 5x - y = 16$, and through $(2, -3)$.

week: .....lesson: 26 Circle Passing Through the Points of Intersection of Two Circles Date : ...\...\.....

Class performance

Activity 1: Shared wireless coverage.

Two known circles represent two stations' wireless coverage. Their intersection points receive both signals.

(1) Find the equation of the line through those points.

(2) Explain what the line represents on the map.

(3) Explain how the intersection points might help select the site of a new service centre.

Activity 2: Finding a site between two stations.

Two possible sites lie at the intersections of distance circles around existing stations.

(1) Use the circle equations to find the points.

(2) Which is more suitable if the new station must be close to both existing stations?

(3) Explain how the intersection coordinates aid the map decision.

Homework

Choose the correct answer:

1. The equation of the line through the intersections of $x^2 + y^2 - 4x - 6y + 9 = 0$ and $x^2 + y^2 + 2x - 2y - 15 = 0$: (A) $3x + 2y - 12 = 0$ (B) $2x + 3y - 12 = 0$ (C) $3x - 2y + 12 = 0$ (D) $2x - 3y - 12 = 0$.

2. Find the equation of the line passing through the intersections of each pair:

(a) $x^2 + y^2 = 13$ and $x^2 + y^2 - 10x + 8y = 5$;

(b) $x^2 + y^2 = 9$ and $x^2 + y^2 + 6x - 12y = 9$.

3. Find the coordinates of the intersection of each pair of circles:

(a) $x^2 + y^2 = 8$ and $(x - 2)^2 + (y - 1)^2 = 4$;

(b) $x^2 + y^2 = 16$ and $(x + 2)^2 + (y - 4)^2 = 16$.

4. Find the intersections for each pair:

(a) $x^2 + y^2 = 10$ and $(x - 1)^2 + (y - 2)^2 = 5$;

(b) $x^2 + y^2 = 25$ and $(x + 3)^2 + (y - 4)^2 = 25$.

5. Find the intersections for each pair:

(a) $x^2 + y^2 = 20$ and $(x - 4)^2 + (y - 2)^2 = 16$;

(b) $x^2 + y^2 = 13$ and $(x + 2)^2 + (y + 3)^2 = 9$.

Weekly Assessment week: ..... Date : ...../...../.....

Group A

First: Choose the correct answer.

A line $(k + 2)x + (k - 1)y - 6 = 0$ passes through a fixed point for every k. Choose its coordinates:

(A) (2, −2) (B) (−2, 2) (C) (2, 2) (D) (−2, −2).

Second: Answer the following questions.

1. Find the chord length cut from the circle $x^2 + y^2 = 25$ by $3x + 4y - 15 = 0$.

2. Find the equation of the tangent to the circle $x^2 + y^2 = 25$ at (3, 4).

3. Determine the relation between $x^2 + y^2 = 25$ and $(x - 8)^2 + y^2 = 9$.

4. Find the circle through the intersections of $x^2 + y^2 - 4x + 2y - 1 = 0$ and $x^2 + y^2 - x - 2y - 1 = 0$ that also passes through (1, 1).

Group B

First: Choose the correct answer.

A line $(k + 3)x + (k - 2)y - 5 = 0$ passes through a fixed point for every k. Choose its coordinates:

(A) (1, −1) (B) (−1, 1) (C) (1, 1) (D) (−1, −1).

Second: Answer the following questions.

1. Find the chord length cut from the circle $x^2 + y^2 = 36$ by $3x + 4y - 15 = 0$.

2. Find the equation of the tangent to the circle $x^2 + y^2 = 36$ at (6, 0).

3. Determine the relation between $x^2 + y^2 = 36$ and $(x - 10)^2 + y^2 = 16$.

4. Find the circle through the intersections of $x^2 + y^2 - 2x + 4y - 4 = 0$ and $x^2 + y^2 + 4x - 2y - 4 = 0$ that also passes through (1, 0).

Group C

First: Choose the correct answer.

A line $(k + 4)x + (k - 3)y - 14 = 0$ passes through a fixed point for every k. Choose its coordinates:

(A) (2, −2) (B) (−2, 2) (C) (2, 2) (D) (−2, −2).

Second: Answer the following questions.

1. Find the chord length cut from the circle $x^2 + y^2 = 100$ by $6x + 8y - 60 = 0$.

2. Find the equation of the tangent to the circle $x^2 + y^2 = 100$ at (6, 8).

3. Determine the relation between $x^2 + y^2 = 49$ and $(x - 5)^2 + y^2 = 16$.

4. Find the circle through the intersections of $x^2 + y^2 - 6x + 2y - 5 = 0$ and $x^2 + y^2 + 2x - 4y - 5 = 0$ that also passes through (1, 1).

Review of the Curriculum

Choose the correct answer

1. A sequence whose the n th term is the sum of the squares of the natural numbers from 1 to n.

If $a_n = 55$, then find n.

(A) 4 (B) 5 (C) 6 (D) 7.

2. If the sum of the first n terms is $S_n = 5n^2 + 2$, find the first term.

(A) 2 (B) 5 (C) 7 (D) 10.

3. What type of sequence satisfies $a_{n+1} = a_n + 6$?

(A) Geometric, ratio 6 (B) Geometric, ratio 4

(C) Arithmetic, difference 6 (D) Arithmetic, difference 4.

4. If $a_{n+1} = a_n + 2n$ and $a_1 = 3$, choose the general term.

(A) $n^2 + n + 1$ (B) $n^2 - n + 3$ (C) $n^2 + 2$ (D) $2n^2 + 1$.

5. In $a_{n+1} = 3a_n + 7$, find the coefficient p and constant q, respectively.

(A) 7, 3 (B) 3, 7 (C) 1, 7 (D) 3, 1.

6. In a proof by mathematical induction, assuming the identity is true at n = k is called what?

(A) Base case (B) Inductive hypothesis (C) Inductive step (D) Final result.

7. To prove $3^n > 2n + 1$ for natural $n \geq 2$ by induction, which value is checked first?

(A) 0 (B) 1 (C) 2 (D) 3.

8. Which expression is divisible by 7 for natural n?

(A) $2^{3n} - 1$ (B) $3^{2n} + 1$ (C) $5^n + 2$ (D) $4^n - 1$.

9. A sequence begins with 2 and satisfies $a_{n+1} = a_n + 4$. Choose its general term.

(A) $4n - 2$ (B) $2n + 4$ (C) $4n + 2$ (D) $2n - 4$.

10. If $3a + ci = 9 - 6i$, find $a + c$.

(A) −3 (B) 3 (C) 9 (D) 6.

11. Evaluate $5i^2$.

(A) 2 (B) −2 (C) 5 (D) −5.

12. The equation $x^2 - 2(k + 2)x + 9 = 0$ has a repeated real root. Find k.

(A) 1 only (B) −5 only (C) 1 or −5 (D) 5 or −1.

13. If α and β are roots of $x^2 - 5x + 3 = 0$, find $(\alpha - \beta)^2$.

(A) 7 (B) 13 (C) 17 (D) 22.

14. If α and β are roots of $4x^2 - 8x + 1 = 0$, find $\alpha + \beta$.

(A) 1 (B) 2 (C) 6 (D) 8.

15. If $x^2 - (k + 1)x + k = 0$ has two distinct positive roots, what is the condition on k?

(A) $k > 0$ only (B) $k > 0$ and $k \neq 1$ (C) $k \geq 1$ (D) $k < 0$.

16. If $F(1) = 4$ and $F(-2) = 1$, find the remainder when $F(x)$ is divided by $x^2 + x - 2$.

(A) $x + 3$ (B) $x - 3$ (C) $3x + 1$ (D) $-x + 3$.

17. Find the remainder when $x^3 + 2x^2 - x + 4$ is divided by $x^2 - 3x + 2$.

(A) $12x - 6$ (B) $6x - 12$ (C) $12x + 6$ (D) $6x + 12$.

18. Which cubic equation has roots 2, −1 and 3?

(A) $x^3 - 4x^2 - x + 6 = 0$ (B) $x^3 + 4x^2 - x + 6 = 0$

(C) $x^3 - 4x^2 + x + 6 = 0$ (D) $x^3 + 4x^2 + x + 6 = 0$.

19. P lies on $y = 2x - 1$ and is equidistant from A(2, 1) and B(5, −2). Find P.

(A) (−2, −5) (B) (2, 3) (C) (−1, −3) (D) (−3, −7).

20. M is the midpoint of A(−4, 6) and B(8, −2). Find M.

(A) (2, 2) (B) (−2, 4) (C) (4, 2) (D) (2, −2).

21. Which line has slope 4 and passes through (2, 3)?

(A) $y = 4x - 5$ (B) $y = 4x + 5$ (C) $y = 2x - 4$ (D) $y = 4x + 3$.

22. Find the shortest distance from (2, 3) to $3x + 4y - 28 = 0$.

(A) 1 (B) 2 (C) 3 (D) 4.

Answer the following

1. An arithmetic sequence has common difference 3 and its eighth term is 25. Find the position of the term equal to 112, and the position of a term whose value exceeds 300.

2. A sequence starts with 2, and its successive differences are 3, 5, 7, ... Find the sequence.

3. If $a_{n+1} = a_n + 3$ and $a_1 = 2$, find the general term and the fourth term.

4. Use mathematical induction to prove that $3^{2n} - 1$ is divisible by 8.

5. A sequence begins with 3 and satisfies $a_{n+1} = a_n + 5$. Prove by induction that $a_n = 5n - 2$.

6. Find real x and y if $4 - 5i = 8x + 2yi$.

7. Evaluate $(5 - 3i^2)(3 + 6i^4)$.

8. Solve $2x^2 - 4x + 10 = 0$ over the complex numbers; verify both roots by substitution.

9. If α and β are roots of $x^2 - 5x + 3 = 0$, find $(\alpha - \beta)^2$. Explain why it is unnecessary to calculate the individual roots.

10. If $F(x) = x^3 - (k + 4)x^2 + (4k + 4)x - 8$ has the repeated root $x = 2$, find k and factor $F(x)$ for that value.

11. Find the point dividing AB externally in the ratio 3:2, where A(2, 5) and B(7, 10).

12. Find the midpoint of line segment AB, A(−6, 4) and B(8, −2).

13. find the coordinates of the symmetric point of C(−2, −6) in the y axis.

14. Find the side lengths of triangle A(1, 2), B(5, 2), C(5, 5), then classify it.

15. Find the equation of line passes through (5, −3) parallel to $y = 2x + 7$.

16. Find the equation of line passes through (−3, 5) parallel to $y = 4x - 2$.

17. Find k if $y = 3x + k$ passes through (2, 9).

18. Find the equation of line passes through (−1, 3) perpendicular to $y = 2x + 6$.

19. Given A(3, −1) and B(9, 7), find the midpoint then find equation of the perpendicular bisector of AB.

20. Find the shortest distance from (4, −2) to $3x + 4y - 10 = 0$.

21. Find the perpendicular distance from (−2, 5) to $x - 2y + 3 = 0$.

22. Find the fixed point common to all lines $(k + 1)x + (k - 2)y - 9 = 0$.

23. Prove that $(k - 3)x + (k + 2)y - 10 = 0$ passes through a fixed point for all k, and find it.

24. Find the fixed point common to all lines $(2k + 1)x + (2k - 3)y - 12 = 0$.

25. Find the equation of the circle centred at (2, −3) with radius 6.

26. Find the equation of the circle centred at (2, 5) tangent to the x axis.

27. Find all k for which $x^2 + y^2 - 8x + 6y + k = 0$ represents a real circle.

28. Find all k for which $x^2 + y^2 + 4x - 12y + k = 0$ represents a real circle.

29. Find k for which $x^2 + y^2 - 10x + 4y + k = 0$ represents a point circle.

30. Find the intersections of $(x - 2)^2 + (y + 1)^2 = 25$ and $y = 2$.

31. Determine the number of intersections of $x^2 + y^2 - 6x + 4y - 3 = 0$ and $y = 2x - 1$; find any intersection points.

32. Find the intersections of $(x - 3)^2 + (y - 1)^2 = 25$ and $y = 1$.

33. Determine the number of intersections of $x^2 + y^2 - 4x + 2y - 4 = 0$ and $y = 2x - 3$; find the points.

34. A circle centred at (2, 3) has radius 13 units. The line $y = 8$ cuts a chord. Find its length.

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